Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.6-Inverse-cosecant/158-5.6.1-u-a+b-arccsc-c-x-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [178]. This is test number [158].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (178)	0.00 (0)
Mathematica	100.00 (178)	0.00 (0)
Maple	82.58 (147)	17.42 (31)
Fricas	69.10 (123)	30.90 (55)
Giac	52.81 (94)	47.19 (84)
Sympy	37.64 (67)	62.36 (111)
Maxima	35.39 (63)	64.61 (115)
Mupad	32.02 (57)	67.98 (121)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

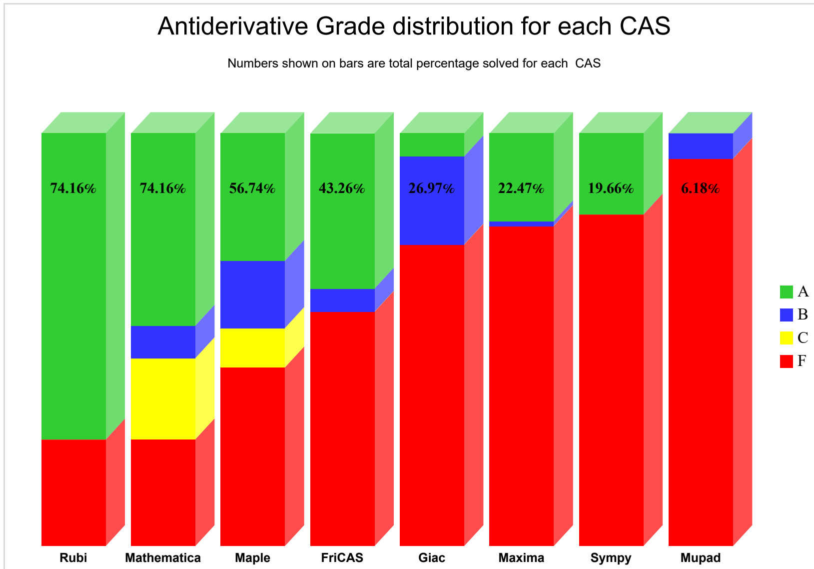
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

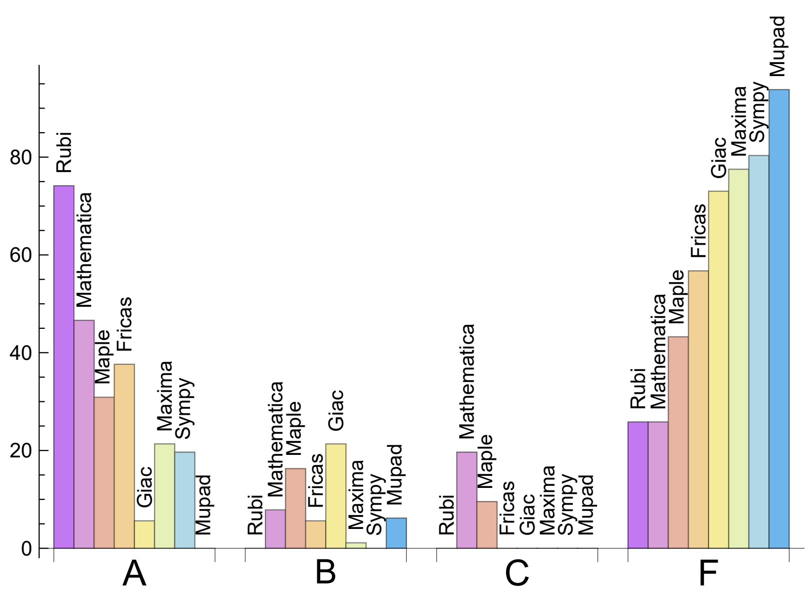
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.157	0.000	0.000	25.843
Mathematica	46.629	7.865	19.663	25.843
Fricas	37.640	5.618	0.000	56.742
Maple	30.899	16.292	9.551	43.258
Maxima	21.348	1.124	0.000	77.528
Sympy	19.663	0.000	0.000	80.337
Giac	5.618	21.348	0.000	73.034
Mupad	0.000	6.180	0.000	93.820

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	31	100.00	0.00	0.00
Fricas	55	92.73	7.27	0.00
Giac	84	65.48	4.76	29.76
Maxima	115	40.87	0.00	59.13
Sympy	111	69.37	30.63	0.00
Mupad	121	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.34
Maxima	0.69
Rubi	0.72
Mupad	1.12
Giac	2.14
Maple	5.54
Mathematica	5.64
Sympy	22.69

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	35.07	1.21	27.00	1.17
Sympy	129.01	1.36	36.00	1.16
Maxima	156.00	4.29	98.00	1.41
Rubi	209.55	1.00	133.50	1.00
Fricas	249.81	1.87	85.00	1.24
Mathematica	306.05	1.22	128.50	1.09
Giac	321.71	2.82	48.00	1.25
Maple	342.34	1.54	170.00	1.19

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

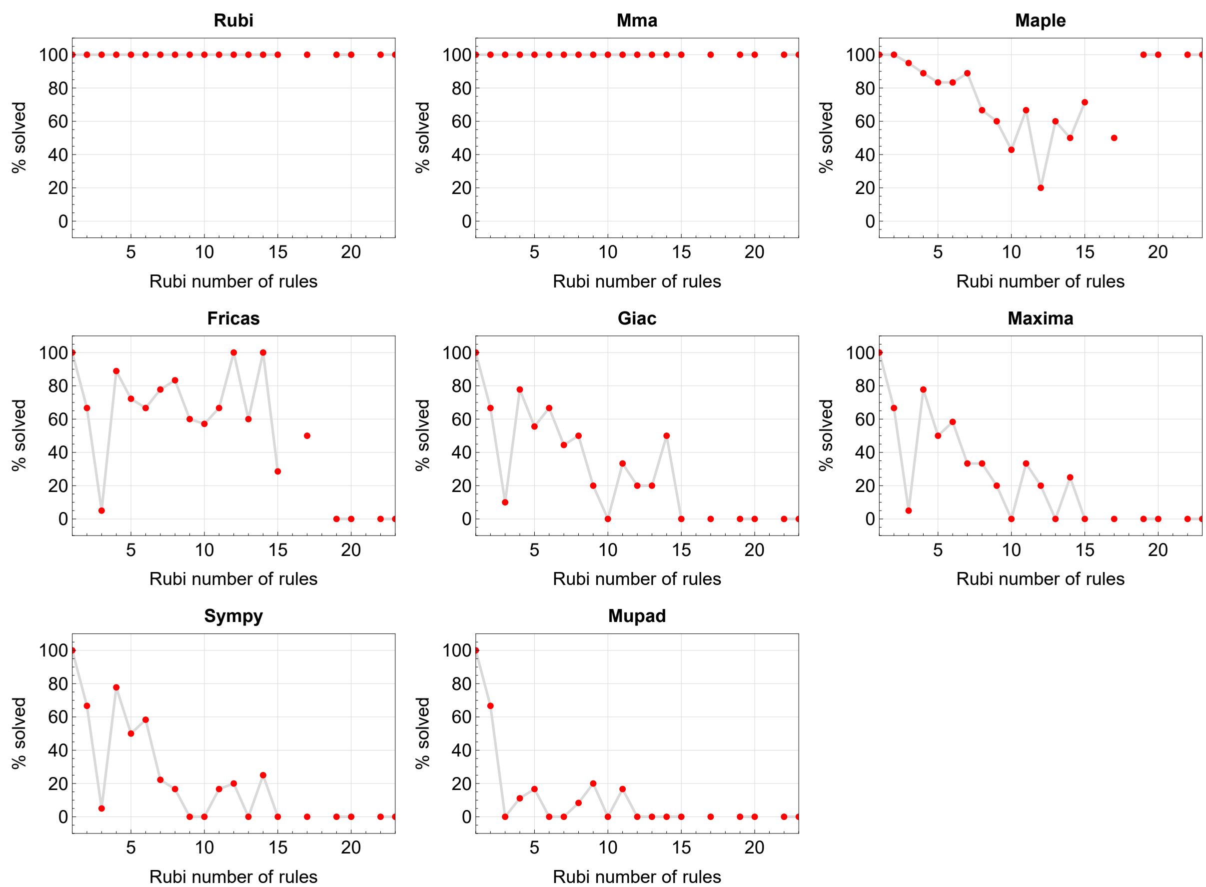


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

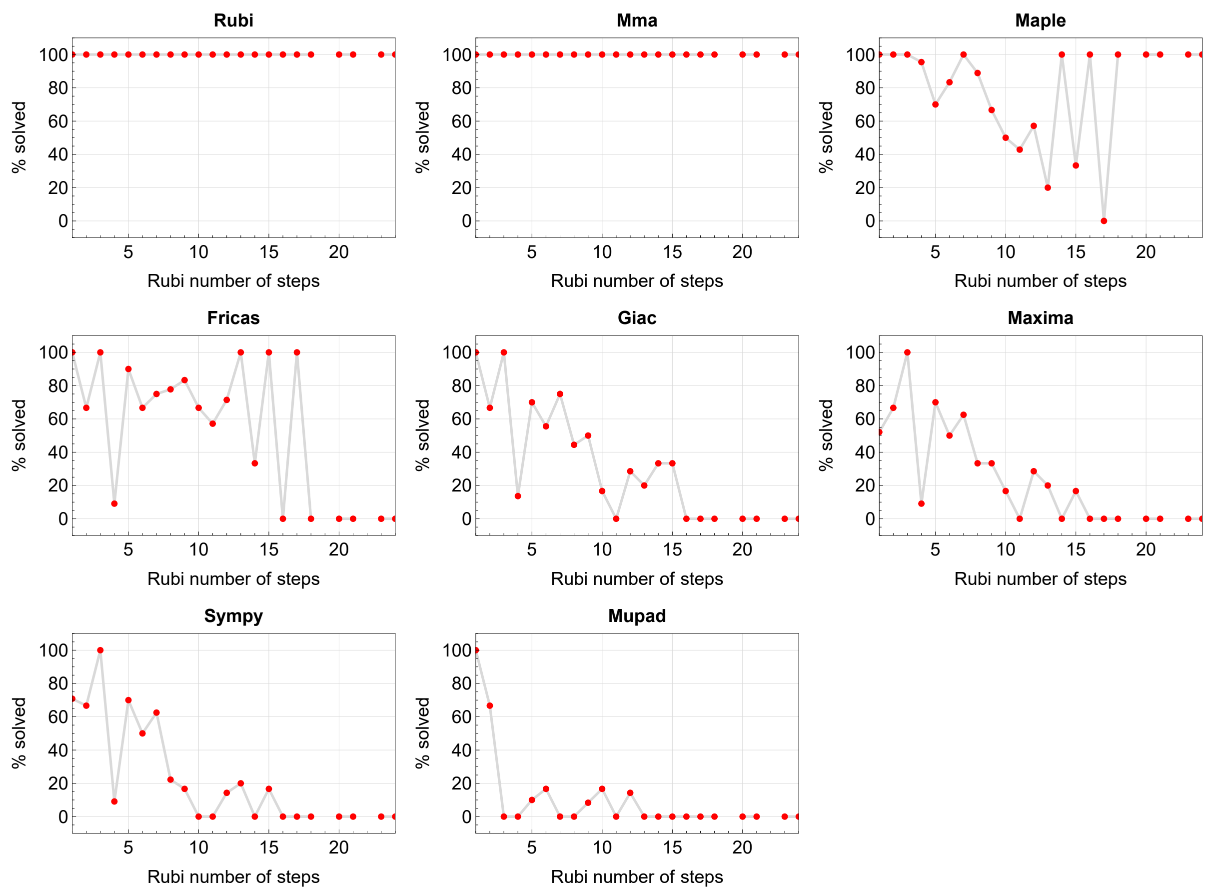


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

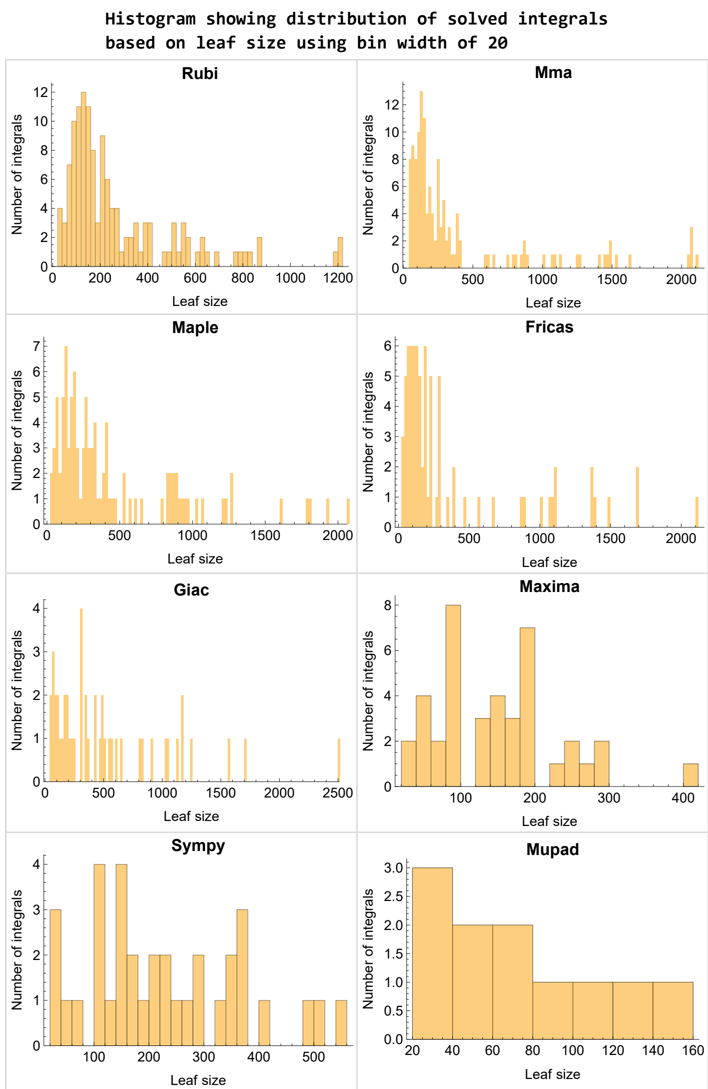


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

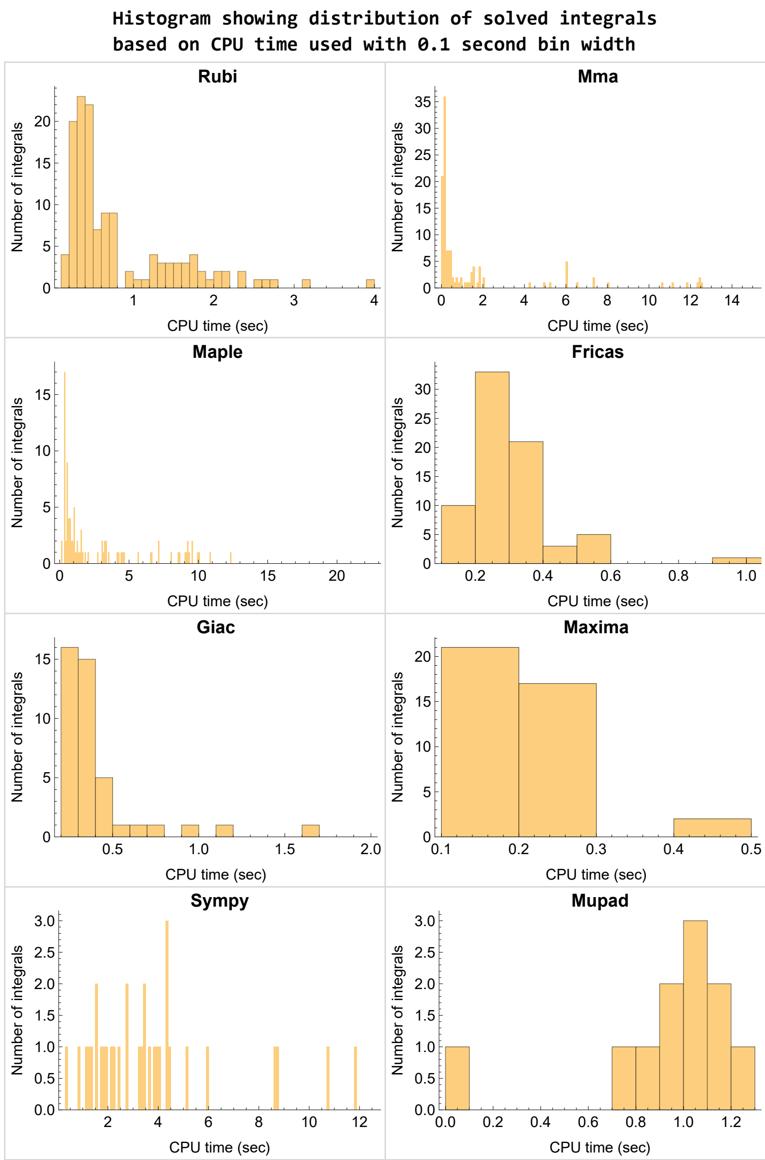


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

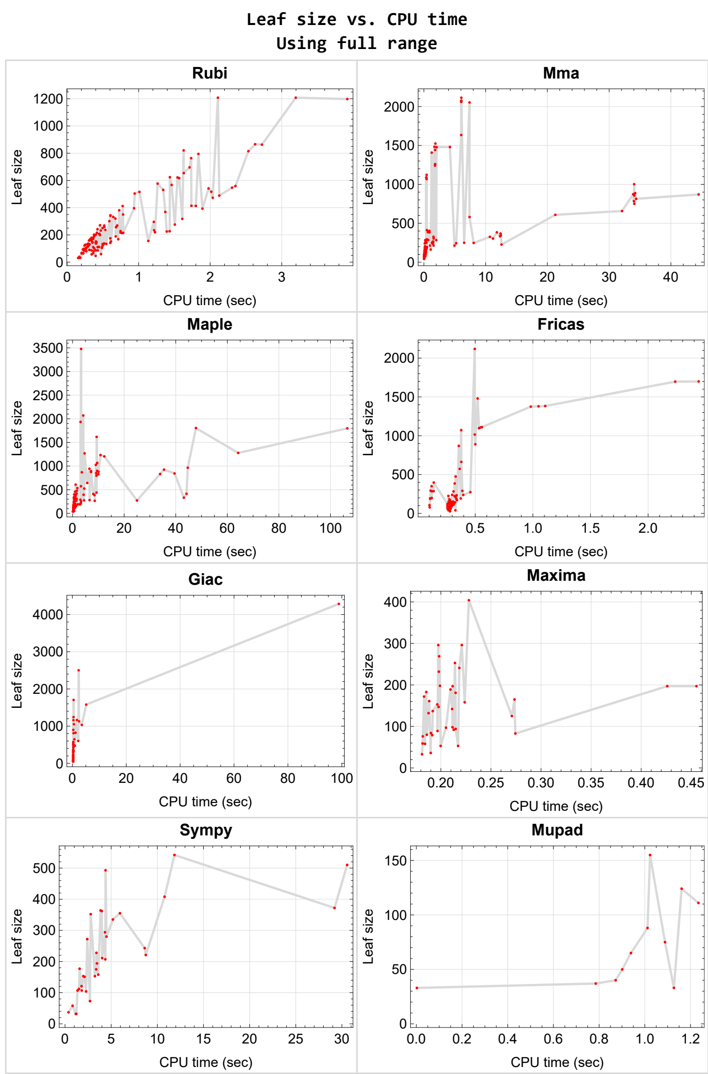


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {175}

Mathematica {25, 51, 53, 57, 60, 72, 75, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117}

Maple {98, 99, 101, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

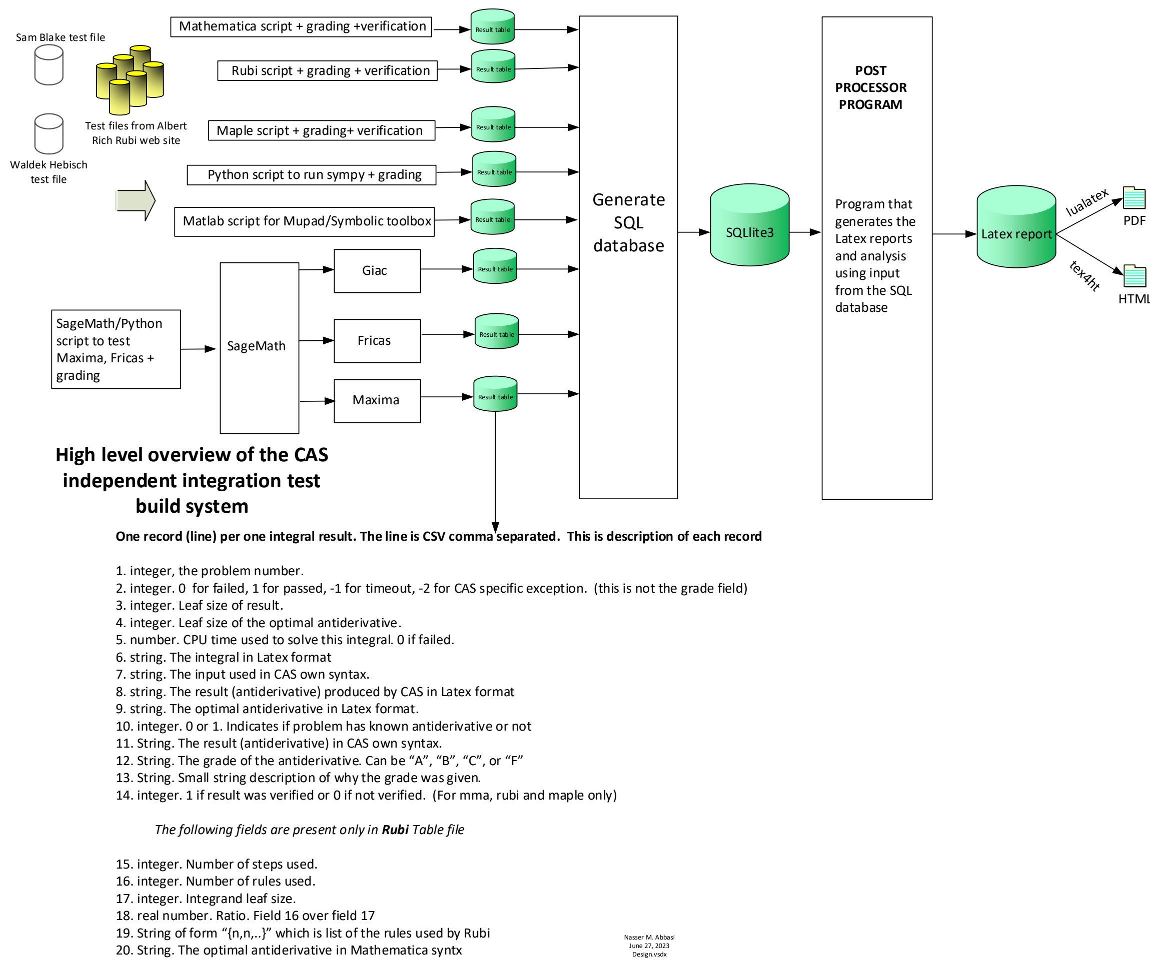
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 59, 60, 66, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 108, 109, 110, 115, 116, 117, 145, 154, 163, 165, 166, 167, 174, 175, 176 }

B grade { 25, 53, 72, 98, 99, 100, 101, 102, 103, 104, 106, 107, 111, 114 }

C grade { 51, 52, 56, 57, 58, 63, 64, 65, 69, 70, 71, 105, 112, 113, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 164 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 18, 22, 24, 25, 27, 36, 37, 38, 46, 47, 49, 53, 57, 58, 59, 60, 63, 64, 65, 66, 69, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 96, 97 }

B grade { 10, 12, 17, 19, 20, 21, 23, 26, 28, 29, 30, 31, 32, 44, 45, 48, 50, 51, 52, 56, 70, 71, 72, 75, 88, 95, 105, 112, 113 }

C grade { 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117 }

F normal fail { 41, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 29, 30, 31, 32, 44, 45, 46, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 105, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 163, 164, 174, 175, 176 }

B grade { 7, 17, 47, 49, 50, 112, 113, 156, 157, 158 }

C grade { }

F normal fail { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 52, 53, 57, 59, 60, 63, 65, 66, 69, 70, 71, 72, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 165, 166, 167 }

F(-1) timedout fail { 56, 58, 64, 75 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 17, 20, 29, 44, 45, 46, 47, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95 }

B grade { 15, 22 }

C grade { }

F normal fail { 8, 16, 18, 19, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 86, 87, 96, 97, 99, 101, 103, 104, 105, 106, 111, 112, 113, 114, 120, 129, 140, 149, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 98, 100, 102, 107, 108, 109, 110, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162 }

2.1.6 Giac

A grade { 9, 10, 11, 12, 36, 37, 38, 80, 81, 92 }

B grade { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 17, 20, 21, 22, 23, 29, 30, 31, 32, 44, 45, 46, 47, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91, 93, 94, 95 }

C grade { }

F normal fail { 16, 18, 24, 25, 26, 27, 28, 41, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 87, 97, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 176 }

F(-1) timedout fail { 103, 104, 107, 111 }

F(-2) exception fail { 8, 19, 48, 49, 50, 86, 96, 98, 99, 100, 101, 102, 105, 106, 108, 109, 110, 112, 113, 114, 115, 116, 117, 174, 175 }

2.1.7 Mupad

A grade { }

B grade { 6, 7, 8, 9, 10, 20, 29, 47, 79, 86, 87 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 44, 45, 46, 47, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95 }

B grade { }

C grade { }

F normal fail { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 71, 72, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 105, 107, 108, 109, 110, 118, 119, 120, 126, 127, 129, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 166, 167, 176 }

F(-1) timedout fail { 54, 61, 67, 69, 70, 73, 74, 75, 104, 106, 111, 112, 113, 114, 115, 116, 117, 128, 132, 133, 136, 137, 151, 159, 160, 161, 162, 163, 164, 165, 169, 170, 174, 175 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	122	107	177	161	115	221	646	0
N.S.	1	1.07	0.94	1.55	1.41	1.01	1.94	5.67	0.00
time (sec)	N/A	0.265	0.102	0.368	0.188	0.313	8.747	0.747	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	97	72	79	80	62	153	518	0
N.S.	1	1.09	0.81	0.89	0.90	0.70	1.72	5.82	0.00
time (sec)	N/A	0.251	0.092	0.362	0.186	0.287	1.981	0.315	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	97	141	132	106	175	480	0
N.S.	1	1.04	1.09	1.58	1.48	1.19	1.97	5.39	0.00
time (sec)	N/A	0.243	0.056	0.342	0.188	0.288	3.376	0.626	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	68	62	70	59	52	107	352	0
N.S.	1	1.06	0.97	1.09	0.92	0.81	1.67	5.50	0.00
time (sec)	N/A	0.222	0.078	0.345	0.182	0.273	1.352	0.304	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	85	94	97	94	107	310	0
N.S.	1	1.00	1.33	1.47	1.52	1.47	1.67	4.84	0.00
time (sec)	N/A	0.216	0.045	0.329	0.205	0.291	1.809	0.452	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	61	36	39	58	182	40
N.S.	1	1.00	1.28	1.56	0.92	1.00	1.49	4.67	1.03
time (sec)	N/A	0.193	0.028	0.341	0.190	0.328	0.820	0.288	0.872

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	37	53	64	32	62	33
N.S.	1	1.00	1.87	1.19	1.71	2.06	1.03	2.00	1.06
time (sec)	N/A	0.171	0.038	0.131	0.200	0.278	1.224	0.270	1.127

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	87	53	136	0	0	0	0	65
N.S.	1	1.36	0.83	2.12	0.00	0.00	0.00	0.00	1.02
time (sec)	N/A	0.432	0.043	0.890	0.000	0.000	0.000	0.000	0.939

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	41	59	33	26	37	42	37
N.S.	1	1.00	1.28	1.84	1.03	0.81	1.16	1.31	1.16
time (sec)	N/A	0.203	0.029	0.352	0.181	0.281	0.375	0.297	0.785

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	64	66	96	83	40	121	66	50
N.S.	1	1.25	1.29	1.88	1.63	0.78	2.37	1.29	0.98
time (sec)	N/A	0.231	0.035	0.372	0.274	0.278	1.787	0.286	0.901

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	65	59	71	58	39	112	87	0
N.S.	1	1.08	0.98	1.18	0.97	0.65	1.87	1.45	0.00
time (sec)	N/A	0.242	0.056	0.352	0.184	0.268	1.510	0.276	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	96	78	138	125	53	194	117	0
N.S.	1	1.26	1.03	1.82	1.64	0.70	2.55	1.54	0.00
time (sec)	N/A	0.251	0.050	0.359	0.271	0.270	3.458	0.277	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	86	69	79	76	50	158	149	0
N.S.	1	1.05	0.84	0.96	0.93	0.61	1.93	1.82	0.00
time (sec)	N/A	0.255	0.066	0.329	0.182	0.279	3.601	0.303	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	128	88	174	165	63	243	174	0
N.S.	1	1.27	0.87	1.72	1.63	0.62	2.41	1.72	0.00
time (sec)	N/A	0.274	0.079	0.361	0.274	0.284	8.632	0.283	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	116	124	170	197	146	0	811	0
N.S.	1	1.08	1.16	1.59	1.84	1.36	0.00	7.58	0.00
time (sec)	N/A	0.518	0.137	1.028	0.426	0.296	0.000	0.429	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	133	213	268	0	0	0	0	0
N.S.	1	0.96	1.53	1.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	0.997	1.439	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	65	89	122	84	111	0	427	0
N.S.	1	1.18	1.62	2.22	1.53	2.02	0.00	7.76	0.00
time (sec)	N/A	0.394	0.166	1.007	0.190	0.287	0.000	0.350	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	81	147	176	0	0	0	0	0
N.S.	1	0.96	1.75	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	0.197	0.752	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	105	137	338	0	0	0	0	0
N.S.	1	1.15	1.51	3.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.527	0.107	1.091	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	57	71	115	79	57	0	104	88
N.S.	1	1.14	1.42	2.30	1.58	1.14	0.00	2.08	1.76
time (sec)	N/A	0.368	0.102	0.761	0.191	0.260	0.000	0.296	1.012

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	90	102	184	0	82	0	163	0
N.S.	1	1.02	1.16	2.09	0.00	0.93	0.00	1.85	0.00
time (sec)	N/A	0.338	0.078	0.589	0.000	0.259	0.000	0.301	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	112	108	153	197	93	0	224	0
N.S.	1	1.10	1.06	1.50	1.93	0.91	0.00	2.20	0.00
time (sec)	N/A	0.479	0.156	1.345	0.455	0.265	0.000	0.287	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	140	148	265	0	120	0	304	0
N.S.	1	1.04	1.10	1.98	0.00	0.90	0.00	2.27	0.00
time (sec)	N/A	0.434	0.119	1.270	0.000	0.259	0.000	0.301	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	215	285	417	0	0	0	0	0
N.S.	1	1.04	1.38	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.843	0.704	1.561	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	220	214	580	535	0	0	0	0	0
N.S.	1	0.97	2.64	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.819	7.394	1.897	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	134	182	303	0	0	0	0	0
N.S.	1	1.06	1.44	2.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.601	0.469	1.589	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	137	265	378	0	0	0	0	0
N.S.	1	0.95	1.84	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.563	0.281	1.155	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	141	242	608	0	0	0	0	0
N.S.	1	1.14	1.95	4.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.670	0.154	1.204	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	135	197	147	98	0	195	155
N.S.	1	1.08	1.69	2.46	1.84	1.22	0.00	2.44	1.94
time (sec)	N/A	0.471	0.135	0.890	0.198	0.259	0.000	0.318	1.023

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	142	186	318	0	150	0	302	0
N.S.	1	1.14	1.49	2.54	0.00	1.20	0.00	2.42	0.00
time (sec)	N/A	0.418	0.156	1.072	0.000	0.270	0.000	0.316	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	189	204	299	0	173	0	428	0
N.S.	1	1.11	1.20	1.76	0.00	1.02	0.00	2.52	0.00
time (sec)	N/A	0.718	0.196	1.513	0.000	0.280	0.000	0.314	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	269	283	479	0	225	0	576	0
N.S.	1	1.29	1.36	2.30	0.00	1.08	0.00	2.77	0.00
time (sec)	N/A	0.753	0.229	1.655	0.000	0.267	0.000	0.315	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.50
time (sec)	N/A	0.178	2.624	0.842	0.268	0.255	0.358	33.774	0.787

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.60
time (sec)	N/A	0.179	2.557	0.590	0.294	0.245	0.400	11.266	0.790

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.43
time (sec)	N/A	0.183	0.236	0.505	0.290	0.275	0.873	1.776	0.841

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	46	43	48	0	0	0	54	0
N.S.	1	0.98	0.91	1.02	0.00	0.00	0.00	1.15	0.00
time (sec)	N/A	0.440	0.061	0.593	0.000	0.000	0.000	0.282	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	59	56	58	0	0	0	95	0
N.S.	1	0.94	0.89	0.92	0.00	0.00	0.00	1.51	0.00
time (sec)	N/A	0.519	0.059	0.503	0.000	0.000	0.000	0.289	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	110	91	102	0	0	0	200	0
N.S.	1	0.94	0.78	0.87	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	0.499	0.139	0.508	0.000	0.000	0.000	0.287	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1279	44	15	18	22
N.S.	1	1.00	1.12	1.00	79.94	2.75	0.94	1.12	1.38
time (sec)	N/A	0.198	4.271	0.862	15.475	0.286	21.443	0.883	1.086

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	551	30	15	18	22
N.S.	1	1.00	1.12	1.00	34.44	1.88	0.94	1.12	1.38
time (sec)	N/A	0.201	2.799	0.828	6.868	0.273	9.777	0.614	1.119

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	83	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.38
time (sec)	N/A	0.206	0.662	3.506	0.313	0.261	1.137	0.963	0.788

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	684	32	15	18	22
N.S.	1	1.00	1.12	1.00	42.75	2.00	0.94	1.12	1.38
time (sec)	N/A	0.203	1.392	1.998	1.740	0.273	5.312	1.732	0.819

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	168	165	401	269	290	362	1130	0
N.S.	1	1.01	0.99	2.40	1.61	1.74	2.17	6.77	0.00
time (sec)	N/A	0.796	0.170	0.630	0.198	0.388	3.977	2.480	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	130	122	304	198	209	228	602	0
N.S.	1	1.06	0.99	2.47	1.61	1.70	1.85	4.89	0.00
time (sec)	N/A	0.547	0.110	0.570	0.199	0.329	3.402	2.191	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	87	113	110	92	129	104	341	0
N.S.	1	1.05	1.36	1.33	1.11	1.55	1.25	4.11	0.00
time (sec)	N/A	0.366	0.138	0.483	0.213	0.289	2.280	0.444	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	58	37	53	64	32	62	33
N.S.	1	1.00	1.87	1.19	1.71	2.06	1.03	2.00	1.06
time (sec)	N/A	0.182	0.030	0.163	0.217	0.276	1.168	0.283	0.002

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	411	868	0	0	0	0	0
N.S.	1	1.00	1.60	3.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.741	0.494	3.599	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	110	141	192	0	475	0	0	0
N.S.	1	1.08	1.38	1.88	0.00	4.66	0.00	0.00	0.00
time (sec)	N/A	0.377	0.150	3.020	0.000	0.330	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	199	250	573	0	1111	0	0	0
N.S.	1	1.16	1.45	3.33	0.00	6.46	0.00	0.00	0.00
time (sec)	N/A	0.489	0.312	3.244	0.000	0.558	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	496	547	870	1204	0	0	0	0	0
N.S.	1	1.10	1.75	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.515	44.493	12.327	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	473	368	826	0	0	0	0	0
N.S.	1	1.17	0.91	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.183	12.419	9.987	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	315	350	657	386	0	0	0	0	0
N.S.	1	1.11	2.09	1.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.872	32.090	8.517	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	73	21	0	21	25
N.S.	1	1.00	1.10	0.90	3.48	1.00	0.00	1.00	1.19
time (sec)	N/A	0.247	38.504	0.716	0.689	0.255	0.000	1.133	0.909

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	88	21	20	21	25
N.S.	1	1.00	1.10	0.90	4.19	1.00	0.95	1.00	1.19
time (sec)	N/A	0.251	5.295	0.582	0.676	0.271	22.263	1.148	0.871

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	396	333	798	0	0	0	0	0
N.S.	1	1.06	0.90	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.022	12.335	9.266	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	714	489	873	1233	0	0	0	0	0
N.S.	1	0.68	1.22	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.348	33.902	10.845	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	530	414	784	850	0	0	0	0	0
N.S.	1	0.78	1.48	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.865	34.012	9.159	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	276	383	410	0	0	0	0	0
N.S.	1	0.80	1.11	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.643	0.911	8.052	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	212	237	243	252	0	0	0	0	0
N.S.	1	1.12	1.15	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.575	5.235	3.325	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	67	29	0	21	25
N.S.	1	1.00	1.10	0.90	3.19	1.38	0.00	1.00	1.19
time (sec)	N/A	0.255	3.686	0.681	0.904	0.265	0.000	0.679	0.865

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	89	31	20	21	25
N.S.	1	1.00	1.10	0.90	4.24	1.48	0.95	1.00	1.19
time (sec)	N/A	0.253	6.466	0.630	0.638	0.269	19.500	0.701	0.865

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	393	814	880	0	0	0	0	0
N.S.	1	0.71	1.48	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.041	34.378	10.080	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	318	750	439	0	0	0	0	0
N.S.	1	0.86	2.03	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.754	34.083	9.296	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	225	226	280	0	0	0	0	0
N.S.	1	0.95	0.95	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.532	12.570	6.606	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	137	124	215	0	0	0	0	0
N.S.	1	1.15	1.04	1.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.439	0.198	3.235	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	97	40	0	21	25
N.S.	1	1.00	1.10	0.90	4.62	1.90	0.00	1.00	1.19
time (sec)	N/A	0.258	11.987	0.783	0.697	0.290	0.000	0.684	0.890

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	145	42	20	21	25
N.S.	1	1.00	1.10	0.90	6.90	2.00	0.95	1.00	1.19
time (sec)	N/A	0.269	15.768	0.834	0.703	0.274	84.989	0.696	0.913

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	602	559	887	1067	0	0	0	0	0
N.S.	1	0.93	1.47	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.543	34.213	9.564	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	517	856	1026	0	0	0	0	0
N.S.	1	1.18	1.95	2.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.204	34.074	9.030	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	413	345	900	0	0	0	0	0
N.S.	1	1.32	1.10	2.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.002	12.464	9.512	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	298	320	608	875	0	0	0	0	0
N.S.	1	1.07	2.04	2.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	21.287	7.162	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	177	51	0	21	25
N.S.	1	1.00	1.10	0.90	8.43	2.43	0.00	1.00	1.19
time (sec)	N/A	0.266	30.013	0.907	0.780	0.265	0.000	0.695	0.900

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	179	53	0	21	25
N.S.	1	1.00	1.10	0.90	8.52	2.52	0.00	1.00	1.19
time (sec)	N/A	0.284	27.563	0.869	0.760	0.260	0.000	0.696	0.912

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	540	516	1002	1618	0	0	0	0	0
N.S.	1	0.96	1.86	3.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.112	34.051	9.336	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	167	140	328	296	191	408	1166	0
N.S.	1	0.81	0.68	1.59	1.44	0.93	1.98	5.66	0.00
time (sec)	N/A	0.355	0.164	0.629	0.197	0.374	10.788	1.665	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	136	121	254	232	170	294	822	0
N.S.	1	0.84	0.75	1.58	1.44	1.06	1.83	5.11	0.00
time (sec)	N/A	0.323	0.130	0.630	0.198	0.341	4.328	1.162	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	101	149	132	153	141	153	473	0
N.S.	1	0.93	1.37	1.21	1.40	1.29	1.40	4.34	0.00
time (sec)	N/A	0.266	0.169	0.314	0.196	0.325	3.234	0.943	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	84	104	116	89	125	73	1052	75
N.S.	1	0.97	1.20	1.33	1.02	1.44	0.84	12.09	0.86
time (sec)	N/A	0.279	0.096	0.329	0.197	0.297	2.705	0.565	1.089

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	101	69	108	94	66	151	136	0
N.S.	1	0.96	0.66	1.03	0.90	0.63	1.44	1.30	0.00
time (sec)	N/A	0.286	0.088	0.322	0.215	0.283	2.118	0.303	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	131	94	127	137	88	280	245	0
N.S.	1	0.86	0.62	0.84	0.90	0.58	1.84	1.61	0.00
time (sec)	N/A	0.304	0.138	0.327	0.192	0.280	4.492	0.292	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	159	110	145	172	109	372	367	0
N.S.	1	0.81	0.56	0.74	0.87	0.55	1.89	1.86	0.00
time (sec)	N/A	0.339	0.133	0.352	0.183	0.285	29.200	0.292	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	161	115	139	183	127	364	1244	0
N.S.	1	0.82	0.59	0.71	0.93	0.65	1.86	6.35	0.00
time (sec)	N/A	0.374	0.173	0.637	0.185	0.309	3.857	0.426	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	131	97	121	142	106	272	900	0
N.S.	1	0.86	0.63	0.79	0.93	0.69	1.78	5.88	0.00
time (sec)	N/A	0.346	0.161	0.722	0.211	0.305	2.408	0.339	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	116	78	217	98	85	177	556	0
N.S.	1	0.84	0.57	1.57	0.71	0.62	1.28	4.03	0.00
time (sec)	N/A	0.317	0.101	0.738	0.212	0.294	1.579	0.328	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	152	108	190	0	0	0	0	111
N.S.	1	1.23	0.87	1.53	0.00	0.00	0.00	0.00	0.90
time (sec)	N/A	0.652	0.082	2.043	0.000	0.000	0.000	0.000	1.235

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	175	125	191	0	0	0	0	124
N.S.	1	1.28	0.91	1.39	0.00	0.00	0.00	0.00	0.91
time (sec)	N/A	0.683	0.088	2.759	0.000	0.000	0.000	0.000	1.161

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	215	184	459	404	273	542	1579	0
N.S.	1	0.85	0.73	1.82	1.60	1.08	2.15	6.27	0.00
time (sec)	N/A	0.452	0.231	0.974	0.228	0.458	11.857	5.134	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	179	151	339	296	237	355	1033	0
N.S.	1	0.94	0.79	1.77	1.55	1.24	1.86	5.41	0.00
time (sec)	N/A	0.375	0.144	0.511	0.221	0.396	5.950	3.505	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	146	134	250	197	232	207	2502	0
N.S.	1	0.90	0.82	1.53	1.21	1.42	1.27	15.35	0.00
time (sec)	N/A	0.374	0.124	0.521	0.212	0.338	4.362	2.318	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	147	125	230	158	222	211	4288	0
N.S.	1	0.94	0.80	1.46	1.01	1.41	1.34	27.31	0.00
time (sec)	N/A	0.387	0.175	0.469	0.224	0.321	4.024	98.895	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	171	127	175	181	126	335	314	0
N.S.	1	0.93	0.69	0.96	0.99	0.69	1.83	1.72	0.00
time (sec)	N/A	0.398	0.176	0.502	0.215	0.276	5.190	0.297	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	201	153	207	241	158	510	491	0
N.S.	1	0.83	0.63	0.86	1.00	0.66	2.12	2.04	0.00
time (sec)	N/A	0.437	0.165	0.534	0.218	0.283	30.569	0.303	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	207	159	198	253	186	493	1706	0
N.S.	1	0.86	0.66	0.82	1.05	0.77	2.04	7.05	0.00
time (sec)	N/A	0.485	0.202	1.046	0.214	0.336	4.391	0.450	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	160	124	352	189	152	352	1160	0
N.S.	1	0.82	0.64	1.81	0.97	0.78	1.81	5.95	0.00
time (sec)	N/A	0.379	0.172	0.972	0.210	0.308	2.782	0.392	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	217	157	288	0	0	0	0	0
N.S.	1	1.17	0.84	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.820	0.283	3.061	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	228	194	274	0	0	0	0	0
N.S.	1	1.21	1.03	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.817	0.504	4.461	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	565	617	1260	415	0	0	0	0	0
N.S.	1	1.09	2.23	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.646	1.823	44.193	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	507	567	1123	394	0	0	0	0	0
N.S.	1	1.12	2.21	0.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.523	0.451	4.277	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	577	1068	272	0	0	0	0	0
N.S.	1	1.09	2.02	0.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.358	0.450	24.994	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	479	531	1089	1934	0	0	0	0	0
N.S.	1	1.11	2.27	4.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.408	0.384	3.107	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	572	624	1241	332	0	0	0	0	0
N.S.	1	1.09	2.17	0.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.575	1.806	43.161	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	628	696	1480	643	0	0	0	0	0
N.S.	1	1.11	2.36	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.863	4.220	5.692	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	593	654	1442	524	0	0	0	0	0
N.S.	1	1.10	2.43	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.759	1.835	4.549	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	118	286	265	0	385	0	0	0
N.S.	1	0.88	2.13	1.98	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.320	0.553	8.608	0.000	0.322	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	566	622	1408	2071	0	0	0	0	0
N.S.	1	1.10	2.49	3.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.620	1.275	4.158	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	F	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	803	863	1634	965	0	0	0	0	0
N.S.	1	1.07	2.03	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.942	6.045	44.659	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	765	821	1482	844	0	0	0	0	0
N.S.	1	1.07	1.94	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.763	1.705	39.557	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	762	815	1477	832	0	0	0	0	0
N.S.	1	1.07	1.94	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.696	2.059	33.948	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	806	866	1525	925	0	0	0	0	0
N.S.	1	1.07	1.89	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.785	1.863	35.461	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	727	795	2053	1269	0	0	0	0	0
N.S.	1	1.09	2.82	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.930	7.390	4.667	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	144	390	945	0	1015	0	0	0
N.S.	1	0.92	2.48	6.02	0.00	6.46	0.00	0.00	0.00
time (sec)	N/A	0.365	0.761	6.541	0.000	0.496	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	189	385	894	0	890	0	0	0
N.S.	1	0.98	1.99	4.63	0.00	4.61	0.00	0.00	0.00
time (sec)	N/A	0.408	0.643	7.185	0.000	0.501	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	704	764	2114	3479	0	0	0	0	0
N.S.	1	1.09	3.00	4.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.827	6.064	3.307	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1144	1208	2067	1804	0	0	0	0	0
N.S.	1	1.06	1.81	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.322	6.070	47.847	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1144	1208	2075	1278	0	0	0	0	0
N.S.	1	1.06	1.81	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.467	6.064	64.233	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	1134	1198	2060	1798	0	0	0	0	0
N.S.	1	1.06	1.82	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.252	6.048	106.523	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	403	369	326	0	0	1699	0	0	0
N.S.	1	0.92	0.81	0.00	0.00	4.22	0.00	0.00	0.00
time (sec)	N/A	1.506	1.564	0.000	0.000	2.439	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	269	263	0	0	1379	0	0	0
N.S.	1	0.91	0.89	0.00	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.576	1.442	0.000	0.000	1.049	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	179	213	0	0	1098	0	0	0
N.S.	1	0.92	1.09	0.00	0.00	5.63	0.00	0.00	0.00
time (sec)	N/A	0.395	1.573	0.000	0.000	0.533	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	1.17
time (sec)	N/A	0.276	6.453	0.493	0.000	0.258	10.391	0.349	1.275

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.278	10.945	0.764	0.000	0.260	14.183	0.369	1.407

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.271	10.764	0.526	0.000	0.259	118.153	0.343	1.394

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.217	17.908	0.439	0.000	0.266	47.880	0.357	1.266

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.271	1.862	0.316	0.000	0.264	7.153	0.356	1.526

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	301	247	0	0	192	0	0	0
N.S.	1	0.92	0.75	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.662	8.064	0.000	0.000	0.108	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	453	412	325	0	0	292	0	0	0
N.S.	1	0.91	0.72	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.860	10.695	0.000	0.000	0.113	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	341	304	0	0	1697	0	0	0
N.S.	1	0.91	0.81	0.00	0.00	4.54	0.00	0.00	0.00
time (sec)	N/A	0.696	1.586	0.000	0.000	2.235	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	242	248	0	0	1375	0	0	0
N.S.	1	0.92	0.95	0.00	0.00	5.25	0.00	0.00	0.00
time (sec)	N/A	0.442	1.322	0.000	0.000	0.982	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	1.17
time (sec)	N/A	0.283	7.682	2.917	0.000	0.259	72.838	0.474	1.243

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.298	11.890	2.283	0.000	0.274	63.269	0.363	1.493

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.00	1.00	1.17
time (sec)	N/A	0.288	11.079	1.111	0.000	0.261	0.000	0.342	1.514

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	0	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.00	1.00	1.20
time (sec)	N/A	0.226	18.491	1.258	0.000	0.286	0.000	0.387	1.293

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.276	36.147	2.253	0.000	0.267	98.964	0.419	1.686

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.287	5.649	0.400	0.000	0.255	65.772	0.370	1.461

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	416	381	303	0	0	286	0	0	0
N.S.	1	0.92	0.73	0.00	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.826	11.197	0.000	0.000	0.138	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	554	504	383	0	0	397	0	0	0
N.S.	1	0.91	0.69	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	1.042	11.822	0.000	0.000	0.140	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	296	282	0	0	1383	0	0	0
N.S.	1	0.92	0.88	0.00	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	1.336	2.010	0.000	0.000	1.107	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	207	239	0	0	1107	0	0	0
N.S.	1	0.92	1.06	0.00	0.00	4.92	0.00	0.00	0.00
time (sec)	N/A	0.497	1.421	0.000	0.000	0.546	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	128	107	0	0	870	0	0	0
N.S.	1	0.97	0.81	0.00	0.00	6.59	0.00	0.00	0.00
time (sec)	N/A	0.338	0.326	0.000	0.000	0.358	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	1.17
time (sec)	N/A	0.275	1.222	0.518	0.000	0.259	7.655	0.354	1.318

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	1.17
time (sec)	N/A	0.289	5.254	0.839	0.000	0.268	27.192	0.355	1.392

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.274	36.978	0.555	0.000	0.269	53.630	0.366	1.422

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.214	0.724	0.545	0.000	0.257	15.699	0.351	1.170

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	232	140	0	0	105	0	0	0
N.S.	1	0.94	0.57	0.00	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.523	0.183	0.000	0.000	0.103	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	362	332	249	0	0	198	0	0	0
N.S.	1	0.92	0.69	0.00	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.718	6.527	0.000	0.000	0.123	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	252	234	263	0	0	1480	0	0	0
N.S.	1	0.93	1.04	0.00	0.00	5.87	0.00	0.00	0.00
time (sec)	N/A	1.336	1.528	0.000	0.000	0.521	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	152	162	0	0	1072	0	0	0
N.S.	1	0.97	1.04	0.00	0.00	6.87	0.00	0.00	0.00
time (sec)	N/A	0.446	1.110	0.000	0.000	0.378	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	78	0	0	283	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.58	0.00	0.00	0.00
time (sec)	N/A	0.298	0.249	0.000	0.000	0.312	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	1.17
time (sec)	N/A	0.289	8.843	2.148	0.000	0.272	75.350	0.359	1.316

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.00	1.00	1.17
time (sec)	N/A	0.305	11.368	5.027	0.000	0.273	0.000	0.365	1.486

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.287	14.727	1.467	0.000	0.259	122.610	0.381	1.380

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.277	5.864	1.264	0.000	0.266	32.452	0.422	1.149

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	112	0	0	76	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.298	0.198	0.000	0.000	0.105	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	259	213	0	0	188	0	0	0
N.S.	1	0.94	0.77	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.568	4.957	0.000	0.000	0.110	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	219	240	0	0	2119	0	0	0
N.S.	1	0.90	0.99	0.00	0.00	8.72	0.00	0.00	0.00
time (sec)	N/A	1.338	1.466	0.000	0.000	0.496	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	148	162	0	0	663	0	0	0
N.S.	1	0.91	0.99	0.00	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.448	0.412	0.000	0.000	0.379	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	126	130	0	0	573	0	0	0
N.S.	1	0.91	0.94	0.00	0.00	4.15	0.00	0.00	0.00
time (sec)	N/A	0.330	0.350	0.000	0.000	0.365	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	1.17
time (sec)	N/A	0.297	16.092	3.111	0.000	0.273	0.000	0.366	1.267

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	1.17
time (sec)	N/A	0.317	18.717	3.412	0.000	0.277	0.000	0.372	1.460

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.292	15.999	1.654	0.000	0.272	0.000	0.401	1.427

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.283	15.004	1.543	0.000	0.279	0.000	0.446	1.303

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	276	251	185	0	0	284	0	0	0
N.S.	1	0.91	0.67	0.00	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.525	0.314	0.000	0.000	0.124	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	271	249	0	0	350	0	0	0
N.S.	1	0.92	0.84	0.00	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.503	0.421	0.000	0.000	0.121	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	585	541	402	0	0	0	0	0	0
N.S.	1	0.92	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.151	0.884	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	371	344	293	0	0	0	0	0	0
N.S.	1	0.93	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.647	0.480	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	202	171	0	0	0	0	0	0
N.S.	1	0.94	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.406	0.341	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.26
time (sec)	N/A	0.257	2.203	4.670	0.572	0.268	35.125	0.319	0.819

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	1.26
time (sec)	N/A	0.257	4.734	3.287	0.573	0.266	0.000	0.327	0.803

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	1.16
time (sec)	N/A	0.280	1.063	2.732	0.551	0.286	0.000	0.372	0.897

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.275	0.145	1.367	0.395	0.282	51.607	0.344	0.810

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.270	1.088	1.158	0.408	0.272	21.901	0.357	0.897

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.96	1.00	1.16
time (sec)	N/A	0.275	1.222	2.210	0.424	0.279	134.541	0.368	0.982

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	401	227	194	0	0	238	0	0	0
N.S.	1	0.57	0.48	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	1.570	0.271	0.000	0.000	0.300	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	156	159	0	0	181	0	0	0
N.S.	1	0.58	0.59	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	1.188	0.339	0.000	0.000	0.276	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	101	138	0	0	124	0	0	0
N.S.	1	0.80	1.10	0.00	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.414	0.192	0.000	0.000	0.264	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	88	37	34	26	30
N.S.	1	1.00	1.08	0.92	3.38	1.42	1.31	1.00	1.15
time (sec)	N/A	0.269	0.441	0.372	0.518	0.254	11.160	0.359	1.817

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	112	39	36	26	30
N.S.	1	1.00	1.08	0.92	4.31	1.50	1.38	1.00	1.15
time (sec)	N/A	0.288	8.123	3.034	0.496	0.257	83.530	0.362	1.547

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [52] had the largest ratio of [1.15789000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.07	12	0.583
2	A	4	4	1.09	12	0.333
3	A	7	6	1.04	12	0.500
4	A	3	3	1.06	12	0.250
5	A	6	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	1	1	1.00	8	0.125
8	A	10	9	1.36	12	0.750
9	A	2	2	1.00	12	0.167
10	A	5	4	1.25	12	0.333
11	A	5	4	1.08	12	0.333
12	A	6	5	1.26	12	0.417
13	A	5	4	1.05	12	0.333
14	A	7	6	1.27	12	0.500
15	A	10	9	1.08	14	0.643
16	A	9	8	0.96	14	0.571
17	A	8	7	1.18	12	0.583
18	A	7	6	0.96	10	0.600
19	A	10	9	1.15	14	0.643
20	A	9	8	1.14	14	0.571
21	A	6	5	1.02	14	0.357
22	A	9	8	1.10	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	8	7	1.04	14	0.500
24	A	16	15	1.04	14	1.071
25	A	11	10	0.97	14	0.714
26	A	12	11	1.06	12	0.917
27	A	8	7	0.95	10	0.700
28	A	11	10	1.14	14	0.714
29	A	12	11	1.08	14	0.786
30	A	9	8	1.14	14	0.571
31	A	14	13	1.11	14	0.929
32	A	15	14	1.29	14	1.000
33	N/A	1	0	1.00	12	0.000
34	N/A	1	0	1.00	10	0.000
35	N/A	1	0	1.00	14	0.000
36	A	7	6	0.98	14	0.429
37	A	9	8	0.94	14	0.571
38	A	4	3	0.94	14	0.214
39	N/A	1	0	1.00	16	0.000
40	N/A	1	0	1.00	16	0.000
41	A	4	3	1.00	14	0.214
42	N/A	1	0	1.00	16	0.000
43	N/A	1	0	1.00	16	0.000
44	A	15	14	1.01	16	0.875
45	A	13	12	1.06	16	0.750
46	A	12	11	1.05	14	0.786
47	A	1	1	1.00	8	0.125
48	A	2	2	1.00	16	0.125
49	A	8	7	1.08	16	0.438
50	A	9	8	1.16	16	0.500
51	A	24	23	1.10	21	1.095
52	A	23	22	1.17	19	1.158
53	A	16	15	1.11	18	0.833
54	N/A	1	0	1.00	21	0.000
55	N/A	1	0	1.00	21	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	18	17	1.06	18	0.944
57	A	16	15	0.68	21	0.714
58	A	14	13	0.78	21	0.619
59	A	11	10	0.80	19	0.526
60	A	4	4	1.12	18	0.222
61	N/A	1	0	1.00	21	0.000
62	N/A	1	0	1.00	21	0.000
63	A	16	15	0.71	21	0.714
64	A	14	13	0.86	21	0.619
65	A	12	11	0.95	19	0.579
66	A	8	7	1.15	18	0.389
67	N/A	1	0	1.00	21	0.000
68	N/A	1	0	1.00	21	0.000
69	A	23	22	0.93	21	1.048
70	A	23	22	1.18	21	1.048
71	A	21	20	1.32	19	1.053
72	A	16	15	1.07	18	0.833
73	N/A	1	0	1.00	21	0.000
74	N/A	1	0	1.00	21	0.000
75	A	20	19	0.96	18	1.056
76	A	8	7	0.81	19	0.368
77	A	7	6	0.84	19	0.316
78	A	6	5	0.93	16	0.312
79	A	6	5	0.97	19	0.263
80	A	4	4	0.96	19	0.211
81	A	5	5	0.86	19	0.263
82	A	6	6	0.81	19	0.316
83	A	6	5	0.82	19	0.263
84	A	6	5	0.86	19	0.263
85	A	5	4	0.84	17	0.235
86	A	6	5	1.23	19	0.263
87	A	6	5	1.28	19	0.263
88	A	9	8	0.85	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	7	6	0.94	18	0.333
90	A	9	8	0.90	21	0.381
91	A	7	6	0.94	21	0.286
92	A	5	5	0.93	21	0.238
93	A	6	6	0.83	21	0.286
94	A	6	5	0.86	21	0.238
95	A	5	4	0.82	19	0.211
96	A	6	5	1.17	21	0.238
97	A	6	5	1.21	21	0.238
98	A	4	3	1.09	21	0.143
99	A	4	3	1.12	19	0.158
100	A	4	3	1.09	18	0.167
101	A	4	3	1.11	21	0.143
102	A	4	3	1.09	21	0.143
103	A	4	3	1.11	21	0.143
104	A	4	3	1.10	21	0.143
105	A	6	5	0.88	19	0.263
106	A	4	3	1.10	21	0.143
107	A	4	3	1.07	21	0.143
108	A	4	3	1.07	21	0.143
109	A	4	3	1.07	18	0.167
110	A	4	3	1.07	21	0.143
111	A	4	3	1.09	21	0.143
112	A	7	6	0.92	21	0.286
113	A	8	7	0.98	19	0.368
114	A	4	3	1.09	21	0.143
115	A	4	3	1.06	21	0.143
116	A	4	3	1.06	21	0.143
117	A	4	3	1.06	18	0.167
118	A	15	14	0.92	23	0.609
119	A	13	12	0.91	23	0.522
120	A	10	9	0.92	21	0.429
121	N/A	1	0	1.00	23	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
122	N/A	1	0	1.00	23	0.000
123	N/A	1	0	1.00	23	0.000
124	N/A	1	0	1.00	20	0.000
125	N/A	1	0	1.00	23	0.000
126	A	13	13	0.92	23	0.565
127	A	15	15	0.91	23	0.652
128	A	15	14	0.91	23	0.609
129	A	12	11	0.92	21	0.524
130	N/A	1	0	1.00	23	0.000
131	N/A	1	0	1.00	23	0.000
132	N/A	1	0	1.00	23	0.000
133	N/A	1	0	1.00	20	0.000
134	N/A	1	0	1.00	23	0.000
135	N/A	1	0	1.00	23	0.000
136	A	15	15	0.92	23	0.652
137	A	17	17	0.91	23	0.739
138	A	13	12	0.92	23	0.522
139	A	11	10	0.92	23	0.435
140	A	9	8	0.97	21	0.381
141	N/A	1	0	1.00	23	0.000
142	N/A	1	0	1.00	23	0.000
143	N/A	1	0	1.00	23	0.000
144	N/A	1	0	1.00	20	0.000
145	A	12	12	0.94	23	0.522
146	A	13	13	0.92	23	0.565
147	A	11	10	0.93	23	0.435
148	A	9	8	0.97	23	0.348
149	A	5	4	1.00	21	0.190
150	N/A	1	0	1.00	23	0.000
151	N/A	1	0	1.00	23	0.000
152	N/A	1	0	1.00	23	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	N/A	1	0	1.00	23	0.000
154	A	5	5	1.00	20	0.250
155	A	12	12	0.94	23	0.522
156	A	11	10	0.90	23	0.435
157	A	9	8	0.91	23	0.348
158	A	6	5	0.91	21	0.238
159	N/A	1	0	1.00	23	0.000
160	N/A	1	0	1.00	23	0.000
161	N/A	1	0	1.00	23	0.000
162	N/A	1	0	1.00	23	0.000
163	A	10	10	0.91	23	0.435
164	A	11	11	0.92	20	0.550
165	A	6	6	0.92	23	0.261
166	A	6	6	0.93	23	0.261
167	A	5	5	0.94	21	0.238
168	N/A	1	0	1.00	23	0.000
169	N/A	1	0	1.00	23	0.000
170	N/A	1	0	1.00	25	0.000
171	N/A	1	0	1.00	25	0.000
172	N/A	1	0	1.00	25	0.000
173	N/A	1	0	1.00	25	0.000
174	A	8	7	0.57	26	0.269
175	A	10	9	0.58	26	0.346
176	A	9	8	0.80	26	0.308
177	N/A	1	0	1.00	26	0.000
178	N/A	1	0	1.00	26	0.000

CHAPTER 3

Listing of integrals

3.1 $\int x^6(a+b\csc^{-1}(cx))\,dx$

3.1.1 Optimal result

Integrand size = 12, antiderivative size = 114

$$\int x^6(a+b\csc^{-1}(cx))\,dx=\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^2}{112c^5}+\frac{5b\sqrt{1-\frac{1}{c^2x^2}}x^4}{168c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7(a+b\csc^{-1}(cx))+\frac{5b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{112c^7}$$

output
```
1/7*x^7*(a+b*arccsc(c*x))+5/112*b*arctanh((1-1/c^2/x^2)^(1/2))/c^7+5/112*b
*x^2*(1-1/c^2/x^2)^(1/2)/c^5+5/168*b*x^4*(1-1/c^2/x^2)^(1/2)/c^3+1/42*b*x^
6*(1-1/c^2/x^2)^(1/2)/c
```

3.1.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.94

$$\int x^6(a+b\csc^{-1}(cx))\,dx=\frac{ax^7}{7}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{5x^2}{112c^5}+\frac{5x^4}{168c^3}+\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\csc^{-1}(cx)+\frac{5b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

input
```
Integrate[x^6*(a + b*ArcCsc[c*x]),x]
```

output
```
(a*x^7)/7 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) + (5*x^4)/
(168*c^3) + x^6/(42*c)) + (b*x^7*ArcCsc[c*x])/7 + (5*b*Log[x*(1 + Sqrt[(-1
 + c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

3.1.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5744, 798, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+b \csc^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 5744$$

$$\frac{b \int \frac{x^5}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{7c}+\frac{1}{7}x^7 \left(a+b \csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 798$$

$$\frac{1}{7}x^7 \left(a+b \csc^{-1}(cx)\right)-\frac{b \int \frac{x^8}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{14c}$$

$$\Big\downarrow \; 52$$

$$\frac{1}{7}x^7 \left(a+b \csc^{-1}(cx)\right)-\frac{b \left(\frac{5 \int \frac{x^6}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{6c^2}-\frac{1}{3}x^6 \sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}$$

$$\Big\downarrow \; 52$$

$$\frac{1}{7}x^7 \left(a+b \csc^{-1}(cx)\right)-\frac{b \left(\frac{5 \left(\frac{3 \int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{4c^2}-\frac{1}{2}x^4 \sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6 \sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}$$

$$\Big\downarrow \; 52$$

$$\frac{1}{7}x^7(a+b\csc^{-1}(cx))-\frac{b\left(\frac{5\left(\frac{3\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}-x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{7}x^7(a+b\csc^{-1}(cx))-\frac{b\left(\frac{5\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int \frac{1}{c^2-\frac{c^2}{x^4}}d\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{7}x^7(a+b\csc^{-1}(cx))-\frac{b\left(\frac{5\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{14c}$$

input
```
Int[x^6*(a + b*ArcCsc[c*x]),x]
```

output
```
(x^7*(a + b*ArcCsc[c*x]))/7 - (b*(-1/3*(Sqrt[1 - 1/(c^2*x^2)]*x^6) + (5*(-
1/2*(Sqrt[1 - 1/(c^2*x^2)]*x^4) + (3*(-(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTa
nh[Sqrt[1 - 1/(c^2*x^2)]]/c^2))/(4*c^2)))/(6*c^2)))/(14*c)
```

3.1.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.55

method	result
parts	$\frac{a\,x^{7}}{7}+\frac{b\,x^{7}\operatorname{arccsc}\left(c x\right)}{7}+\frac{b\left(c^{2} x^{2}-1\right) x^{4}}{42 c^{3} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\left(c^{2} x^{2}-1\right) x^{2}}{168 c^{5} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\left(c^{2} x^{2}-1\right)}{112 c^{7} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\sqrt{c^{2} x^{2}-1}\,\ln\left(c x+\sqrt{c^{2} x^{2}-1}\right)}{112 c^{8} \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, x}$
derivativedivides	$\frac{\frac{a\,c^{7} x^{7}}{7}+\frac{b\,c^{7} x^{7}\operatorname{arccsc}\left(c x\right)}{7}+\frac{b\left(c^{2} x^{2}-1\right) c^{4} x^{4}}{42 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\left(c^{2} x^{2}-1\right) c^{2} x^{2}}{168 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\left(c^{2} x^{2}-1\right)}{112 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\sqrt{c^{2} x^{2}-1}\,\ln\left(c x+\sqrt{c^{2} x^{2}-1}\right)}{112 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}}{c^{7}}$
default	$\frac{\frac{a\,c^{7} x^{7}}{7}+\frac{b\,c^{7} x^{7}\operatorname{arccsc}\left(c x\right)}{7}+\frac{b\left(c^{2} x^{2}-1\right) c^{4} x^{4}}{42 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\left(c^{2} x^{2}-1\right) c^{2} x^{2}}{168 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\left(c^{2} x^{2}-1\right)}{112 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}}+\frac{5 b\sqrt{c^{2} x^{2}-1}\,\ln\left(c x+\sqrt{c^{2} x^{2}-1}\right)}{112 \sqrt{\frac{c^{2} x^{2}-1}{c^{2} x^{2}}}\, c x}}{c^{7}}$

input
```
int(x^6*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/7*a*x^7+1/7*b*x^7*arccsc(c*x)+1/42*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)*x^4+5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2+5/112
*b/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+5/112*b/c^8*(c^2*x^2-1)^(1/
2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{48\,ac^7x^7-96\,bc^7\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+48\,(bc^7x^7-bc^7)\,\mathrm{arccsc}\,(cx)-15\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{336\,c^7}$$

input
```
integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/336*(48*a*c^7*x^7 - 96*b*c^7*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 48*(b*c^
7*x^7 - b*c^7)*arccsc(c*x) - 15*b*log(-c*x + sqrt(c^2*x^2 - 1)) + (8*b*c^5
*x^5 + 10*b*c^3*x^3 + 15*b*c*x)*sqrt(c^2*x^2 - 1))/c^7
```

3.1.6 Sympy [A] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^7}{7}+\frac{bx^7\,\mathrm{acsc}\,(cx)}{7}$$
$$+\frac{b\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\,\mathrm{acosh}\,(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\,\mathrm{asin}\,(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**6*(a+b*acsc(c*x)),x)
```

output
```
a*x**7/7 + b*x**7*acsc(c*x)/7 + b*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)
) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1))
 - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x*
*2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x*
*2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-
c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.41

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{7}\,ax^7+\frac{1}{672}\left(96\,x^7\,\mathrm{arccsc}\,(cx)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\,\log\left(\sqrt{-\frac{1}{c^2}\cdots}\right.}{c^6}}{c}\right.$$

input
```
integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*x^7 + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^(5/2) -
40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2)
 - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*
log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c
^6)/c)*b
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(96) = 192.

Time = 0.75 (sec) , antiderivative size = 646, normalized size of antiderivative = 5.67

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{2688}\left(\frac{3\,bx^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{3\,ax^7\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^7}{c}+\frac{bx^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6}{c^2}+\frac{21\cdots}{\cdots}\right.$$

input
```
integrate(x^6*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/2688*(3*b*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 3*a*x^7
*(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c
^2 + 21*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 21*a*x^
5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^3 + 9*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1
)^4/c^4 + 63*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^5 + 63
*a*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^5 + 45*b*x^2*(sqrt(-1/(c^2*x^2) +
1) + 1)^2/c^6 + 105*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^7 +
 105*a*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7 + 120*b*log(sqrt(-1/(c^2*x^2) +
1) + 1)/c^8 - 120*b*log(1/(abs(c)*abs(x)))/c^8 + 105*b*arcsin(1/(c*x))/(c^
9*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 105*a/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) +
 1)) - 45*b/(c^10*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 63*b*arcsin(1/(c*x
))/(c^11*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 63*a/(c^11*x^3*(sqrt(-1/(c^
2*x^2) + 1) + 1)^3) - 9*b/(c^12*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 21*b
*arcsin(1/(c*x))/(c^13*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 21*a/(c^13*x^
5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) - b/(c^14*x^6*(sqrt(-1/(c^2*x^2) + 1) +
1)^6) + 3*b*arcsin(1/(c*x))/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7) + 3*
a/(c^15*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))*c
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^6\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^6\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^6*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^6*(a + b*asin(1/(c*x))), x)
```

3.2 $\int x^5(a+b\csc^{-1}(cx))\,dx$

3.2.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{4b\sqrt{1-\frac{1}{c^2x^2}}\,x}{45c^5}+\frac{2b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\csc^{-1}(cx)\right)$$

output
```
1/6*x^6*(a+b*arccsc(c*x))+4/45*b*x*(1-1/c^2/x^2)^(1/2)/c^5+2/45*b*x^3*(1-1
/c^2/x^2)^(1/2)/c^3+1/30*b*x^5*(1-1/c^2/x^2)^(1/2)/c
```

3.2.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^6}{6}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{4x}{45c^5}+\frac{2x^3}{45c^3}+\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\csc^{-1}(cx)$$

input
```
Integrate[x^5*(a + b*ArcCsc[c*x]),x]
```

output
```
(a*x^6)/6 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((4*x)/(45*c^5) + (2*x^3)/(45
*c^3) + x^5/(30*c)) + (b*x^6*ArcCsc[c*x])/6
```

3.2.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5744, 803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b \csc^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 5744$$

$$\frac{b \int \frac{x^4}{\sqrt{1-\frac{1}{c^2 x^2}}}dx}{6c}+\frac{1}{6}x^6 \left(a+b \csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 803$$

$$\frac{b \left(\frac{4 \int \frac{x^2}{\sqrt{1-\frac{1}{c^2 x^2}}}dx}{5c^2}+\frac{1}{5}x^5 \sqrt{1-\frac{1}{c^2 x^2}}\right)}{6c}+\frac{1}{6}x^6 \left(a+b \csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 803$$

$$\frac{b \left(\frac{4 \left(\frac{2 \int \frac{1}{\sqrt{1-\frac{1}{c^2 x^2}}}dx}{3c^2}+\frac{1}{3}x^3 \sqrt{1-\frac{1}{c^2 x^2}}\right)}{5c^2}+\frac{1}{5}x^5 \sqrt{1-\frac{1}{c^2 x^2}}\right)}{6c}+\frac{1}{6}x^6 \left(a+b \csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 746$$

$$\frac{1}{6}x^6 \left(a+b \csc^{-1}(cx)\right)+\frac{b \left(\frac{1}{5}x^5 \sqrt{1-\frac{1}{c^2 x^2}}+\frac{4 \left(\frac{2x \sqrt{1-\frac{1}{c^2 x^2}}}{3c^2}+\frac{1}{3}x^3 \sqrt{1-\frac{1}{c^2 x^2}}\right)}{5c^2}\right)}{6c}$$

input
```
Int[x^5*(a + b*ArcCsc[c*x]),x]
```

output
```
(b*((Sqrt[1 - 1/(c^2*x^2)]*x^5)/5 + (4*((2*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*c^2
) + (Sqrt[1 - 1/(c^2*x^2)]*x^3)/3))/(5*c^2)))/(6*c) + (x^6*(a + b*ArcCsc[c
*x]))/6
```

3.2.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{x^6 a}{6}+\frac{b\left(\frac{c^6 x^6 \operatorname{arccsc}(cx)}{6}+\frac{(c^2x^2-1)(3c^4x^4+4c^2x^2+8)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$	79
derivativedivides	$\frac{\frac{c^6x^6a}{6}+b\left(\frac{c^6 x^6 \operatorname{arccsc}(cx)}{6}+\frac{(c^2x^2-1)(3c^4x^4+4c^2x^2+8)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$	83
default	$\frac{\frac{c^6x^6a}{6}+b\left(\frac{c^6 x^6 \operatorname{arccsc}(cx)}{6}+\frac{(c^2x^2-1)(3c^4x^4+4c^2x^2+8)}{90\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^6}$	83

input
```
int(x^5*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*a+b/c^6*(1/6*c^6*x^6*arccsc(c*x)+1/90*(c^2*x^2-1)*(3*c^4*x^4+4*c^2
*x^2+8)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{15\,bc^6x^6\operatorname{arccsc}\left(cx\right)+15\,ac^6x^6+\left(3\,bc^4x^4+4\,bc^2x^2+8\,b\right)\sqrt{c^2x^2-1}}{90\,c^6}$$

input
```
integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*b*c^6*x^6*arccsc(c*x) + 15*a*c^6*x^6 + (3*b*c^4*x^4 + 4*b*c^2*x^2
 + 8*b)*sqrt(c^2*x^2 - 1))/c^6
```

3.2.6 Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.72

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{ax^6}{6}+\frac{bx^6\operatorname{acsc}\left(cx\right)}{6}+\frac{b\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

input
```
integrate(x**5*(a+b*acsc(c*x)),x)
```

output
```
a*x**6/6 + b*x**6*acsc(c*x)/6 + b*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c
) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5)
, Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(
-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*
c)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\operatorname{arccsc}\left(cx\right)+\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)b$$

input
```
integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/90*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2)
 + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)
*b
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(75) = 150.

Time = 0.32 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.82

$$\int x^5\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{5760}\left(\frac{15\,bx^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{15\,ax^6\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^6}{c}+\frac{6\,bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c^2}\right.$$

input
```
integrate(x^5*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/5760*(15*b*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*a*x
^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)
^5/c^2 + 90*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*
a*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^3 + 50*b*x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3/c^4 + 225*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^
5 + 225*a*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 300*b*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^6 + 300*b*arcsin(1/(c*x))/c^7 + 300*a/c^7 - 300*b/(c^8*x*(
sqrt(-1/(c^2*x^2) + 1) + 1)) + 225*b*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^
2*x^2) + 1) + 1)^2) + 225*a/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 50*
b/(c^10*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*b*arcsin(1/(c*x))/(c^11*x
^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 90*a/(c^11*x^4*(sqrt(-1/(c^2*x^2) + 1
) + 1)^4) - 6*b/(c^12*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 15*b*arcsin(1/
(c*x))/(c^13*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6) + 15*a/(c^13*x^6*(sqrt(-1
/(c^2*x^2) + 1) + 1)^6))*c
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^5 (a + b \csc^{-1}(cx)) \, dx = \int x^5 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^5*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^5*(a + b*asin(1/(c*x))), x)
```

3.3 $\int x^4(a+b\csc^{-1}(cx))\,dx$

3.3.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^4(a+b\csc^{-1}(cx))\,dx=\frac{3b\sqrt{1-\frac{1}{c^2x^2}}x^2}{40c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^4}{20c}+\frac{1}{5}x^5(a+b\csc^{-1}(cx))+\frac{3b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{40c^5}$$

output
```
1/5*x^5*(a+b*arccsc(c*x))+3/40*b*arctanh((1-1/c^2/x^2)^(1/2))/c^5+3/40*b*x
^2*(1-1/c^2/x^2)^(1/2)/c^3+1/20*b*x^4*(1-1/c^2/x^2)^(1/2)/c
```

3.3.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int x^4(a+b\csc^{-1}(cx))\,dx=\frac{ax^5}{5}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{3x^2}{40c^3}+\frac{x^4}{20c}\right)+\frac{1}{5}bx^5\csc^{-1}(cx)+\frac{3b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}$$

input
```
Integrate[x^4*(a + b*ArcCsc[c*x]),x]
```

output
```
(a*x^5)/5 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*((3*x^2)/(40*c^3) + x^4/(20*c
)) + (b*x^5*ArcCsc[c*x])/5 + (3*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)
])])/(40*c^5)
```

3.3.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5744, 798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b \csc^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5744}$$

$$\frac{b \int \frac{x^3}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{5c}+\frac{1}{5}x^5 \left(a+b \csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{5}x^5 \left(a+b \csc^{-1}(cx)\right)-\frac{b \int \frac{x^6}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{10c}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{5}x^5 \left(a+b \csc^{-1}(cx)\right)-\frac{b \left(\frac{3 \int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{4c^2}-\frac{1}{2}x^4 \sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{5}x^5 \left(a+b \csc^{-1}(cx)\right)-\frac{b \left(\frac{3 \left(\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}-x^2 \sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4 \sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{b\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int\frac{1}{c^2-\frac{c^2}{x^4}}d\sqrt{1-\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{5}x^5\left(a+b\csc^{-1}(cx)\right)-\frac{b\left(\frac{3\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}$$

input
```
Int[x^4*(a + b*ArcCsc[c*x]),x]
```

output
```
(x^5*(a + b*ArcCsc[c*x]))/5 - (b*(-1/2*(Sqrt[1 - 1/(c^2*x^2)]*x^4) + (3*(-
(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTanh[Sqrt[1 - 1/(c^2*x^2)]]/c^2))/(4*c^2)
))/(10*c)
```

3.3.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.3.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

method	result	size
parts	$\frac{a x^5}{5}+\frac{x^5 b \operatorname{arccsc}(cx)}{5}+\frac{b(c^2x^2-1)x^2}{20c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b(c^2x^2-1)}{40c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\sqrt{c^2x^2-1}\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$	141
derivativedivides	$\frac{\frac{a c^5x^5}{5}+\frac{\operatorname{arccsc}(cx)b c^5x^5}{5}+\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\sqrt{c^2x^2-1}\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148
default	$\frac{\frac{a c^5x^5}{5}+\frac{\operatorname{arccsc}(cx)b c^5x^5}{5}+\frac{b\left(c^2x^2-1\right)c^2x^2}{20\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\left(c^2x^2-1\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3b\sqrt{c^2x^2-1}\ln\left(cx+\sqrt{c^2x^2-1}\right)}{40\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c^5}$	148

input
```
int(x^4*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/5*x^5*b*arccsc(c*x)+1/20*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)*x^2+3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+3/40*b/c^6
*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{8\,ac^5x^5-16\,bc^5\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+8\left(bc^5x^5-bc^5\right)\operatorname{arccsc}(cx)-3\,b\log\left(-cx+\sqrt{c^2x^2-1}\right)+(2}{40\,c^5}$$

input
```
integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/40*(8*a*c^5*x^5 - 16*b*c^5*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 8*(b*c^5*x
^5 - b*c^5)*arccsc(c*x) - 3*b*log(-c*x + sqrt(c^2*x^2 - 1)) + (2*b*c^3*x^3
 + 3*b*c*x)*sqrt(c^2*x^2 - 1))/c^5
```

3.3.6 Sympy [A] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.97

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{ax^5}{5}+\frac{bx^5\operatorname{acsc}(cx)}{5}+\frac{b\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate(x**4*(a+b*acsc(c*x)),x)
```

output
```
a*x**5/5 + b*x**5*acsc(c*x)/5 + b*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)
) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*
acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 +
1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 +
1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.48

$$\int x^4\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{5}\,ax^5+\frac{1}{80}\left(16\,x^5\operatorname{arccsc}(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)b$$

input
```
integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*
sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1)
 + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2)
+ 1) - 1)/c^4)/c)*b
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(75) = 150.

Time = 0.63 (sec) , antiderivative size = 480, normalized size of antiderivative = 5.39

$$\int x^4 \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{320}\left(\frac{2\,bx^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{2\,ax^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c}+\frac{bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c^2}+\frac{10\,b}{}\right.$$

input
```
integrate(x^4*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/320*(2*b*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 2*a*x^5*
(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^
2 + 10*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 10*a*x^3
*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 8*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2/c^4 + 20*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 20*a*x*
(sqrt(-1/(c^2*x^2) + 1) + 1)/c^5 + 24*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^
6 - 24*b*log(1/(abs(c)*abs(x)))/c^6 + 20*b*arcsin(1/(c*x))/(c^7*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)) + 20*a/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 8*b/(c
^8*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 10*b*arcsin(1/(c*x))/(c^9*x^3*(sq
rt(-1/(c^2*x^2) + 1) + 1)^3) + 10*a/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^
3) - b/(c^10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 2*b*arcsin(1/(c*x))/(c^
11*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 2*a/(c^11*x^5*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^5))*c
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a+b\csc^{-1}(cx)\right)\,dx=\int x^4\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^4*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^4*(a + b*asin(1/(c*x))), x)
```

3.4 $\int x^3(a+b\csc^{-1}(cx))\,dx$

3.4.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^3(a+b\csc^{-1}(cx))\,dx=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4(a+b\csc^{-1}(cx))$$

output
```
1/4*x^4*(a+b*arccsc(c*x))+1/6*b*x*(1-1/c^2/x^2)^(1/2)/c^3+1/12*b*x^3*(1-1/
c^2/x^2)^(1/2)/c
```

3.4.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x^3(a+b\csc^{-1}(cx))\,dx=\frac{ax^4}{4}+b\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\left(\frac{x}{6c^3}+\frac{x^3}{12c}\right)+\frac{1}{4}bx^4\csc^{-1}(cx)$$

input
```
Integrate[x^3*(a + b*ArcCsc[c*x]),x]
```

output
```
(a*x^4)/4 + b*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)]*(x/(6*c^3) + x^3/(12*c)) + (b
*x^4*ArcCsc[c*x])/4
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5744, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \csc^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 5744$$

$$\frac{b \int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{4c}+\frac{1}{4}x^4 \left(a+b \csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 803$$

$$\frac{b \left(\frac{2 \int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3c^2}+\frac{1}{3}x^3 \sqrt{1-\frac{1}{c^2x^2}}\right)}{4c}+\frac{1}{4}x^4 \left(a+b \csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 746$$

$$\frac{1}{4}x^4 \left(a+b \csc^{-1}(cx)\right)+\frac{b \left(\frac{2x\sqrt{1-\frac{1}{c^2x^2}}}{3c^2}+\frac{1}{3}x^3 \sqrt{1-\frac{1}{c^2x^2}}\right)}{4c}$$

input
```
Int[x^3*(a + b*ArcCsc[c*x]),x]
```

output
```
(b*((2*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*c^2) + (Sqrt[1 - 1/(c^2*x^2)]*x^3)/3))/
(4*c) + (x^4*(a + b*ArcCsc[c*x]))/4
```

3.4.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

method	result	size
parts	$\frac{x^4 a}{4}+\frac{b\left(\frac{c^4x^4\,\mathrm{arccsc}(cx)}{4}+\frac{\left(c^2x^2-1\right)\left(c^2x^2+2\right)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	70
derivativedivides	$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\,\mathrm{arccsc}(cx)}{4}+\frac{\left(c^2x^2-1\right)\left(c^2x^2+2\right)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	74
default	$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\,\mathrm{arccsc}(cx)}{4}+\frac{\left(c^2x^2-1\right)\left(c^2x^2+2\right)}{12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^4}$	74

input
```
int(x^3*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*a+b/c^4*(1/4*c^4*x^4*arccsc(c*x)+1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^
2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int x^3 \left(a+b \csc^{-1}(cx)\right) \, dx = \frac{3 \, bc^4x^4 \operatorname{arccsc}(cx) + 3 \, ac^4x^4 + (bc^2x^2 + 2 \, b)\sqrt{c^2x^2 - 1}}{12 \, c^4}$$

input
```
integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*b*c^4*x^4*arccsc(c*x) + 3*a*c^4*x^4 + (b*c^2*x^2 + 2*b)*sqrt(c^2*x
^2 - 1))/c^4
```

3.4.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^3 \left(a+b \csc^{-1}(cx)\right) \, dx = \frac{ax^4}{4} + \frac{bx^4 \operatorname{acsc}(cx)}{4} + \frac{b \left(\begin{cases} \frac{x^2\sqrt{c^2x^2-1}}{3c} + \frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2| > 1 \\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c} + \frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise} \end{cases}\right)}{4c}$$

input
```
integrate(x**3*(a+b*acsc(c*x)),x)
```

output
```
a*x**4/4 + b*x**4*acsc(c*x)/4 + b*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c
) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**
2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int x^3 \left(a+b \csc^{-1}(cx)\right) \, dx = \frac{1}{4} \, ax^4 + \frac{1}{12} \left(3 \, x^4 \operatorname{arccsc}(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 3 \, x\sqrt{-\frac{1}{c^2x^2} + 1}}{c^3}\right) b$$

input
```
integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) +
3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(54) = 108$.

Time = 0.30 (sec) , antiderivative size = 352, normalized size of antiderivative = 5.50

$$\int x^3(a + b\csc^{-1}(cx))\ dx$$
$$= \frac{1}{192}\left(\frac{3\,bx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{3\,ax^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c} + \frac{2\,bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^2} + \frac{12}{-}\right.$$

input
```
integrate(x^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/192*(3*b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*x^4*
(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 2*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/
c^2 + 12*b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a*x
^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 18*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1
)/c^4 + 18*b*arcsin(1/(c*x))/c^5 + 18*a/c^5 - 18*b/(c^6*x*(sqrt(-1/(c^2*x^
2) + 1) + 1)) + 12*b*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2) + 12*a/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 2*b/(c^8*x^3*(sqrt(-
1/(c^2*x^2) + 1) + 1)^3) + 3*b*arcsin(1/(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^4) + 3*a/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4))*c
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a + b\csc^{-1}(cx))\ dx = \int x^3\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\ dx$$

input
```
int(x^3*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^3*(a + b*asin(1/(c*x))), x)
```

3.5 $\int x^2(a+b\csc^{-1}(cx))\,dx$

3.5.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2}{6c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)+\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

output
```
1/3*x^3*(a+b*arccsc(c*x))+1/6*b*arctanh((1-1/c^2/x^2)^(1/2))/c^3+1/6*b*x^2
*(1-1/c^2/x^2)^(1/2)/c
```

3.5.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^3}{3}+\frac{bx^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+\frac{1}{3}bx^3\csc^{-1}(cx)+\frac{b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(a*x^3)/3 + (b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcCsc[c
*x])/3 + (b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5744, 798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+b\csc^{-1}(cx)) \, dx$$

$$\Big\downarrow \ 5744$$

$$\frac{b\int \frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3c}+\frac{1}{3}x^3 (a+b\csc^{-1}(cx))$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}x^3 (a+b\csc^{-1}(cx))-\frac{b\int \frac{x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{6c}$$

$$\Big\downarrow \ 52$$

$$\frac{1}{3}x^3 (a+b\csc^{-1}(cx))-\frac{b\left(\frac{\int \frac{x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}-x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}x^3 (a+b\csc^{-1}(cx))-\frac{b\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int \frac{1}{c^2-\frac{c^2}{x^4}}d\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}x^3 (a+b\csc^{-1}(cx))-\frac{b\left(x^2\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}\right)}{6c}$$

input
```
Int[x^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(x^3*(a + b*ArcCsc[c*x]))/3 - (b*(-(Sqrt[1 - 1/(c^2*x^2)]*x^2) - ArcTanh[S
qrt[1 - 1/(c^2*x^2)]]/c^2))/(6*c)
```

3.5.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
parts	$\frac{x^{3}a}{3}+\frac{b\left(\frac{c^{3}x^{3}\operatorname{arccsc}(cx)}{3}+\frac{\sqrt{c^{2}x^{2}-1}\left(cx\sqrt{c^{2}x^{2}-1}+\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)\right)}{6\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{3}}$	94
derivativedivides	$\frac{\frac{a\,c^{3}x^{3}}{3}+b\left(\frac{c^{3}x^{3}\operatorname{arccsc}(cx)}{3}+\frac{\sqrt{c^{2}x^{2}-1}\left(cx\sqrt{c^{2}x^{2}-1}+\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)\right)}{6\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{3}}$	98
default	$\frac{\frac{a\,c^{3}x^{3}}{3}+b\left(\frac{c^{3}x^{3}\operatorname{arccsc}(cx)}{3}+\frac{\sqrt{c^{2}x^{2}-1}\left(cx\sqrt{c^{2}x^{2}-1}+\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)\right)}{6\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c^{3}}$	98

input
```
int(x^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*a+b/c^3*(1/3*c^3*x^3*arccsc(c*x)+1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x
^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3x^3-4\,bc^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcx+2\left(bc^3x^3-bc^3\right)\operatorname{arccsc}(cx)-b\log\left(-cx+\sqrt{\right.}}{6\,c^3}$$

input
```
integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*x^3 - 4*b*c^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^2
 - 1)*b*c*x + 2*(b*c^3*x^3 - b*c^3)*arccsc(c*x) - b*log(-c*x + sqrt(c^2*x^
2 - 1)))/c^3
```

3.5.6 Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.67

$$\int x^2 (a + b\csc^{-1}(cx))\,dx = \frac{ax^3}{3} + \frac{bx^3\operatorname{acsc}(cx)}{3} + \frac{b\left(\begin{cases} \frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2| > 1 \\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise} \end{cases}\right)}{3c}$$

input
```
integrate(x**2*(a+b*acsc(c*x)),x)
```

output
```
a*x**3/3 + b*x**3*acsc(c*x)/3 + b*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) +
 acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 +
 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int x^2 (a + b\csc^{-1}(cx))\,dx$$
$$= \frac{1}{3}ax^3 + \frac{1}{12}\left(4x^3\operatorname{arccsc}(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

input
```
integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c
^2*x^2) + 1) - 1)/c^2)/c)*b
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(54) = 108.

Time = 0.45 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.84

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{24}\left(\frac{bx^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{ax^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c}+\frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^2}+\frac{3\,bx\left(\sqrt{\right.}\right.}{}\right.$$

input
```
integrate(x^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/24*(b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3/c + b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 3
*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 3*a*x*(sqrt(-1/(c^
2*x^2) + 1) + 1)/c^3 + 4*b*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*log(1
/(abs(c)*abs(x)))/c^4 + 3*b*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1)
 + 1)) + 3*a/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b/(c^6*x^2*(sqrt(-1/(c
^2*x^2) + 1) + 1)^2) + b*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^3) + a/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^2*(a + b*asin(1/(c*x))), x)
```

3.6 $\int x(a+b\csc^{-1}(cx))\,dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 39

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)$$

output
```
1/2*x^2*(a+b*arccsc(c*x))+1/2*b*x*(1-1/c^2/x^2)^(1/2)/c
```

3.6.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{ax^2}{2}+\frac{bx\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bx^2\csc^{-1}(cx)$$

input
```
Integrate[x*(a + b*ArcCsc[c*x]),x]
```

output
```
(a*x^2)/2 + (b*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcCsc[c*x
])/2
```

3.6.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5744, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \; 5744$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 746$$

$$\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2c}$$

input
```
Int[x*(a + b*ArcCsc[c*x]),x]
```

output
```
(b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsc[c*x]))/2
```

3.6.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.6.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.56

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\left(\frac{c^{2}x^{2}\operatorname{arccsc}\left(c x\right)}{2}+\frac{c^{2}x^{2}-1}{2\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c x}\right)}{c^{2}}$	61
derivativedivides	$\frac{\frac{a\,c^{2}x^{2}}{2}+b\left(\frac{c^{2}x^{2}\operatorname{arccsc}\left(c x\right)}{2}+\frac{c^{2}x^{2}-1}{2\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c x}\right)}{c^{2}}$	65
default	$\frac{\frac{a\,c^{2}x^{2}}{2}+b\left(\frac{c^{2}x^{2}\operatorname{arccsc}\left(c x\right)}{2}+\frac{c^{2}x^{2}-1}{2\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,c x}\right)}{c^{2}}$	65

input
```
int(x*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arccsc(c*x)+1/2/((c^2*x^2-1)/c^2/x^2)^(1/2)/c
/x*(c^2*x^2-1))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{bc^2x^2\operatorname{arccsc}\left(cx\right)+ac^2x^2+\sqrt{c^2x^2-1}b}{2\,c^2}$$

input
```
integrate(x*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/2*(b*c^2*x^2*arccsc(c*x) + a*c^2*x^2 + sqrt(c^2*x^2 - 1)*b)/c^2
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int x(a+b\csc^{-1}(cx))\,dx=\frac{ax^2}{2}+\frac{bx^2\operatorname{acsc}(cx)}{2}+\frac{b\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

input
```
integrate(x*(a+b*acsc(c*x)),x)
```

output
```
a*x**2/2 + b*x**2*acsc(c*x)/2 + b*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c*
*2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x(a+b\csc^{-1}(cx))\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)b$$

input
```
integrate(x*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(33) = 66.

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.67

$$\int x(a+b\csc^{-1}(cx))\,dx$$
$$=\frac{1}{8}\left(\frac{bx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{ax^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c}+\frac{2\,bx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{2\,b\arcsin}{c^3}\right.$$

input
```
integrate(x*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/8*(b*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a*x^2*(sqrt(
-1/(c^2*x^2) + 1) + 1)^2/c + 2*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 2*b*
arcsin(1/(c*x))/c^3 + 2*a/c^3 - 2*b/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
 b*arcsin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a/(c^5*x^2*(
sqrt(-1/(c^2*x^2) + 1) + 1)^2))*c
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{2}+\frac{b\,x\,\sqrt{1-\frac{1}{c^2\,x^2}}}{2\,c}$$

input
```
int(x*(a + b*asin(1/(c*x))),x)
```

output
```
(a*x^2)/2 + (b*x^2*asin(1/(c*x)))/2 + (b*x*(1 - 1/(c^2*x^2))^(1/2))/(2*c)
```

3.7 $\int (a+b\csc^{-1}(cx))\,dx$

3.7.1 Optimal result

Integrand size = 8, antiderivative size = 31

$$\int (a+b\csc^{-1}(cx))\,dx = ax+bx\csc^{-1}(cx)+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

output
```
a*x+b*x*arccsc(c*x)+b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

3.7.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int (a+b\csc^{-1}(cx))\,dx = ax+bx\csc^{-1}(cx)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[a + b*ArcCsc[c*x],x]
```

output
```
a*x + b*x*ArcCsc[c*x] + (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
 c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

3.7.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\csc^{-1}(cx)$$

input
```
Int[a + b*ArcCsc[c*x],x]
```

output
```
a*x + b*x*ArcCsc[c*x] + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result	size
default	$ax+bx\,\operatorname{arccsc}(cx)+\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	37
parts	$ax+bx\,\operatorname{arccsc}(cx)+\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	37
derivativedivides	$\frac{acx+b\left(\operatorname{arccsc}(cx)cx+\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{c}$	40

input
```
int(a+b*arccsc(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arccsc(c*x)+b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{acx - 2\,bc\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + (bcx-bc)\operatorname{arccsc}(cx) - b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{c}$$

input
```
integrate(a+b*arccsc(c*x),x, algorithm="fricas")
```

output
```
(a*c*x - 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arccsc(c*x
) - b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

3.7.6 Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = ax + b\left(x\operatorname{acsc}(cx) + \frac{\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2| > 1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}}{c}\right)$$

input
```
integrate(a+b*acsc(c*x),x)
```

output
```
a*x + b*(x*acsc(c*x) + Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asi
n(c*x), True))/c)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = ax+\frac{\left(2\,cx\operatorname{arccsc}\left(cx\right)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\,c}$$

input
```
integrate(a+b*arccsc(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt
(-1/(c^2*x^2) + 1) + 1))*b/c
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{2}\,bc\left(\frac{2\,x\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right)+ax$$

input
```
integrate(a+b*arccsc(c*x),x, algorithm="giac")
```

output
```
1/2*b*c*(2*x*arcsin(1/(c*x))/c + (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-s
qrt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

3.7.9 Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx=a\,x+b\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)+\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

input
```
int(a + b*asin(1/(c*x)),x)
```

output
```
a*x + b*x*asin(1/(c*x)) + (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

3.8 $\int \frac{a+b\csc^{-1}(cx)}{x}\,dx$

3.8.1 Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = \frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b} - \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + \frac{1}{2}ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

output
```
1/2*I*(a+b*arccsc(c*x))^2/b-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1
/2))^2)+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = -b\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + a\log(x) + \frac{1}{2}ib\left(\csc^{-1}(cx)^2 + \text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcCsc[c*x])/x,x]
```

output
```
-(b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])]) + a*Log[x] + (I/2)*b*(ArcC
sc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

3.8.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5742, 5136, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx$$

$$\Big\downarrow \text{5742}$$

$$-\int x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{5136}$$

$$-\int c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d\arcsin\left(\frac{1}{cx}\right)$$

$$\Big\downarrow \text{3042}$$

$$-\int -\left(\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\tan\left(\arcsin\left(\frac{1}{cx}\right)+\frac{\pi}{2}\right)\right)d\arcsin\left(\frac{1}{cx}\right)$$

$$\Big\downarrow \text{25}$$

$$\int \tan\left(\arcsin\left(\frac{1}{cx}\right)+\frac{\pi}{2}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d\arcsin\left(\frac{1}{cx}\right)$$

$$\Big\downarrow \text{4200}$$

$$\frac{i\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)^2}{2b}-2i\int -\frac{e^{2i\arcsin\left(\frac{1}{cx}\right)}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{1-e^{2i\arcsin\left(\frac{1}{cx}\right)}}d\arcsin\left(\frac{1}{cx}\right)$$

$$\Big\downarrow \text{25}$$

$$2i\int \frac{e^{2i\arcsin\left(\frac{1}{cx}\right)}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{1-e^{2i\arcsin\left(\frac{1}{cx}\right)}}d\arcsin\left(\frac{1}{cx}\right)+\frac{i\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)^2}{2b}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)-\frac{1}{2}ib\int\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)d\arcsin\left(\frac{1}{cx}\right)\right)+\frac{i\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)^2}{2b}$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)-\frac{1}{4}b\int x\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)de^{2i\arcsin\left(\frac{1}{cx}\right)}\right)+\frac{i\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)^2}{2b}$$

$$\Big\downarrow \; 2838$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{4}b\operatorname{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)\right)+\frac{i\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)^2}{2b}$$

input
```
Int[(a + b*ArcCsc[c*x])/x,x]
```

output
```
((I/2)*(a + b*ArcSin[1/(c*x)])^2)/b + (2*I)*((I/2)*(a + b*ArcSin[1/(c*x)])
*Log[1 - E^((2*I)*ArcSin[1/(c*x)])] + (b*PolyLog[2, E^((2*I)*ArcSin[1/(c*x
)])])/4)
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5136
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n*Cot[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

rule 5742
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a + b
*ArcSin[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

3.8.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.12

method	result
parts	$a\ln\left(x\right)+b\left(\frac{i\,\mathrm{arccsc}\left(cx\right)^{2}}{2}-\mathrm{arccsc}\left(cx\right)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)+i\,\mathrm{polylog}\left(2,\frac{i}{cx}+\sqrt{1\right.\right.$
derivativedivides	$a\ln\left(cx\right)+b\left(\frac{i\,\mathrm{arccsc}\left(cx\right)^{2}}{2}-\mathrm{arccsc}\left(cx\right)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)+i\,\mathrm{polylog}\left(2,\frac{i}{cx}+\sqrt{1\right.\right.$
default	$a\ln\left(cx\right)+b\left(\frac{i\,\mathrm{arccsc}\left(cx\right)^{2}}{2}-\mathrm{arccsc}\left(cx\right)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^{2}x^{2}}}\right)+i\,\mathrm{polylog}\left(2,\frac{i}{cx}+\sqrt{1\right.\right.$

input
```
int((a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(1/2*I*arccsc(c*x)^2-arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))
+I*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^
2)^(1/2))+I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))
```

3.8.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/x, x)
```

3.8.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = \int \frac{a+b\,\mathrm{acsc}\,(cx)}{x}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x,x)
```

output
```
Integral((a + b*acsc(c*x))/x, x)
```

3.8.7 Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

output
```
(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) +
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b + a*log(x)
```

3.8.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{a+b\csc^{-1}(cx)}{x}\,dx = \frac{b\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,1\mathrm{i}}{2}+\frac{b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2\,1\mathrm{i}}{2}+a\,\ln\left(x\right)-b\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right)$$

input
```
int((a + b*asin(1/(c*x)))/x,x)
```

output
```
(b*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 + (b*asin(1/(c*x))^2*1i)/2 + a*
log(x) - b*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(c*x))
```

3.9 $\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx$

3.9.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx=-bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{x}$$

output
```
(-a-b*arccsc(c*x))/x-b*c*(1-1/c^2/x^2)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx=-\frac{a}{x}-bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\csc^{-1}(cx)}{x}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/x^2,x]
```

output
```
-(a/x) - b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/x
```

3.9.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5744, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx$$

$$\Big\downarrow \text{5744}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^3}dx}{c}-\frac{a+b\csc^{-1}(cx)}{x}$$

$$\Big\downarrow \text{793}$$

$$-\frac{a+b\csc^{-1}(cx)}{x}-bc\sqrt{1-\frac{1}{c^2x^2}}$$

input
```
Int[(a + b*ArcCsc[c*x])/x^2,x]
```

output
```
-(b*c*Sqrt[1 - 1/(c^2*x^2)]) - (a + b*ArcCsc[c*x])/x
```

3.9.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.9.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)$	59
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	63
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}\right)\right)$	63

input
```
int((a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*c*(-1/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^
2-1))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx=-\frac{b\operatorname{arccsc}(cx)+\sqrt{c^2x^2-1}b+a}{x}$$

input
```
integrate((a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

output
```
-(b*arccsc(c*x) + sqrt(c^2*x^2 - 1)*b + a)/x
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx=\begin{cases}-\frac{a}{x}-bc\sqrt{1-\frac{1}{c^2x^2}}-\frac{b\operatorname{acsc}(cx)}{x} & \text{for } c\neq 0\\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*acsc(c*x))/x**2,x)
```

output
```
Piecewise((-a/x - b*c*sqrt(1 - 1/(c**2*x**2)) - b*acsc(c*x)/x, Ne(c, 0)),
(-(a + zoo*b)/x, True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx = -\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)b-\frac{a}{x}$$

input
```
integrate((a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

output
```
-(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b - a/x
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx = -\left(b\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b\arcsin\left(\frac{1}{cx}\right)}{cx}+\frac{a}{cx}\right)c$$

input
```
integrate((a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

output
```
-(b*sqrt(-1/(c^2*x^2) + 1) + b*arcsin(1/(c*x))/(c*x) + a/(c*x))*c
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx = -\frac{a}{x}-b\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}-\frac{b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x}$$

input
```
int((a + b*asin(1/(c*x)))/x^2,x)
```

output
```
- a/x - b*c*(1 - 1/(c^2*x^2))^(1/2) - (b*asin(1/(c*x)))/x
```

3.9. $\int \frac{a+b\csc^{-1}(cx)}{x^2}\,dx$

3.10 $\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx$

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx=-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2\csc^{-1}(cx)-\frac{a+b\csc^{-1}(cx)}{2x^2}$$

output
```
1/4*b*c^2*arccsc(c*x)+1/2*(-a-b*arccsc(c*x))/x^2-1/4*b*c*(1-1/c^2/x^2)^(1/
2)/x
```

3.10.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{bc\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x}-\frac{b\csc^{-1}(cx)}{2x^2}+\frac{1}{4}bc^2\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcCsc[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*c*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcCsc[c*x])/
(2*x^2) + (b*c^2*ArcSin[1/(c*x)])/4
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5744, 858, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx$$

$$\Big\downarrow \text{5744}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}dx}{2c}-\frac{a+b\csc^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{858}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x}}{2c}-\frac{a+b\csc^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{b\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)}{2c}-\frac{a+b\csc^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{b\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)}{2c}-\frac{a+b\csc^{-1}(cx)}{2x^2}$$

input
```
Int[(a + b*ArcCsc[c*x])/x^3,x]
```

output
```
-1/2*(a + b*ArcCsc[c*x])/x^2 + (b*(-1/2*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x + (c
^3*ArcSin[1/(c*x)])/2))/(2*c)
```

3.10.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

method	result	size
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\operatorname{arccsc}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2-\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3c^3}\right)$	96
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arccsc}(cx)}{2c^2x^2}-\frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)\right)$	99
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arccsc}(cx)}{2c^2x^2}-\frac{\sqrt{c^2x^2-1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^2x^2+\sqrt{c^2x^2-1}\right)}{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}\right)\right)$	99

input
```
int((a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arccsc(c*x)+1/4*(c^2*x^2-1)^(1/2)*(arctan(1
/(c^2*x^2-1)^(1/2))*c^2*x^2-(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1/2)
/x^3/c^3)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = \frac{(bc^2x^2-2\,b)\,\mathrm{arccsc}\,(cx)-\sqrt{c^2x^2-1}b-2\,a}{4\,x^2}$$

input
```
integrate((a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

output
```
1/4*((b*c^2*x^2 - 2*b)*arccsc(c*x) - sqrt(c^2*x^2 - 1)*b - 2*a)/x^2
```

3.10.6 Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.37

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{b\,\mathrm{acsc}\,(cx)}{2x^2} - \frac{b\left(\begin{cases}\frac{ic^3\,\mathrm{acosh}\left(\frac{1}{cx}\right)}{2}-\frac{ic^2}{2x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{2x^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|}>1\\ -\frac{c^3\,\mathrm{asin}\left(\frac{1}{cx}\right)}{2}+\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise}\end{cases}\right)}{2c}$$

input
```
integrate((a+b*acsc(c*x))/x**3,x)
```

output
```
-a/(2*x**2) - b*acsc(c*x)/(2*x**2) - b*Piecewise((I*c**3*acosh(1/(c*x))/2
- I*c**2/(2*x*sqrt(-1 + 1/(c**2*x**2))) + I/(2*x**3*sqrt(-1 + 1/(c**2*x**2
))), 1/Abs(c**2*x**2) > 1), (-c**3*asin(1/(c*x))/2 + c**2*sqrt(1 - 1/(c**2
*x**2))/(2*x), True))/(2*c)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = \frac{1}{4}\,b\left(\frac{\frac{c^4x\sqrt{-\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}-c^3\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c}-\frac{2\,\mathrm{arccsc}\,(cx)}{x^2}\right) - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/4*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3
*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/2*a/x^2
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{a+b\csc^{-1}(cx)}{x^3}\,dx = -\frac{1}{4}\left(2\,bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+2\,ac\left(\frac{1}{c^2x^2}-1\right)+bc\arcsin\left(\frac{1}{cx}\right)+\frac{b\sqrt{-\frac{1}{c^2x^2}+1}}{x}\right)c$$

input
```
integrate((a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

output
```
-1/4*(2*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 2*a*c*(1/(c^2*x^2) - 1) +
b*c*arcsin(1/(c*x)) + b*sqrt(-1/(c^2*x^2) + 1)/x)*c
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{a + b\csc^{-1}(cx)}{x^3} \, dx = -\frac{a}{2\,x^2} - \frac{b\,c^2 \operatorname{asin}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2} - 1\right)}{4} - \frac{b\,c\,\sqrt{1 - \frac{1}{c^2\,x^2}}}{4\,x}$$

input
```
int((a + b*asin(1/(c*x)))/x^3,x)
```

output
```
- a/(2*x^2) - (b*c^2*asin(1/(c*x))*(2/(c^2*x^2) - 1))/4 - (b*c*(1 - 1/(c^2
*x^2))^(1/2))/(4*x)
```

3.11 $\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx$

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx=-\frac{1}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}+\frac{1}{9}bc^3\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\csc^{-1}(cx)}{3x^3}$$

output
```
1/9*b*c^3*(1-1/c^2/x^2)^(3/2)+1/3*(-a-b*arccsc(c*x))/x^3-1/3*b*c^3*(1-1/c^
2/x^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx=-\frac{a}{3x^3}+b\left(-\frac{2c^3}{9}-\frac{c}{9x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\csc^{-1}(cx)}{3x^3}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 + b*((-2*c^3)/9 - c/(9*x^2))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (
b*ArcCsc[c*x])/(3*x^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5744, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx$$

$$\Big\downarrow \text{5744}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^5}dx}{3c}-\frac{a+b\csc^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x^2}}{6c}-\frac{a+b\csc^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{53}$$

$$\frac{b\int \left(\frac{c^2}{\sqrt{1-\frac{1}{c^2x^2}}}-c^2\sqrt{1-\frac{1}{c^2x^2}}\right)d\frac{1}{x^2}}{6c}-\frac{a+b\csc^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{2}{3}c^4\left(1-\frac{1}{c^2x^2}\right)^{3/2}-2c^4\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c}-\frac{a+b\csc^{-1}(cx)}{3x^3}$$

input
```
Int[(a + b*ArcCsc[c*x])/x^4,x]
```

output
```
(b*(-2*c^4*Sqrt[1 - 1/(c^2*x^2)] + (2*c^4*(1 - 1/(c^2*x^2))^(3/2))/3))/(6*
c) - (a + b*ArcCsc[c*x])/(3*x^3)
```

3.11.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\operatorname{arccsc}(cx)}{3c^3x^3}-\frac{(c^2x^2-1)\,(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)$	71
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arccsc}(cx)}{3c^3x^3}-\frac{(c^2x^2-1)\,(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)\right)$	75
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arccsc}(cx)}{3c^3x^3}-\frac{(c^2x^2-1)\,(2c^2x^2+1)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)\right)$	75

input
```
int((a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = -\frac{3\,b\,\mathrm{arccsc}\,(cx)+(2\,bc^2x^2+b)\sqrt{c^2x^2-1}+3\,a}{9\,x^3}$$

input
```
integrate((a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*b*arccsc(c*x) + (2*b*c^2*x^2 + b)*sqrt(c^2*x^2 - 1) + 3*a)/x^3
```

3.11.6 Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.87

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{b\,\mathrm{acsc}\,(cx)}{3x^3}-\frac{b\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((a+b*acsc(c*x))/x**4,x)
```

output
```
-a/(3*x**3) - b*acsc(c*x)/(3*x**3) - b*Piecewise((2*c**3*sqrt(c**2*x**2 -
1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*
sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*
c)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx = \frac{1}{9}\,b\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right)-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/9*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3
*arccsc(c*x)/x^3) - 1/3*a/x^3
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx$$
$$=\frac{1}{9}\left(bc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,bc^2\sqrt{-\frac{1}{c^2x^2}+1}-\frac{3\,bc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{3\,bc\arcsin\left(\frac{1}{cx}\right)}{x}-\frac{3\,a}{cx^3}\right)c$$

input
```
integrate((a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

output
```
1/9*(b*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 3*b*c^2*sqrt(-1/(c^2*x^2) + 1) - 3*b
*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 3*b*c*arcsin(1/(c*x))/x - 3*a/(c*
x^3))*c
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^4}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/x^4,x)
```

output
```
int((a + b*asin(1/(c*x)))/x^4, x)
```

3.12 $\int \frac{a+b\csc^{-1}(cx)}{x^5} \, dx$

3.12.1 Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{a+b\csc^{-1}(cx)}{x^5} \, dx = -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}}{32x} + \frac{3}{32}bc^4\csc^{-1}(cx) - \frac{a+b\csc^{-1}(cx)}{4x^4}$$

output
```
3/32*b*c^4*arccsc(c*x)+1/4*(-a-b*arccsc(c*x))/x^4-1/16*b*c*(1-1/c^2/x^2)^(
1/2)/x^3-3/32*b*c^3*(1-1/c^2/x^2)^(1/2)/x
```

3.12.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{a+b\csc^{-1}(cx)}{x^5} \, dx = -\frac{a}{4x^4} + b\left(-\frac{c}{16x^3} - \frac{3c^3}{32x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{b\csc^{-1}(cx)}{4x^4} + \frac{3}{32}bc^4\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcCsc[c*x])/x^5,x]
```

output
```
-1/4*a/x^4 + b*(-1/16*c/x^3 - (3*c^3)/(32*x))*Sqrt[(-1 + c^2*x^2)/(c^2*x^2
)] - (b*ArcCsc[c*x])/(4*x^4) + (3*b*c^4*ArcSin[1/(c*x)])/32
```

3.12.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5744, 858, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx$$

$$\Big\downarrow \text{5744}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^6}dx}{4c}-\frac{a+b\csc^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{858}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x}}{4c}-\frac{a+b\csc^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{262}$$

$$\frac{b\left(\frac{3}{4}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c}-\frac{a+b\csc^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{262}$$

$$\frac{b\left(\frac{3}{4}c^2\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c}-\frac{a+b\csc^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{223}$$

$$\frac{b\left(\frac{3}{4}c^2\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)}{4c}-\frac{a+b\csc^{-1}(cx)}{4x^4}$$

input
```
Int[(a + b*ArcCsc[c*x])/x^5,x]
```

output
```
-1/4*(a + b*ArcCsc[c*x])/x^4 + (b*(-1/4*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^3 +
(3*c^2*(-1/2*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x + (c^3*ArcSin[1/(c*x)])/2))/4))
/(4*c)
```

3.12.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(67) = 134$.

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

method	result	s
parts	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arccsc}(cx)}{4x^4}+\frac{3bc^3\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{3bc(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}-\frac{b(c^2x^2-1)}{16c\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5}$	1
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arccsc}(cx)}{4c^4x^4}+\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}-\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$	1
default	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\,\mathrm{arccsc}(cx)}{4c^4x^4}+\frac{3b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{3b(c^2x^2-1)}{32\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}-\frac{b(c^2x^2-1)}{16\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}\right)$	1

input
```
int((a+b*arccsc(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4-1/4*b/x^4*arccsc(c*x)+3/32*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-3/32*b*c*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)/x^3-1/16*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x^5
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx = \frac{(3\,bc^4x^4-8\,b)\,\mathrm{arccsc}\,(cx)-(3\,bc^2x^2+2\,b)\sqrt{c^2x^2-1}-8\,a}{32\,x^4}$$

input
```
integrate((a+b*arccsc(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/32*((3*b*c^4*x^4 - 8*b)*arccsc(c*x) - (3*b*c^2*x^2 + 2*b)*sqrt(c^2*x^2 -
 1) - 8*a)/x^4
```

3.12.6 Sympy [A] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.55

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx$$
$$= -\frac{a}{4x^4}-\frac{b\,\mathrm{acsc}\,(cx)}{4x^4} - \frac{b\left(\begin{cases}\frac{3ic^5\,\mathrm{acosh}\,\left(\frac{1}{cx}\right)}{8}-\frac{3ic^4}{8x\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{ic^2}{8x^3\sqrt{-1+\frac{1}{c^2x^2}}}+\frac{i}{4x^5\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|}>1\\ -\frac{3c^5\,\mathrm{asin}\,\left(\frac{1}{cx}\right)}{8}+\frac{3c^4}{8x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{c^2}{8x^3\sqrt{1-\frac{1}{c^2x^2}}}-\frac{1}{4x^5\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise}\end{cases}\right)}{4c}$$

input
```
integrate((a+b*acsc(c*x))/x**5,x)
```

output
```
-a/(4*x**4) - b*acsc(c*x)/(4*x**4) - b*Piecewise((3*I*c**5*acosh(1/(c*x))/
8 - 3*I*c**4/(8*x*sqrt(-1 + 1/(c**2*x**2))) + I*c**2/(8*x**3*sqrt(-1 + 1/(
c**2*x**2))) + I/(4*x**5*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1),
 (-3*c**5*asin(1/(c*x))/8 + 3*c**4/(8*x*sqrt(1 - 1/(c**2*x**2))) - c**2/(8
*x**3*sqrt(1 - 1/(c**2*x**2))) - 1/(4*x**5*sqrt(1 - 1/(c**2*x**2))), True)
)/(4*c)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.64

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx$$
$$= -\frac{1}{32}b\left(\frac{3\,c^5\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right)+\frac{3\,c^8x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+5\,c^6x\sqrt{-\frac{1}{c^2x^2}+1}}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c}+\frac{8\,\mathrm{arccsc}\,(cx)}{x^4}\right)$$
$$-\frac{a}{4\,x^4}$$

input
```
integrate((a+b*arccsc(c*x))/x^5,x, algorithm="maxima")
```

output
```
-1/32*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2*x
^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) - 1
)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/4*a/x^4
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{a+b\csc^{-1}(cx)}{x^5}\,dx =$$
$$-\frac{1}{32}\left(8\,bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+16\,bc^3\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+5\,bc^3\arcsin\left(\frac{1}{cx}\right)-\frac{2\,bc^2(-$$

input
```
integrate((a+b*arccsc(c*x))/x^5,x, algorithm="giac")
```

output
```
-1/32*(8*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 16*b*c^3*(1/(c^2*x^2)
 - 1)*arcsin(1/(c*x)) + 5*b*c^3*arcsin(1/(c*x)) - 2*b*c^2*(-1/(c^2*x^2) +
1)^(3/2)/x + 5*b*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 8*a/(c*x^4))*c
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x^5}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/x^5,x)
```

output
```
int((a + b*asin(1/(c*x)))/x^5, x)
```

3.13 $\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx$

3.13.1 Optimal result

Integrand size = 12, antiderivative size = 82

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx=-\frac{1}{5}bc^5\sqrt{1-\frac{1}{c^2x^2}}+\frac{2}{15}bc^5\left(1-\frac{1}{c^2x^2}\right)^{3/2}-\frac{1}{25}bc^5\left(1-\frac{1}{c^2x^2}\right)^{5/2}-\frac{a+b\csc^{-1}(cx)}{5x^5}$$

output
```
2/15*b*c^5*(1-1/c^2/x^2)^(3/2)-1/25*b*c^5*(1-1/c^2/x^2)^(5/2)+1/5*(-a-b*ar
ccsc(c*x))/x^5-1/5*b*c^5*(1-1/c^2/x^2)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx=-\frac{a}{5x^5}+b\left(-\frac{8c^5}{75}-\frac{c}{25x^4}-\frac{4c^3}{75x^2}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\csc^{-1}(cx)}{5x^5}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/x^6,x]
```

output
```
-1/5*a/x^5 + b*((-8*c^5)/75 - c/(25*x^4) - (4*c^3)/(75*x^2))*Sqrt[(-1 + c^
2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/(5*x^5)
```

3.13.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5744, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx$$

$$\Big\downarrow \; 5744$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^7}dx}{5c}-\frac{a+b\csc^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \; 798$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x^2}}{10c}-\frac{a+b\csc^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \; 53$$

$$\frac{b\int \left(\left(1-\frac{1}{c^2x^2}\right)^{3/2}c^4-2\sqrt{1-\frac{1}{c^2x^2}}c^4+\frac{c^4}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x^2}}{10c}-\frac{a+b\csc^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\left(-\frac{2}{5}c^6\left(1-\frac{1}{c^2x^2}\right)^{5/2}+\frac{4}{3}c^6\left(1-\frac{1}{c^2x^2}\right)^{3/2}-2c^6\sqrt{1-\frac{1}{c^2x^2}}\right)}{10c}-\frac{a+b\csc^{-1}(cx)}{5x^5}$$

input
```
Int[(a + b*ArcCsc[c*x])/x^6,x]
```

output
```
(b*(-2*c^6*Sqrt[1 - 1/(c^2*x^2)] + (4*c^6*(1 - 1/(c^2*x^2))^(3/2))/3 - (2*
c^6*(1 - 1/(c^2*x^2))^(5/2))/5))/(10*c) - (a + b*ArcCsc[c*x])/(5*x^5)
```

3.13.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.13.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arccsc}(cx)}{5c^5x^5}-\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)$	79
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arccsc}(cx)}{5c^5x^5}-\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arccsc}(cx)}{5c^5x^5}-\frac{(c^2x^2-1)\,(8c^4x^4+4c^2x^2+3)}{75\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)\right)$	83

input
```
int((a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arccsc(c*x)-1/75*(c^2*x^2-1)*(8*c^4*x^4+4*c
^2*x^2+3)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^6/x^6)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.61

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{15\,b\,\mathrm{arccsc}\left(cx\right)+\left(8\,bc^4x^4+4\,bc^2x^2+3\,b\right)\sqrt{c^2x^2-1}+15\,a}{75\,x^5}$$

input
```
integrate((a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*(15*b*arccsc(c*x) + (8*b*c^4*x^4 + 4*b*c^2*x^2 + 3*b)*sqrt(c^2*x^2 -
 1) + 15*a)/x^5
```

3.13.6 Sympy [A] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.93

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{b\,\mathrm{acsc}\left(cx\right)}{5x^5} - \frac{b\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for}\ \left|c^2x^2\right|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$

input
```
integrate((a+b*acsc(c*x))/x**6,x)
```

output
```
-a/(5*x**5) - b*acsc(c*x)/(5*x**5) - b*Piecewise((8*c**5*sqrt(c**2*x**2 -
1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(
5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*
c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), T
rue))/(5*c)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{1}{75}\,b\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\left(cx\right)}{x^5}\right) - \frac{a}{5\,x^5}$$

input
```
integrate((a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/75*b*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2)
 + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) - 1/5*a/x^5
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(68) = 136.

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.82

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = -\frac{1}{75}\left(3\,bc^4\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-10\,bc^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{15\,bc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+15\,bc^4\sqrt{}\right.$$

input
```
integrate((a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/75*(3*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 10*b*c^4*(-1/(
c^2*x^2) + 1)^(3/2) + 15*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 15*
b*c^4*sqrt(-1/(c^2*x^2) + 1) + 30*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/
x + 15*b*c^3*arcsin(1/(c*x))/x + 15*a/(c*x^5))*c
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^6}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^6}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/x^6,x)
```

output
```
int((a + b*asin(1/(c*x)))/x^6, x)
```

3.14 $\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx$

3.14.1 Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx=-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{36x^5}-\frac{5bc^3\sqrt{1-\frac{1}{c^2x^2}}}{144x^3}-\frac{5bc^5\sqrt{1-\frac{1}{c^2x^2}}}{96x}+\frac{5}{96}bc^6\csc^{-1}(cx)-\frac{a+b\csc^{-1}(cx)}{6x^6}$$

output
```
5/96*b*c^6*arccsc(c*x)+1/6*(-a-b*arccsc(c*x))/x^6-1/36*b*c*(1-1/c^2/x^2)^(
1/2)/x^5-5/144*b*c^3*(1-1/c^2/x^2)^(1/2)/x^3-5/96*b*c^5*(1-1/c^2/x^2)^(1/2
)/x
```

3.14.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx=-\frac{a}{6x^6}+b\left(-\frac{c}{36x^5}-\frac{5c^3}{144x^3}-\frac{5c^5}{96x}\right)\sqrt{\frac{-1+c^2x^2}{c^2x^2}}-\frac{b\csc^{-1}(cx)}{6x^6}+\frac{5}{96}bc^6\arcsin\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcCsc[c*x])/x^7,x]
```

output
```
-1/6*a/x^6 + b*(-1/36*c/x^5 - (5*c^3)/(144*x^3) - (5*c^5)/(96*x))*Sqrt[(-1
 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsc[c*x])/(6*x^6) + (5*b*c^6*ArcSin[1/(c*x)
])/96
```

3.14.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5744, 858, 262, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}dx$$

$$\Big\downarrow \text{5744}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^8}dx}{6c}-\frac{a+b\csc^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{858}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^6}d\frac{1}{x}}{6c}-\frac{a+b\csc^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{262}$$

$$\frac{b\left(\frac{5}{6}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^4}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}-\frac{a+b\csc^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{262}$$

$$\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}-\frac{a+b\csc^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{262}$$

$$\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}-\frac{a+b\csc^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{223}$$

$$\frac{b\left(\frac{5}{6}c^2\left(\frac{3}{4}c^2\left(\frac{1}{2}c^3\arcsin\left(\frac{1}{cx}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{2x}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{4x^3}\right)-\frac{c^2\sqrt{1-\frac{1}{c^2x^2}}}{6x^5}\right)}{6c}-\frac{a+b\csc^{-1}(cx)}{6x^6}$$

input
```
Int[(a + b*ArcCsc[c*x])/x^7,x]
```

output
```
-1/6*(a + b*ArcCsc[c*x])/x^6 + (b*(-1/6*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^5 +
(5*c^2*(-1/4*(c^2*Sqrt[1 - 1/(c^2*x^2)])/x^3 + (3*c^2*(-1/2*(c^2*Sqrt[1 -
1/(c^2*x^2)])/x + (c^3*ArcSin[1/(c*x)])/2))/4))/6))/(6*c)
```

3.14.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.14.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

method	result
parts	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arccsc}(cx)}{6x^6}+\frac{5bc^5\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{5bc^3(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^3}-\frac{5bc(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^5}-\frac{b(c^2x^2-}{36c\sqrt{\frac{c^2x^2}{c^2x}}}$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6}-\frac{b\,\mathrm{arccsc}(cx)}{6c^6x^6}+\frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{5b(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}-\frac{5b(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}-\frac{}{3}\right.$
default	$c^6\left(-\frac{a}{6c^6x^6}-\frac{b\,\mathrm{arccsc}(cx)}{6c^6x^6}+\frac{5b\sqrt{c^2x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}-\frac{5b(c^2x^2-1)}{96\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}-\frac{5b(c^2x^2-1)}{144\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^5x^5}-\frac{}{3}\right.$

input
```
int((a+b*arccsc(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6-1/6*b/x^6*arccsc(c*x)+5/96*b*c^5*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*arctan(1/(c^2*x^2-1)^(1/2))-5/96*b*c^3*(c^2*x^2-1)/((c^2
*x^2-1)/c^2/x^2)^(1/2)/x^3-5/144*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/
2)/x^5-1/36*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^7
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = \frac{3\left(5\,bc^6x^6-16\,b\right)\mathrm{arccsc}\left(cx\right)-\left(15\,bc^4x^4+10\,bc^2x^2+8\,b\right)\sqrt{c^2x^2-1}-48\,a}{288\,x^6}$$

input
```
integrate((a+b*arccsc(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/288*(3*(5*b*c^6*x^6 - 16*b)*arccsc(c*x) - (15*b*c^4*x^4 + 10*b*c^2*x^2 +
 8*b)*sqrt(c^2*x^2 - 1) - 48*a)/x^6
```

3.14.6 Sympy [A] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.41

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} - \frac{b\,\mathrm{acsc}\,(cx)}{6x^6} - \frac{b\left(\begin{cases} \frac{5ic^7\,\mathrm{acosh}\left(\frac{1}{cx}\right)}{16} - \frac{5ic^6}{16x\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{5ic^4}{48x^3\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{ic^2}{24x^5\sqrt{-1+\frac{1}{c^2x^2}}} + \frac{i}{6x^7\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for } \frac{1}{|c^2x^2|} > 1 \\ -\frac{5c^7\,\mathrm{asin}\left(\frac{1}{cx}\right)}{16} + \frac{5c^6}{16x\sqrt{1-\frac{1}{c^2x^2}}} - \frac{5c^4}{48x^3\sqrt{1-\frac{1}{c^2x^2}}} - \frac{c^2}{24x^5\sqrt{1-\frac{1}{c^2x^2}}} - \frac{1}{6x^7\sqrt{1-\frac{1}{c^2x^2}}} & \text{otherwise} \end{cases}\right)}{6c}$$

input
```
integrate((a+b*acsc(c*x))/x**7,x)
```

output
```
-a/(6*x**6) - b*acsc(c*x)/(6*x**6) - b*Piecewise((5*I*c**7*acosh(1/(c*x))/
16 - 5*I*c**6/(16*x*sqrt(-1 + 1/(c**2*x**2))) + 5*I*c**4/(48*x**3*sqrt(-1
+ 1/(c**2*x**2))) + I*c**2/(24*x**5*sqrt(-1 + 1/(c**2*x**2))) + I/(6*x**7*
sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1), (-5*c**7*asin(1/(c*x))/1
6 + 5*c**6/(16*x*sqrt(1 - 1/(c**2*x**2))) - 5*c**4/(48*x**3*sqrt(1 - 1/(c*
*2*x**2))) - c**2/(24*x**5*sqrt(1 - 1/(c**2*x**2))) - 1/(6*x**7*sqrt(1 - 1
/(c**2*x**2))), True))/(6*c)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.63

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = -\frac{1}{288}b\left(\frac{15\,c^7\arctan\left(cx\sqrt{-\frac{1}{c^2x^2}+1}\right) - \frac{15\,c^{12}x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+40\,c^{10}x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\,c^8x\sqrt{-\frac{1}{c^2x^2}+1}}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3-3\,c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2+3\,c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1}}{c} + \frac{48\,\mathrm{arccsc}}{x^6}\right. - \frac{a}{6\,x^6}$$

input
```
integrate((a+b*arccsc(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/288*b*((15*c^7*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) - (15*c^12*x^5*(-1/(c
^2*x^2) + 1)^(5/2) + 40*c^10*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(
-1/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^2*x^2) - 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) -
 1)^2 + 3*c^2*x^2*(1/(c^2*x^2) - 1) - 1))/c + 48*arccsc(c*x)/x^6) - 1/6*a/
x^6
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(85) = 170.

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.72

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx =$$

$$-\frac{1}{288}\left(48\,bc^5\left(\frac{1}{c^2x^2}-1\right)^3\arcsin\left(\frac{1}{cx}\right)+144\,bc^5\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+144\,bc^5\left(\frac{1}{c^2x^2}-1\right)\text{arcsi}\right.$$

input
```
integrate((a+b*arccsc(c*x))/x^7,x, algorithm="giac")
```

output
```
-1/288*(48*b*c^5*(1/(c^2*x^2) - 1)^3*arcsin(1/(c*x)) + 144*b*c^5*(1/(c^2*x
^2) - 1)^2*arcsin(1/(c*x)) + 144*b*c^5*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) +
 33*b*c^5*arcsin(1/(c*x)) + 8*b*c^4*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2)
+ 1)/x - 26*b*c^4*(-1/(c^2*x^2) + 1)^(3/2)/x + 33*b*c^4*sqrt(-1/(c^2*x^2)
+ 1)/x + 48*a/(c*x^6))*c
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^7}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^7}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/x^7,x)
```

output
```
int((a + b*asin(1/(c*x)))/x^7, x)
```

3.15 $\int x^3(a+b\csc^{-1}(cx))^2\,dx$

3.15.1 Optimal result

Integrand size = 14, antiderivative size = 107

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{b^2x^2}{12c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x(a+b\csc^{-1}(cx))}{3c^3}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x^3(a+b\csc^{-1}(cx))}{6c}+\frac{1}{4}x^4\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{3c^4}$$

output
```
1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arccsc(c*x))^2+1/3*b^2*ln(x)/c^4+1/3*b*x*(a+
b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/c^3+1/6*b*x^3*(a+b*arccsc(c*x))*(1-1/c^
2/x^2)^(1/2)/c
```

3.15.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.16

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx$$
$$=\frac{cx\left(b^2cx+3a^2c^3x^3+2ab\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)\right)+2bcx\left(3ac^3x^3+b\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)\right)\csc^{-1}(cx)+3b^2c}{12c^4}$$

input
```
Integrate[x^3*(a + b*ArcCsc[c*x])^2,x]
```

output
```
(c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))
 + 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2))*ArcCsc[c*
x] + 3*b^2*c^4*x^4*ArcCsc[c*x]^2 + 4*b^2*Log[x])/(12*c^4)
```

3.15.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {5746, 4910, 3042, 4673, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \csc^{-1}(cx)\right)^2 \, dx$$

$$\Big\downarrow \text{5746}$$

$$-\frac{\int c^5 \sqrt{1-\frac{1}{c^2x^2}} x^5 \left(a+b\csc^{-1}(cx)\right)^2 d\csc^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{4910}$$

$$-\frac{\frac{1}{2} b \int c^4 x^4 \left(a+b\csc^{-1}(cx)\right) d\csc^{-1}(cx)-\frac{1}{4} c^4 x^4 \left(a+b\csc^{-1}(cx)\right)^2}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{2} b \int \left(a+b\csc^{-1}(cx)\right) \csc\left(\csc^{-1}(cx)\right)^4 d\csc^{-1}(cx)-\frac{1}{4} c^4 x^4 \left(a+b\csc^{-1}(cx)\right)^2}{c^4}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{\frac{1}{2} b \left(\frac{2}{3} \int c^2 x^2 \left(a+b\csc^{-1}(cx)\right) d\csc^{-1}(cx)-\frac{1}{3} c^3 x^3 \sqrt{1-\frac{1}{c^2x^2}} \left(a+b\csc^{-1}(cx)\right)-\frac{1}{6} b c^2 x^2\right)-\frac{1}{4} c^4 x^4 \left(a+b\csc^{-1}(c\right.}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{2} b \left(\frac{2}{3} \int \left(a+b\csc^{-1}(cx)\right) \csc\left(\csc^{-1}(cx)\right)^2 d\csc^{-1}(cx)-\frac{1}{3} c^3 x^3 \sqrt{1-\frac{1}{c^2x^2}} \left(a+b\csc^{-1}(cx)\right)-\frac{1}{6} b c^2 x^2\right)-\frac{1}{4} c^4 x^4 \left(a\right.}{c^4}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\left(b\int c\sqrt{1-\frac{1}{c^2x^2}}xd\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)-\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))-\frac{1}{6}bc}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\left(b\int -\tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)-\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(c}{c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\left(-b\int \tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)-\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(c}{c^4}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\left(b\log\left(\frac{1}{cx}\right)-cx\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)-\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))-\frac{1}{6}bc^2x^2\right)-\frac{1}{4}c^4x^4(a+}{c^4}$$

input
```
Int[x^3*(a + b*ArcCsc[c*x])^2,x]
```

output
```
-((-1/4*(c^4*x^4*(a + b*ArcCsc[c*x])^2) + (b*(-1/6*(b*c^2*x^2) - (c^3*Sqrt
[1 - 1/(c^2*x^2)]*x^3*(a + b*ArcCsc[c*x]))/3 + (2*(-(c*Sqrt[1 - 1/(c^2*x^2
)]*x*(a + b*ArcCsc[c*x])) + b*Log[1/(c*x)]))/3))/2)/c^4)
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.15.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.59

method	result
parts	$\frac{a^2x^4}{4}+\frac{b^2\left(\frac{\operatorname{arccsc}(cx)^2c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}+\frac{\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)}{c^4}+\frac{2ab\left(\frac{c^4x^4\operatorname{arccsc}(c\ldots}{4}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{\operatorname{arccsc}(cx)^2c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}+\frac{\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)+2ab\left(\frac{c^4x^4\operatorname{arccsc}(cx)}{4}+\ldots\right.}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{\operatorname{arccsc}(cx)^2c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{6}+\frac{c^2x^2}{12}+\frac{\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}-\frac{\ln\left(\frac{1}{cx}\right)}{3}\right)+2ab\left(\frac{c^4x^4\operatorname{arccsc}(cx)}{4}+\ldots\right.}{c^4}$

input
```
int(x^3*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+b^2/c^4*(1/4*arccsc(c*x)^2*c^4*x^4+1/6*arccsc(c*x)*((c^2*x^2-1
)/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2+1/3*arccsc(c*x)*c*x*((c^2*x^2-1)/c^2
/x^2)^(1/2)-1/3*ln(1/c/x))+2*a*b/c^4*(1/4*c^4*x^4*arccsc(c*x)+1/12*(c^2*x^
2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx$$
$$=\frac{3\,b^2c^4x^4\operatorname{arccsc}\left(cx\right)^2+3\,a^2c^4x^4-12\,abc^4\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+b^2c^2x^2+4\,b^2\log\left(x\right)+6\left(abc^4x^4-\right.}{12\,c^4}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*c^4*x^4*arccsc(c*x)^2 + 3*a^2*c^4*x^4 - 12*a*b*c^4*arctan(-c*x
 + sqrt(c^2*x^2 - 1)) + b^2*c^2*x^2 + 4*b^2*log(x) + 6*(a*b*c^4*x^4 - a*b*
c^4)*arccsc(c*x) + 2*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)*arccsc(c
*x))*sqrt(c^2*x^2 - 1))/c^4
```

3.15.6 Sympy [F]

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int x^3(a+b\operatorname{acsc}\left(cx\right))^2\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))**2,x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))**2, x)
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(93) = 186$.

Time = 0.43 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.84

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx = \frac{1}{4}\,b^2x^4\operatorname{arccsc}\left(cx\right)^2 + \frac{1}{4}\,a^2x^4 + \frac{1}{6}\left(3\,x^4\operatorname{arccsc}\left(cx\right) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)ab + \frac{\left(2\,c^4x^4\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right) + 2\,c^2x^2\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right) + \left(c^2x^2+2\,\log\left(x^2\right)\right)\sqrt{cx+1}\right.}{12\,\sqrt{cx+1}\sqrt{cx-1}c^4}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arccsc(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arccsc(c*x) + (c^2*x^
3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a*b + 1/12*(
2*c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*c^2*x^2*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1)) + (c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*
x - 1) - 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqr
t(c*x - 1)*c^4)
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(93) = 186$.

Time = 0.43 (sec) , antiderivative size = 811, normalized size of antiderivative = 7.58

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^2\,dx = \frac{1}{192}\left(\frac{3\,b^2x^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)^2}{c} + \frac{6\,abx^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c} + \frac{3\,a^2x^4\left(\sqrt{-\frac{1}{c^2x^2}\right.}{c}\right.$$

input
```
integrate(x^3*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

output
```
1/192*(3*b^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))^2/c + 6*a*
b*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a^2*x^4*(sqrt(-
1/(c^2*x^2) + 1) + 1)^4/c + 4*b^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsi
n(1/(c*x))/c^2 + 4*a*b*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 12*b^2*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))^2/c^3 + 24*a*b*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a^2*x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2/c^3 + 4*b^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 36*b^2*
x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^4 + 36*a*b*x*(sqrt(-1/(c^
2*x^2) + 1) + 1)/c^4 + 18*b^2*arcsin(1/(c*x))^2/c^5 + 36*a*b*arcsin(1/(c*x
))/c^5 - 128*b^2*log(2)/c^5 + 64*b^2*log(2*sqrt(-1/(c^2*x^2) + 1) + 2)/c^5
 - 64*b^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^5 - 64*b^2*log(1/(abs(c)*abs(x
)))/c^5 + 18*a^2/c^5 + 8*b^2/c^5 - 36*b^2*arcsin(1/(c*x))/(c^6*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)) - 36*a*b/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*b^
2*arcsin(1/(c*x))^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*a*b*arcs
in(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 12*a^2/(c^7*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2) + 4*b^2/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)
^2) - 4*b^2*arcsin(1/(c*x))/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 4*a
*b/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3*b^2*arcsin(1/(c*x))^2/(c^9
*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 6*a*b*arcsin(1/(c*x))/(c^9*x^4*(sqr
t(-1/(c^2*x^2) + 1) + 1)^4) + 3*a^2/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + ...
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

input
```
int(x^3*(a + b*asin(1/(c*x)))^2,x)
```

output
```
int(x^3*(a + b*asin(1/(c*x)))^2, x)
```

3.16 $\int x^2(a+b\csc^{-1}(cx))^2\,dx$

3.16.1 Optimal result

Integrand size = 14, antiderivative size = 139

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{b^2x}{3c^2}+\frac{b\sqrt{1-\frac{1}{c^2x^2}}x^2(a+b\csc^{-1}(cx))}{3c}+\frac{1}{3}x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2b(a+b\csc^{-1}(cx))\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{3c^3}+\frac{ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{3c^3}$$

output
```
1/3*b^2*x/c^2+1/3*x^3*(a+b*arccsc(c*x))^2+2/3*b*(a+b*arccsc(c*x))*arctanh(
I/c/x+(1-1/c^2/x^2)^(1/2))/c^3-1/3*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1
/2))/c^3+1/3*I*b^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+1/3*b*x^2*(a+b
*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/c
```

3.16.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.53

$$\int x^2 \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$
$$= \frac{1}{3}\left(a^2x^3+2abx^3\csc^{-1}(cx)+\frac{ab\left(-cx+c^3x^3-\sqrt{-1+c^2x^2}\log\left(-cx+\sqrt{-1+c^2x^2}\right)\right)}{c^4\sqrt{1-\frac{1}{c^2x^2}}x}\right.$$
$$-\frac{ib^2\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c^3}$$
$$+\frac{b^2\left(cx+c^3x^3\csc^{-1}(cx)^2+\csc^{-1}(cx)\left(c^2\sqrt{1-\frac{1}{c^2x^2}}x^2-\log\left(1-e^{i\csc^{-1}(cx)}\right)+\log\left(1+e^{i\csc^{-1}(cx)}\right)\right)+\right.}{c^3}$$

input
```
Integrate[x^2*(a + b*ArcCsc[c*x])^2,x]
```

output
```
(a^2*x^3 + 2*a*b*x^3*ArcCsc[c*x] + (a*b*(-(c*x) + c^3*x^3 - Sqrt[-1 + c^2*
x^2]*Log[-(c*x) + Sqrt[-1 + c^2*x^2]]))/(c^4*Sqrt[1 - 1/(c^2*x^2)]*x) - (I
*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])])/c^3 + (b^2*(c*x + c^3*x^3*ArcCsc[c*x]
^2 + ArcCsc[c*x]*(c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2 - Log[1 - E^(I*ArcCsc[c*x]
)] + Log[1 + E^(I*ArcCsc[c*x])]) + I*PolyLog[2, E^(I*ArcCsc[c*x])]))/c^3)/
3
```

3.16.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5746, 4910, 3042, 4673, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$
$$\Big\downarrow \; 5746$$
$$-\frac{\int c^4\sqrt{1-\frac{1}{c^2x^2}}x^4\left(a+b\csc^{-1}(cx)\right)^2 d\csc^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{4910}$$

$$-\frac{\frac{2}{3}b\int c^3x^3\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^2}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2}{3}b\int \left(a+b\csc^{-1}(cx)\right)\csc\left(\csc^{-1}(cx)\right)^3d\csc^{-1}(cx)-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^2}{c^3}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{\frac{2}{3}b\left(\frac{1}{2}\int cx\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)-\frac{bcx}{2}\right)-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^2}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{2}{3}b\left(\frac{1}{2}\int \left(a+b\csc^{-1}(cx)\right)\csc\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)-\frac{bcx}{2}\right)-\frac{1}{3}c^3x^3\left(a+b\right.}{c^3}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2}{3}b\left(\frac{1}{2}\left(-b\int\log\left(1-e^{i\csc^{-1}(cx)}\right)d\csc^{-1}(cx)+b\int\log\left(1+e^{i\csc^{-1}(cx)}\right)d\csc^{-1}(cx)\right.\right.}{c^3}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2}{3}b\left(\frac{1}{2}\left(ib\int e^{-i\csc^{-1}(cx)}\log\left(1-e^{i\csc^{-1}(cx)}\right)de^{i\csc^{-1}(cx)}-ib\int e^{-i\csc^{-1}(cx)}\log\left(1+e\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^2+\frac{2}{3}b\left(\frac{1}{2}\left(-2\text{arctanh}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)+ib\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)-ib\right.\right.}{c^3}$$

input
```
Int[x^2*(a + b*ArcCsc[c*x])^2,x]
```

output
```
-((-1/3*(c^3*x^3*(a + b*ArcCsc[c*x])^2) + (2*b*(-1/2*(b*c*x) - (c^2*Sqrt[1
 - 1/(c^2*x^2)]*x^2*(a + b*ArcCsc[c*x]))/2 + (-2*(a + b*ArcCsc[c*x])*ArcTa
nh[E^(I*ArcCsc[c*x])] + I*b*PolyLog[2, -E^(I*ArcCsc[c*x])] - I*b*PolyLog[2
, E^(I*ArcCsc[c*x])])/2))/3)/c^3)
```

3.16. $\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx$

3.16.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.16.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.93

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{\left(c^2x^2\operatorname{arccsc}(cx)^2+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}-\frac{\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{3}+\frac{i\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right.}{3}\cdots}{c^3}$
derivativedivides	$\frac{\frac{c^3x^3a^2}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arccsc}(cx)^2+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}-\frac{\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{3}+\frac{i\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right.}{3}\cdots}{\cdots}$
default	$\frac{\frac{c^3x^3a^2}{3}+b^2\left(\frac{\left(c^2x^2\operatorname{arccsc}(cx)^2+\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+1\right)cx}{3}-\frac{\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{3}+\frac{i\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right.}{3}\cdots}{\cdots}$

input
```
int(x^2*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+b^2/c^3*(1/3*(c^2*x^2*arccsc(c*x)^2+arccsc(c*x)*c*x*((c^2*x^2-
1)/c^2/x^2)^(1/2)+1)*c*x-1/3*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+1
/3*I*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+1/3*arccsc(c*x)*ln(1+I/c/x+(1-1/
c^2/x^2)^(1/2))-1/3*I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))+2*a*b/c^3*(1/
3*c^3*x^3*arccsc(c*x)+1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x^2-1)^(1/2)+ln(c*x+
(c^2*x^2-1)^(1/2)))/((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x)
```

3.16.5 Fricas [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int(b\operatorname{arccsc}(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arccsc(c*x)^2 + 2*a*b*x^2*arccsc(c*x) + a^2*x^2, x)
```

3.16.6 Sympy [F]

$$\int x^2 \left(a + b \csc^{-1}(cx)\right)^2 \, dx = \int x^2 (a + b \operatorname{acsc}(cx))^2 \, dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))**2, x)
```

3.16.7 Maxima [F]

$$\int x^2 \left(a + b \csc^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arccsc}(cx) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/6*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(
c^2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/12*(4*x^3*arctan2(1, sqrt(c*x + 1)*sqrt
(c*x - 1))^2 - x^3*log(c^2*x^2)^2 - 2*c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c
*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 + 36*c^2*integrate(1/3*x^4*log(
c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 72*c^2*integrate(1/3*x^4*log(x)/(c^2*x
^2 - 1), x)*log(c) + 36*c^2*integrate(1/3*x^4*log(c^2*x^2)*log(x)/(c^2*x^2
 - 1), x) - 36*c^2*integrate(1/3*x^4*log(x)^2/(c^2*x^2 - 1), x) + 12*c^2*i
ntegrate(1/3*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x) + 6*(2*x/c^2 - log(c*x + 1
)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 36*integrate(1/3*x^2*log(c^2*x^2)/(c^
2*x^2 - 1), x)*log(c) + 72*integrate(1/3*x^2*log(x)/(c^2*x^2 - 1), x)*log(
c) + 24*integrate(1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*x^2*arctan(1/(sqrt(c*x +
 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x) - 36*integrate(1/3*x^2*log(c^2*x^2)*
log(x)/(c^2*x^2 - 1), x) + 36*integrate(1/3*x^2*log(x)^2/(c^2*x^2 - 1), x)
 - 12*integrate(1/3*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2
```

3.16.8 Giac [F]

$$\int x^2 \left(a+b \csc^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arccsc}(cx)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^2*x^2, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b \csc^{-1}(cx)\right)^2 \, dx = \int x^2 \left(a+b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

input
```
int(x^2*(a + b*asin(1/(c*x)))^2,x)
```

output
```
int(x^2*(a + b*asin(1/(c*x)))^2, x)
```

3.17 $\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 55

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

output
```
1/2*x^2*(a+b*arccsc(c*x))^2+b^2*ln(x)/c^2+b*x*(a+b*arccsc(c*x))*(1-1/c^2/x
^2)^(1/2)/c
```

3.17.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx$$
$$=\frac{acx\left(2b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)+2bcx\left(b\sqrt{1-\frac{1}{c^2x^2}}+acx\right)\csc^{-1}(cx)+b^2c^2x^2\csc^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

input
```
Integrate[x*(a + b*ArcCsc[c*x])^2,x]
```

output
```
(a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(b*Sqrt[1 - 1/(c^2*x^
2)] + a*c*x)*ArcCsc[c*x] + b^2*c^2*x^2*ArcCsc[c*x]^2 + 2*b^2*Log[c*x])/(2*
c^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5746, 4910, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\csc^{-1}(cx)\right)^2 dx$$

$$\Big\downarrow \text{5746}$$

$$-\frac{\int c^3\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{4910}$$

$$-\frac{b\int c^2x^2\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \left(a+b\csc^{-1}(cx)\right)\csc\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{b\left(b\int c\sqrt{1-\frac{1}{c^2x^2}}xd\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)\right)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(b\int -\tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)\right)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\left(-b\int \tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)\right)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\left(b\log\left(\frac{1}{cx}\right)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)\right)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^2}{c^2}$$

input
```
Int[x*(a + b*ArcCsc[c*x])^2,x]
```

output
```
-((-1/2*(c^2*x^2*(a + b*ArcCsc[c*x])^2) + b*(-(c*Sqrt[1 - 1/(c^2*x^2)]*x*(
a + b*ArcCsc[c*x])) + b*Log[1/(c*x)]))/c^2)
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(51) = 102$.

Time = 1.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.22

method	result	size
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{c^2x^2\,\mathrm{arccsc}(cx)^2}{2}+\mathrm{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)}{c^2}+\frac{2ab\left(\frac{c^2x^2\,\mathrm{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	122
derivativedivides	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\,\mathrm{arccsc}(cx)^2}{2}+\mathrm{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)+2ab\left(\frac{c^2x^2\,\mathrm{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	123
default	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\,\mathrm{arccsc}(cx)^2}{2}+\mathrm{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}-\ln\left(\frac{1}{cx}\right)\right)+2ab\left(\frac{c^2x^2\,\mathrm{arccsc}(cx)}{2}+\frac{c^2x^2-1}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}\right)}{c^2}$	123

input
```
int(x*(a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arccsc(c*x)^2+arccsc(c*x)*c*x*((c^2*x^2-1
)/c^2/x^2)^(1/2)-ln(1/c/x))+2*a*b/c^2*(1/2*c^2*x^2*arccsc(c*x)+1/2/((c^2*x
^2-1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2-1))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(51) = 102$.

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx$$
$$=\frac{b^2c^2x^2\,\mathrm{arccsc}\,(cx)^2+a^2c^2x^2-4\,abc^2\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\,b^2\log\left(x\right)+2\left(abc^2x^2-abc^2\right)\mathrm{arccsc}\,(}{2\,c^2}$$

input
```
integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2*arccsc(c*x)^2 + a^2*c^2*x^2 - 4*a*b*c^2*arctan(-c*x + sqr
t(c^2*x^2 - 1)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 - a*b*c^2)*arccsc(c*x) + 2
*sqrt(c^2*x^2 - 1)*(b^2*arccsc(c*x) + a*b))/c^2
```

3.17.6 Sympy [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int x(a+b\operatorname{acsc}(cx))^2\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))**2,x)
```

output
```
Integral(x*(a + b*acsc(c*x))**2, x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx=\frac{1}{2}\,b^2x^2\operatorname{arccsc}(cx)^2+\frac{1}{2}\,a^2x^2+\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)ab+\left(\frac{x\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)}{c}+\frac{\log(x)}{c^2}\right)b^2$$

input
```
integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arccsc(c*x)^2 + 1/2*a^2*x^2 + (x^2*arccsc(c*x) + x*sqrt(-1/(c^
2*x^2) + 1)/c)*a*b + (x*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x)/c + log(x)/c^2)
*b^2
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(51) = 102.

Time = 0.35 (sec) , antiderivative size = 427, normalized size of antiderivative = 7.76

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx$$

$$=\frac{1}{8}\left(\frac{b^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)^2}{c}+\frac{2\,abx^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{a^2x^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right.}{c}\right.$$

input
```
integrate(x*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

output
```
1/8*(b^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))^2/c + 2*a*b*x^
2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a^2*x^2*(sqrt(-1/(c^2
*x^2) + 1) + 1)^2/c + 4*b^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))
/c^2 + 4*a*b*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 2*b^2*arcsin(1/(c*x))^2/
c^3 + 4*a*b*arcsin(1/(c*x))/c^3 - 16*b^2*log(2)/c^3 + 8*b^2*log(2*sqrt(-1/
(c^2*x^2) + 1) + 2)/c^3 - 8*b^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 - 8*b^
2*log(1/(abs(c)*abs(x)))/c^3 + 2*a^2/c^3 - 4*b^2*arcsin(1/(c*x))/(c^4*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)) - 4*a*b/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
 b^2*arcsin(1/(c*x))^2/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 2*a*b*ar
csin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a^2/(c^5*x^2*(sqr
t(-1/(c^2*x^2) + 1) + 1)^2))*c
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\csc^{-1}(cx)\right)^2\,dx=\int x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

input
```
int(x*(a + b*asin(1/(c*x)))^2,x)
```

output
```
int(x*(a + b*asin(1/(c*x)))^2, x)
```

3.18 $\int (a+b\csc^{-1}(cx))^2\,dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 84

$$\int (a+b\csc^{-1}(cx))^2\,dx = x(a+b\csc^{-1}(cx))^2+\frac{4b(a+b\csc^{-1}(cx))\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{2ib^2\operatorname{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c}+\frac{2ib^2\operatorname{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c}$$

output
```
x*(a+b*arccsc(c*x))^2+4*b*(a+b*arccsc(c*x))*arctanh(I/c/x+(1-1/c^2/x^2)^(1
/2))/c-2*I*b^2*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c+2*I*b^2*polylog(2,I
/c/x+(1-1/c^2/x^2)^(1/2))/c
```

3.18.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.75

$$\int (a+b\csc^{-1}(cx))^2\,dx = \frac{a^2cx+2abcx\csc^{-1}(cx)+b^2cx\csc^{-1}(cx)^2-2b^2\csc^{-1}(cx)\log\left(1-e^{i\csc^{-1}(cx)}\right)+2b^2\csc^{-1}(cx)\log\left(1+e^{i\csc^{-1}(cx)}\right)\ldots}{\ldots}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^2,x]
```

output
```
(a^2*c*x + 2*a*b*c*x*ArcCsc[c*x] + b^2*c*x*ArcCsc[c*x]^2 - 2*b^2*ArcCsc[c*
x]*Log[1 - E^(I*ArcCsc[c*x])] + 2*b^2*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x]
)] + 2*a*b*Log[Cos[ArcCsc[c*x]/2]] - 2*a*b*Log[Sin[ArcCsc[c*x]/2]] - (2*I)
*b^2*PolyLog[2, -E^(I*ArcCsc[c*x])] + (2*I)*b^2*PolyLog[2, E^(I*ArcCsc[c*x
])])/c
```

3.18.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5740, 4910, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx$$

$$\Big\downarrow \text{5740}$$

$$-\frac{\int c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)}{c}$$

$$\Big\downarrow \text{4910}$$

$$-\frac{2b\int cx\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-cx\left(a+b\csc^{-1}(cx)\right)^2}{c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\int \left(a+b\csc^{-1}(cx)\right)\csc\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)-cx\left(a+b\csc^{-1}(cx)\right)^2}{c}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-cx\left(a+b\csc^{-1}(cx)\right)^2+2b\left(-b\int \log\left(1-e^{i\csc^{-1}(cx)}\right)d\csc^{-1}(cx)+b\int \log\left(1+e^{i\csc^{-1}(cx)}\right)d\csc^{-1}(cx)-2\text{arc}}{c}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{-cx\left(a+b\csc^{-1}(cx)\right)^2+2b\left(ib\int e^{-i\csc^{-1}(cx)}\log\left(1-e^{i\csc^{-1}(cx)}\right)de^{i\csc^{-1}(cx)}-ib\int e^{-i\csc^{-1}(cx)}\log\left(1+e^{i\csc^{-1}(}\right.\right.}{c}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-cx\left(a+b\csc^{-1}(cx)\right)^2+2b\left(-2\operatorname{arctanh}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)+ib\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)-ib\,\text{PolyL}\right.}{c}$$

input
```
Int[(a + b*ArcCsc[c*x])^2,x]
```

output
```
-((-(c*x*(a + b*ArcCsc[c*x])^2) + 2*b*(-2*(a + b*ArcCsc[c*x])*ArcTanh[E^(I
*ArcCsc[c*x])] + I*b*PolyLog[2, -E^(I*ArcCsc[c*x])] - I*b*PolyLog[2, E^(I*
ArcCsc[c*x])]))/c)
```

3.18.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5740
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   Su
bst[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b,
c, n}, x] && IGtQ[n, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.10

method	result
derivativedivides	$\frac{a^2cx+b^2\left(\operatorname{arccsc}(cx)^2cx-2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+2\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-2i\operatorname{dilog}\left(1+\frac{i}{cx}+\sqrt{\right.}\right.}{c}$
default	$\frac{a^2cx+b^2\left(\operatorname{arccsc}(cx)^2cx-2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+2\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-2i\operatorname{dilog}\left(1+\frac{i}{cx}+\sqrt{\right.}\right.}{c}$
parts	$a^2x+\frac{b^2\left(\operatorname{arccsc}(cx)^2cx-2\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+2\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-2i\operatorname{dilog}\left(1+\frac{i}{cx}+\right.\right.}{c}$

input
```
int((a+b*arccsc(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*x+b^2*(arccsc(c*x)^2*c*x-2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)
^(1/2))+2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*I*dilog(1+I/c/x+(1
-1/c^2/x^2)^(1/2))+2*I*dilog(1-I/c/x-(1-1/c^2/x^2)^(1/2)))+2*a*b*(arccsc(c
*x)*c*x+ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))))
```

3.18.5 Fricas [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int \left(b\operatorname{arccsc}\left(cx\right)+a\right)^2dx$$

input
```
integrate((a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2, x)
```

3.18.6 Sympy [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int \left(a+b\,\mathrm{acsc}\,(cx)\right)^2\,dx$$

input
```
integrate((a+b*acsc(c*x))**2,x)
```

output
```
Integral((a + b*acsc(c*x))**2, x)
```

3.18.7 Maxima [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^2\,dx=\int \left(b\,\mathrm{arccsc}\,(cx)+a\right)^2\,dx$$

input
```
integrate((a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*(2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 - 4*c
^2*integrate(x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) + 8*c^2*integrate(x
^2*log(x)/(c^2*x^2 - 1), x)*log(c) - 4*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
 - 1))^2 - 4*c^2*integrate(x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) + 4*c
^2*integrate(x^2*log(x)^2/(c^2*x^2 - 1), x) - 4*c^2*integrate(x^2*log(c^2*
x^2)/(c^2*x^2 - 1), x) + x*log(c^2*x^2)^2 + 2*(log(c*x + 1)/c - log(c*x -
1)/c)*log(c)^2 + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 8*int
egrate(log(x)/(c^2*x^2 - 1), x)*log(c) - 8*integrate(sqrt(c*x + 1)*sqrt(c*
x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x) + 4*integ
rate(log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 4*integrate(log(x)^2/(c^2*x^2
 - 1), x) + 4*integrate(log(c^2*x^2)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c
*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2)
+ 1) + 1))*a*b/c
```

3.18.8 Giac [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^2 \, dx=\int (b \operatorname{arccsc}(cx)+a)^2 \, dx$$

input
```
integrate((a+b*arccsc(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^2, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\csc^{-1}(cx)\right)^2 \, dx=\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

input
```
int((a + b*asin(1/(c*x)))^2,x)
```

output
```
int((a + b*asin(1/(c*x)))^2, x)
```

3.19 $\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx = \frac{i(a+b\csc^{-1}(cx))^3}{3b} - (a+b\csc^{-1}(cx))^2\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ib(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right) - \frac{1}{2}b^2\operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(cx)}\right)$$

output
```
1/3*I*(a+b*arccsc(c*x))^3/b-(a+b*arccsc(c*x))^2*ln(1-(I/c/x+(1-1/c^2/x^2)^
(1/2))^2)+I*b*(a+b*arccsc(c*x))*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1
/2*b^2*polylog(3,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx = -2ab\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + a^2\log(cx) + iab\left(\csc^{-1}(cx)^2 + \operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right) + \frac{1}{24}ib^2\left(\pi^3 - 8\csc^{-1}(cx)^3 + 24i\csc^{-1}(cx)^2\log\left(1-e^{-2i\csc^{-1}(cx)}\right) - 24\csc^{-1}(cx)\operatorname{PolyLog}\left(2,e^{-2i\csc^{-1}(cx)}\right) + 12i\operatorname{PolyLog}\left(3,e^{-2i\csc^{-1}(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcCsc[c*x])^2/x,x]
```

output
```
-2*a*b*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a^2*Log[c*x] + I*a*b*(
ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/24)*b^2*(Pi^3 - 8*
ArcCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 24*
ArcCsc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[3, E^((-2*
I)*ArcCsc[c*x])])
```

3.19.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5746, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}dx$$

$$\Big\downarrow \text{ 5746}$$

$$-\int c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)$$

$$\Big\downarrow \text{ 3042}$$

$$-\int -\left(a+b\csc^{-1}(cx)\right)^2\tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)$$

$$\Big\downarrow \text{ 25}$$

$$\int \tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)$$

$$\Big\downarrow \text{ 4200}$$

$$\frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b}-2i\int -\frac{e^{2i\csc^{-1}(cx)}\left(a+b\csc^{-1}(cx)\right)^2}{1-e^{2i\csc^{-1}(cx)}}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{ 25}$$

$$2i\int \frac{e^{2i\csc^{-1}(cx)}\left(a+b\csc^{-1}(cx)\right)^2}{1-e^{2i\csc^{-1}(cx)}}d\csc^{-1}(cx)+\frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b}$$

$$\Big\downarrow \text{ 2620}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-ib\int\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)d\csc^{-1}(cx)\right)+\frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)-\frac{1}{2}ib\int\text{PolyL}\right.\right.$$
$$\frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b}$$

$$\Big\downarrow \text{2720}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)-\frac{1}{4}b\int e^{-2i\csc}\right.\right.$$
$$\frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b}$$

$$\Big\downarrow \text{7143}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)-\frac{1}{4}b\,\text{PolyLog}\right.\right.$$
$$\frac{i\left(a+b\csc^{-1}(cx)\right)^3}{3b}$$

input
```
Int[(a + b*ArcCsc[c*x])^2/x,x]
```

output
```
((I/3)*(a + b*ArcCsc[c*x])^3)/b + (2*I)*((I/2)*(a + b*ArcCsc[c*x])^2*Log[1
 - E^((2*I)*ArcCsc[c*x])] - I*b*((I/2)*(a + b*ArcCsc[c*x])*PolyLog[2, E^((
2*I)*ArcCsc[c*x])] - (b*PolyLog[3, E^((2*I)*ArcCsc[c*x])])/4))
```

3.19. $\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x}\,dx$

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.19. $\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.19.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(127) = 254.

Time = 1.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.71

method	result
parts	$a^2 \ln(x) + b^2\left(\frac{i \operatorname{arccsc}(cx)^3}{3} - \operatorname{arccsc}(cx)^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2i \operatorname{arccsc}(cx) \operatorname{polylo}\right.$
derivativedivides	$a^2 \ln(cx) + b^2\left(\frac{i \operatorname{arccsc}(cx)^3}{3} - \operatorname{arccsc}(cx)^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2i \operatorname{arccsc}(cx) \operatorname{polyl}\right.$
default	$a^2 \ln(cx) + b^2\left(\frac{i \operatorname{arccsc}(cx)^3}{3} - \operatorname{arccsc}(cx)^2 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 2i \operatorname{arccsc}(cx) \operatorname{polyl}\right.$

input
```
int((a+b*arccsc(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(1/3*I*arccsc(c*x)^3-arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2)^
(1/2))+2*I*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-2*polylog(3,I/
c/x+(1-1/c^2/x^2)^(1/2))-arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+2*I
*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-2*polylog(3,-I/c/x-(1-1
/c^2/x^2)^(1/2)))+2*a*b*(1/2*I*arccsc(c*x)^2-arccsc(c*x)*ln(1-I/c/x-(1-1/c
^2/x^2)^(1/2))+I*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-arccsc(c*x)*ln(1+I/c
/x+(1-1/c^2/x^2)^(1/2))+I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))
```

3.19.5 Fricas [F]

$$\int \frac{(a + b \csc^{-1}(cx))^2}{x} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)^2}{x} \, dx$$

input
```
integrate((a+b*arccsc(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2)/x, x)
```

3.19.6 Sympy [F]

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x}\,dx = \int \frac{(a + b\,\mathrm{acsc}\,(cx))^2}{x}\,dx$$

input
```
integrate((a+b*acsc(c*x))**2/x,x)
```

output
```
Integral((a + b*acsc(c*x))**2/x, x)
```

3.19.7 Maxima [F]

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))^2/x,x, algorithm="maxima")
```

output
```
-1/2*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 + b^2*c^2*inte
grate(x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 2*b^2*c^2*integrate(x^2*
log(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*c^2*integrate(x^2*log(c^2*x^2)*log
(x)/(c^2*x^3 - x), x) - b^2*c^2*integrate(x^2*log(x)^2/(c^2*x^3 - x), x) +
 2*a*b*c^2*integrate(x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3
- x), x) + 1/2*b^2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + b^2
*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2*log(x) - 1/4*b^2*log(c^2*x^2)^2
*log(x) - b^2*integrate(log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + 2*b^2*inte
grate(log(x)/(c^2*x^3 - x), x)*log(c) + 2*b^2*integrate(sqrt(c*x + 1)*sqrt
(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)
 - 2*b^2*integrate(log(c^2*x^2)*log(x)/(c^2*x^3 - x), x) + b^2*integrate(l
og(x)^2/(c^2*x^3 - x), x) - 2*a*b*integrate(arctan(1/(sqrt(c*x + 1)*sqrt(c
*x - 1)))/(c^2*x^3 - x), x) + a^2*log(x)
```

3.19.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))^2/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:ln of unsigned or minus infinity Error: Bad Argument Va
lue
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x}\,dx = \int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*asin(1/(c*x)))^2/x,x)
```

output
```
int((a + b*asin(1/(c*x)))^2/x, x)
```

3.20 $\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}\,dx$

3.20.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}\,dx = \frac{2b^2}{x} - 2bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right) - \frac{(a+b\csc^{-1}(cx))^2}{x}$$

output
```
2*b^2/x-(a+b*arccsc(c*x))^2/x-2*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}\,dx = -\frac{a^2-2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x+2b\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\csc^{-1}(cx)+b^2\csc^{-1}(cx)^2}{x}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^2/x^2,x]
```

output
```
-((a^2 - 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 2*b*(a + b*c*Sqrt[1 - 1
/(c^2*x^2)]*x)*ArcCsc[c*x] + b^2*ArcCsc[c*x]^2)/x)
```

3.20.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5746, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}dx$$

$$\Big\downarrow \text{5746}$$

$$-c\int \sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-c\int (a+b\csc^{-1}(cx))^2\sin\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3777}$$

$$-c\left(2b\int -\frac{a+b\csc^{-1}(cx)}{cx}d\csc^{-1}(cx)+\frac{(a+b\csc^{-1}(cx))^2}{cx}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{(a+b\csc^{-1}(cx))^2}{cx}-2b\int \frac{a+b\csc^{-1}(cx)}{cx}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c\left(\frac{(a+b\csc^{-1}(cx))^2}{cx}-2b\int (a+b\csc^{-1}(cx))\sin\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3777}$$

$$-c\left(\frac{(a+b\csc^{-1}(cx))^2}{cx}-2b\left(b\int \sqrt{1-\frac{1}{c^2x^2}}d\csc^{-1}(cx)-\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c\left(\frac{(a+b\csc^{-1}(cx))^2}{cx}-2b\left(b\int \sin\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$-c\left(\frac{(a+b\csc^{-1}(cx))^2}{cx}-2b\left(\frac{b}{cx}-\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)\right)$$

input
```
Int[(a + b*ArcCsc[c*x])^2/x^2,x]
```

output
```
-(c*((a + b*ArcCsc[c*x])^2/(c*x) - 2*b*(b/(c*x) - Sqrt[1 - 1/(c^2*x^2)]*(a
 + b*ArcCsc[c*x]))))
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(48) = 96$.

Time = 0.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.30

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2abc\Bigg(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x^2}$
derivativedivides	$c\Bigg(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\Bigg(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x}$
default	$c\Bigg(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{cx}+\frac{2}{cx}-2\operatorname{arccsc}(cx)\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)+2ab\Bigg(-\frac{\operatorname{arccsc}(cx)}{cx}-\frac{c^2x^2-1}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2x}$

input
```
int((a+b*arccsc(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*c*(-1/c/x*arccsc(c*x)^2+2/c/x-2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^
2)^(1/2))+2*a*b*c*(-1/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^
2*(c^2*x^2-1))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^2}\,dx = -\frac{b^2\operatorname{arccsc}(cx)^2+2\,ab\operatorname{arccsc}(cx)+a^2-2\,b^2+2\,\sqrt{c^2x^2-1}(b^2\operatorname{arccsc}(cx)+ab)}{x}$$

input
```
integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
-(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2 - 2*b^2 + 2*sqrt(c^2*x^2 - 1
)*(b^2*arccsc(c*x) + a*b))/x
```

3.20.6 Sympy [F]

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\operatorname{acsc}\left(cx\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*acsc(c*x))**2/x**2,x)
```

output
```
Integral((a + b*acsc(c*x))**2/x**2, x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx=-2\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}\left(cx\right)}{x}\right)ab-2\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}\left(cx\right)-\frac{1}{x}\right)b^2-\frac{b^2\operatorname{arccsc}\left(cx\right)^2}{x}-\frac{a^2}{x}$$

input
```
integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-2*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a*b - 2*(c*sqrt(-1/(c^2*x^2)
 + 1)*arccsc(c*x) - 1/x)*b^2 - b^2*arccsc(c*x)^2/x - a^2/x
```

3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(48) = 96.

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.08

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx=-\left(2\,b^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)+2\,ab\sqrt{-\frac{1}{c^2x^2}+1}+\frac{b^2\arcsin\left(\frac{1}{cx}\right)^2}{cx}+\frac{2\,ab\arcsin\left(\frac{1}{cx}\right)}{cx}+\frac{a^2}{cx}-\frac{2\,b^2}{cx}\right)c$$

input
```
integrate((a+b*arccsc(c*x))^2/x^2,x, algorithm="giac")
```

output
```
-(2*b^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) + 2*a*b*sqrt(-1/(c^2*x^2) +
 1) + b^2*arcsin(1/(c*x))^2/(c*x) + 2*a*b*arcsin(1/(c*x))/(c*x) + a^2/(c*x
) - 2*b^2/(c*x))*c
```

3.20.9 Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.76

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^2}\,dx=-\frac{a^2}{x}-\frac{b^2\left(\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-2\right)}{x}-2\,b^2\,c\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}-2\,a\,b\,c\,\left(\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)}{c\,x}\right)$$

input
```
int((a + b*asin(1/(c*x)))^2/x^2,x)
```

output
```
- a^2/x - (b^2*(asin(1/(c*x))^2 - 2))/x - 2*b^2*c*asin(1/(c*x))*(1 - 1/(c^
2*x^2))^(1/2) - 2*a*b*c*((1 - 1/(c^2*x^2))^(1/2) + asin(1/(c*x))/(c*x))
```

3.21 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^3}\,dx$

3.21.1 Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^3}\,dx = \frac{b^2}{4x^2}+\frac{1}{2}abc^2\csc^{-1}(cx)+\frac{1}{4}b^2c^2\csc^{-1}(cx)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2x}-\frac{\left(a+b\csc^{-1}(cx)\right)^2}{2x^2}$$

output
```
1/4*b^2/x^2+1/2*a*b*c^2*arccsc(c*x)+1/4*b^2*c^2*arccsc(c*x)^2-1/2*(a+b*arc
csc(c*x))^2/x^2-1/2*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

3.21.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^3}\,dx = \frac{-2a^2+b^2-2abc\sqrt{1-\frac{1}{c^2x^2}}x-2b\left(2a+bc\sqrt{1-\frac{1}{c^2x^2}}x\right)\csc^{-1}(cx)+b^2\left(-2+c^2x^2\right)\csc^{-1}(cx)^2+2abc^2x^2}{4x^2}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^2/x^3,x]
```

output
```
(-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 2*b*(2*a + b*c*Sqrt[1 -
1/(c^2*x^2)]*x)*ArcCsc[c*x] + b^2*(-2 + c^2*x^2)*ArcCsc[c*x]^2 + 2*a*b*c^2
*x^2*ArcSin[1/(c*x)])/(4*x^2)
```

3.21.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5746, 4904, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{5746}$$

$$-c^2\int \frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{cx}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4904}$$

$$-c^2\left(\frac{(a+b\csc^{-1}(cx))^2}{2c^2x^2}-b\int \frac{a+b\csc^{-1}(cx)}{c^2x^2}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^2\left(\frac{(a+b\csc^{-1}(cx))^2}{2c^2x^2}-b\int (a+b\csc^{-1}(cx))\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3791}$$

$$-c^2\left(\frac{(a+b\csc^{-1}(cx))^2}{2c^2x^2}-b\left(\frac{1}{2}\int (a+b\csc^{-1}(cx))d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{2cx}+\frac{b}{4c^2x^2}\right)\right)$$

$$\Big\downarrow \text{17}$$

$$-c^2\left(\frac{(a+b\csc^{-1}(cx))^2}{2c^2x^2}-b\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{2cx}+\frac{(a+b\csc^{-1}(cx))^2}{4b}+\frac{b}{4c^2x^2}\right)\right)$$

input
```
Int[(a + b*ArcCsc[c*x])^2/x^3,x]
```

output
```
-(c^2*((a + b*ArcCsc[c*x])^2/(2*c^2*x^2) - b*(b/(4*c^2*x^2) - (Sqrt[1 - 1/
(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(2*c*x) + (a + b*ArcCsc[c*x])^2/(4*b))))
```

3.21.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(76) = 152$.

Time = 0.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.09

method	result
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^2}{2c^2x^2}-\frac{\operatorname{arccsc}(cx)\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}+\frac{\operatorname{arccsc}(cx)^2}{4}+\frac{1}{4c^2x^2}\right)+2$
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^2}{2c^2x^2}-\frac{\operatorname{arccsc}(cx)\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}+\frac{\operatorname{arccsc}(cx)^2}{4}+\frac{1}{4c^2x^2}\right)\right.$
default	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^2}{2c^2x^2}-\frac{\operatorname{arccsc}(cx)\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{2cx}+\frac{\operatorname{arccsc}(cx)^2}{4}+\frac{1}{4c^2x^2}\right)\right.$

input
```
int((a+b*arccsc(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+b^2*c^2*(1/2*(c^2*x^2-1)/c^2/x^2*arccsc(c*x)^2-1/2*arccsc(c*x
)*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x+1/4*arccsc(c*x)^2+1/4/
c^2/x^2)+2*a*b*c^2*(-1/2/c^2/x^2*arccsc(c*x)-1/4*(c^2*x^2-1)^(1/2)*(-arcta
n(1/(c^2*x^2-1)^(1/2))*c^2*x^2+(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)^(1
/2)/c^3/x^3)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^3}\,dx$$
$$=\frac{(b^2c^2x^2-2\,b^2)\operatorname{arccsc}(cx)^2-2\,a^2+b^2+2\,(abc^2x^2-2\,ab)\operatorname{arccsc}(cx)-2\,\sqrt{c^2x^2-1}(b^2\operatorname{arccsc}(cx)+ab)}{4\,x^2}$$

input
```
integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
1/4*((b^2*c^2*x^2 - 2*b^2)*arccsc(c*x)^2 - 2*a^2 + b^2 + 2*(a*b*c^2*x^2 -
2*a*b)*arccsc(c*x) - 2*sqrt(c^2*x^2 - 1)*(b^2*arccsc(c*x) + a*b))/x^2
```

3.21.6 Sympy [F]

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^3} dx = \int \frac{(a + b\,\mathrm{acsc}\,(cx))^2}{x^3} dx$$

input
```
integrate((a+b*acsc(c*x))**2/x**3,x)
```

output
```
Integral((a + b*acsc(c*x))**2/x**3, x)
```

3.21.7 Maxima [F]

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^3} dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)^2}{x^3} dx$$

input
```
integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*a*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c
^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/8*(4*(c^
2*(log(c*x + 1) + log(c*x - 1) - 2*log(x))*log(c)^2 - 4*c^2*integrate(1/2*
x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 8*c^2*integrate(1/2*x^2*log(
x)/(c^2*x^5 - x^3), x)*log(c) - 4*c^2*integrate(1/2*x^2*log(c^2*x^2)*log(x
)/(c^2*x^5 - x^3), x) + 4*c^2*integrate(1/2*x^2*log(x)^2/(c^2*x^5 - x^3),
x) + 2*c^2*integrate(1/2*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x) - (c^2*log(c
*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*log(c)^2 + 4*integrate(
1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) - 8*integrate(1/2*log(x)/(c^2*
x^5 - x^3), x)*log(c) + 4*integrate(1/2*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan
(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x) + 4*integrate(1/2*lo
g(c^2*x^2)*log(x)/(c^2*x^5 - x^3), x) - 4*integrate(1/2*log(x)^2/(c^2*x^5
- x^3), x) - 2*integrate(1/2*log(c^2*x^2)/(c^2*x^5 - x^3), x))*x^2 + 4*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - log(c^2*x^2)^2)*b^2/x^2 - 1/2*a^2
/x^2
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(76) = 152.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.85

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^3}\,dx =$$
$$-\frac{1}{8}\left(4\,b^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2+8\,abc\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)+2\,b^2c\arcsin\left(\frac{1}{cx}\right)^2+4\,a^2c\left(\frac{1}{c^2x^2}\right.\right.$$

input
```
integrate((a+b*arccsc(c*x))^2/x^3,x, algorithm="giac")
```

output
```
-1/8*(4*b^2*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^2 + 8*a*b*c*(1/(c^2*x^2) -
 1)*arcsin(1/(c*x)) + 2*b^2*c*arcsin(1/(c*x))^2 + 4*a^2*c*(1/(c^2*x^2) - 1
) - 2*b^2*c*(1/(c^2*x^2) - 1) + 4*a*b*c*arcsin(1/(c*x)) - b^2*c + 4*b^2*sq
rt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 4*a*b*sqrt(-1/(c^2*x^2) + 1)/x)*c
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\csc^{-1}(cx))^2}{x^3}\,dx = \int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*asin(1/(c*x)))^2/x^3,x)
```

output
```
int((a + b*asin(1/(c*x)))^2/x^3, x)
```

3.22 $\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx$

3.22.1 Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx = \frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}-\frac{4}{9}bc^3\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx)) -\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{9x^2}-\frac{(a+b\csc^{-1}(cx))^2}{3x^3}$$

output
```
2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arccsc(c*x))^2/x^3-4/9*b*c^3*(a+b*arcc
sc(c*x))*(1-1/c^2/x^2)^(1/2)-2/9*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)
/x^2
```

3.22.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx = -\frac{9a^2+6abc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)-2b^2(1+6c^2x^2)+6b\left(3a+bc\sqrt{1-\frac{1}{c^2x^2}}x(1+2c^2x^2)\right)\csc^{-1}(cx)+}{27x^3}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^2/x^4,x]
```

output
```
-1/27*(9*a^2 + 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) - 2*b^2*(1
+ 6*c^2*x^2) + 6*b*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2))*Arc
Csc[c*x] + 9*b^2*ArcCsc[c*x]^2)/x^3
```

3.22.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5746, 4904, 3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{5746}$$

$$-c^3\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{c^2x^2}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4904}$$

$$-c^3\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\int \frac{a+b\csc^{-1}(cx)}{c^3x^3}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^3\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\int \left(a+b\csc^{-1}(cx)\right)\sin\left(\csc^{-1}(cx)\right)^3d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3791}$$

$$-c^3\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\int \frac{a+b\csc^{-1}(cx)}{cx}d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{3c^2x^2}+\frac{b}{9c^3x^3}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^3\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\int \left(a+b\csc^{-1}(cx)\right)\sin\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{3c^2x^2}\right.\right.$$

$$\Big\downarrow \text{3777}$$

$$-c^3\left(\frac{(a+b\csc^{-1}(cx))^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\left(b\int\sqrt{1-\frac{1}{c^2x^2}}d\csc^{-1}(cx)-\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+}{3c^2}\right.\right.$$

$\big\downarrow$ 3042

$$-c^3\left(\frac{(a+b\csc^{-1}(cx))^2}{3c^3x^3}-\frac{2}{3}b\left(\frac{2}{3}\left(b\int\sin\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)-\frac{\sqrt{1-}}{}\right.\right.$$

$\big\downarrow$ 3117

$$-c^3\left(\frac{(a+b\csc^{-1}(cx))^2}{3c^3x^3}-\frac{2}{3}b\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))}{3c^2x^2}+\frac{2}{3}\left(\frac{b}{cx}-\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))\right)+\frac{b}{9c^3x^3}\right.\right.$$

input
```
Int[(a + b*ArcCsc[c*x])^2/x^4,x]
```

output
```
-(c^3*((a + b*ArcCsc[c*x])^2/(3*c^3*x^3) - (2*b*(b/(9*c^3*x^3) - (Sqrt[1 -
 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(3*c^2*x^2) + (2*(b/(c*x) - Sqrt[1 - 1/
(c^2*x^2)]*(a + b*ArcCsc[c*x])))/3))/3))
```

3.22.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.22.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.50

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3}-\frac{2\operatorname{arccsc}(cx)\,(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\,c^3\left(-\frac{\operatorname{arccsc}(c}{3c^3x^3}\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3}-\frac{2\operatorname{arccsc}(cx)\,(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arccsc}(}{3c^3x}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{3c^3x^3}-\frac{2\operatorname{arccsc}(cx)\,(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{9c^2x^2}+\frac{2}{27c^3x^3}+\frac{4}{9cx}\right)+2ab\left(-\frac{\operatorname{arccsc}(}{3c^3x}\right.\right.$

input
```
int((a+b*arccsc(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arccsc(c*x)^2-2/9*arccsc(c*x)*(2*c^2*x^
2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4/9/c/x)+2*a*b*c^3*(
-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/c^4/x^4)
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx$$
$$=\frac{12\,b^2c^2x^2-9\,b^2\,\mathrm{arccsc}\,(cx)^2-18\,ab\,\mathrm{arccsc}\,(cx)-9\,a^2+2\,b^2-6\,(2\,abc^2x^2+ab+(2\,b^2c^2x^2+b^2)\,\mathrm{arccsc}\,(c$$
$$27\,x^3$$

input
```
integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
1/27*(12*b^2*c^2*x^2 - 9*b^2*arccsc(c*x)^2 - 18*a*b*arccsc(c*x) - 9*a^2 +
2*b^2 - 6*(2*a*b*c^2*x^2 + a*b + (2*b^2*c^2*x^2 + b^2)*arccsc(c*x))*sqrt(c
^2*x^2 - 1))/x^3
```

3.22.6 Sympy [F]

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx=\int \frac{(a+b\,\mathrm{acsc}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((a+b*acsc(c*x))**2/x**4,x)
```

output
```
Integral((a + b*acsc(c*x))**2/x**4, x)
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.

Time = 0.46 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.93

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^4}\,dx$$
$$=\frac{2}{9}\,ab\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right)-\frac{b^2\,\mathrm{arccsc}\,(cx)^2}{3\,x^3}-\frac{a^2}{3\,x^3}$$
$$-\frac{2\left(6\,c^5x^4\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)-3\,c^3x^2\arctan\left(1,\sqrt{cx+1}\sqrt{cx-1}\right)-(6\,c^3x^2+c)\sqrt{cx+1}\sqrt{cx}\right.}{27\,\sqrt{cx+1}\sqrt{cx-1}cx^3}$$

input
```
integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
2/9*a*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c -
 3*arccsc(c*x)/x^3) - 1/3*b^2*arccsc(c*x)^2/x^3 - 1/3*a^2/x^3 - 2/27*(6*c^
5*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*c^3*x^2*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1)) - (6*c^3*x^2 + c)*sqrt(c*x + 1)*sqrt(c*x - 1) - 3*
c*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1
)*c*x^3)
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(88) = 176.

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.20

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}\,dx$$
$$= \frac{1}{27}\left(6\,b^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)+6\,abc^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-18\,b^2c^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)-\frac{9\,b\ldots}{}\right.$$

input
```
integrate((a+b*arccsc(c*x))^2/x^4,x, algorithm="giac")
```

output
```
1/27*(6*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x)) + 6*a*b*c^2*(-1/(
c^2*x^2) + 1)^(3/2) - 18*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) -
9*b^2*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^2/x - 18*a*b*c^2*sqrt(-1/(c^2*x^
2) + 1) - 18*a*b*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 9*b^2*c*arcsin(1/
(c*x))^2/x + 2*b^2*c*(1/(c^2*x^2) - 1)/x - 18*a*b*c*arcsin(1/(c*x))/x + 14
*b^2*c/x - 9*a^2/(c*x^3))*c
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^4}\,dx$$

input
```
int((a + b*asin(1/(c*x)))^2/x^4,x)
```

output
```
int((a + b*asin(1/(c*x)))^2/x^4, x)
```

3.23 $\int \frac{\left(a+b \csc^{-1}(cx)\right)^2}{x^5} \, dx$

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 134

$$\int \frac{\left(a+b \csc^{-1}(cx)\right)^2}{x^5} \, dx = \frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16} abc^4 \csc^{-1}(cx) + \frac{3}{32} b^2c^4 \csc^{-1}(cx)^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{16x} - \frac{\left(a+b\csc^{-1}(cx)\right)^2}{4x^4}$$

output
```
1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arccsc(c*x)+3/32*b^2*c^4*arccsc
(c*x)^2-1/4*(a+b*arccsc(c*x))^2/x^4-1/8*b*c*(a+b*arccsc(c*x))*(1-1/c^2/x^2
)^(1/2)/x^3-3/16*b*c^3*(a+b*arccsc(c*x))*(1-1/c^2/x^2)^(1/2)/x
```

3.23.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b \csc^{-1}(cx)\right)^2}{x^5} \, dx = \frac{-8a^2+b^2-4abc\sqrt{1-\frac{1}{c^2x^2}}x+3b^2c^2x^2-6abc^3\sqrt{1-\frac{1}{c^2x^2}}x^3-2b\left(8a+bc\sqrt{1-\frac{1}{c^2x^2}}x(2+3c^2x^2)\right)\csc^{-1}}{32x^4}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^2/x^5,x]
```

output
```
(-8*a^2 + b^2 - 4*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 - 6*a*b*c^
3*Sqrt[1 - 1/(c^2*x^2)]*x^3 - 2*b*(8*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 +
3*c^2*x^2))*ArcCsc[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcCsc[c*x]^2 + 6*a*b*c^4*x
^4*ArcSin[1/(c*x)])/(32*x^4)
```

3.23.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5746, 4904, 3042, 3791, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{x^5}dx$$

$$\Big\downarrow \text{5746}$$

$$-c^4\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{c^3x^3}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4904}$$

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\int \frac{a+b\csc^{-1}(cx)}{c^4x^4}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\int \left(a+b\csc^{-1}(cx)\right)\sin\left(\csc^{-1}(cx)\right)^4d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3791}$$

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\int \frac{a+b\csc^{-1}(cx)}{c^2x^2}d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{4c^3x^3}+\frac{b}{16c^4x^4}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\int\left(a+b\csc^{-1}(cx)\right)\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{4c^3x^3}\right.\right.$$

$\big\downarrow$ 3791

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2cx}+\frac{b}{4c^2x^2}\right)\right.\right.$$

$\big\downarrow$ 17

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)}{2cx}+\frac{\left(a+b\csc^{-1}(cx)\right)^2}{4b}+\frac{b}{4c^2x^2}\right)-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{}\right.\right.$$

input
```
Int[(a + b*ArcCsc[c*x])^2/x^5,x]
```

output
```
-(c^4*((a + b*ArcCsc[c*x])^2/(4*c^4*x^4) - (b*(b/(16*c^4*x^4) - (Sqrt[1 -
1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(4*c^3*x^3) + (3*(b/(4*c^2*x^2) - (Sqrt[
1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x]))/(2*c*x) + (a + b*ArcCsc[c*x])^2/(4*b
)))/4))/2))
```

3.23.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(116) = 232$.

Time = 1.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\operatorname{arccsc}(cx)^2}{4c^4x^4}+\frac{\operatorname{arccsc}(cx)\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arccsc}(cx)^2}{32}+\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{4c^4x^4}+\frac{\operatorname{arccsc}(cx)\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arccsc}(cx}{32}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arccsc}(cx)^2}{4c^4x^4}+\frac{\operatorname{arccsc}(cx)\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{16c^3x^3}-\frac{3\operatorname{arccsc}(cx}{32}\right.\right.$

input
```
int((a+b*arccsc(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4+b^2*c^4*(-1/4/c^4/x^4*arccsc(c*x)^2+1/16*arccsc(c*x)*(3*c^3*x
^3*arccsc(c*x)-3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c^2/x^
2)^(1/2))/c^3/x^3-3/32*arccsc(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-1/2*a*
b/x^4*arccsc(c*x)+3/16*a*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/
2)/x*arctan(1/(c^2*x^2-1)^(1/2))-3/16*a*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)/x^3-1/8*a*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx$$
$$= \frac{3\,b^2c^2x^2 + (3\,b^2c^4x^4 - 8\,b^2)\,\mathrm{arccsc}\,(cx)^2 - 8\,a^2 + b^2 + 2\,(3\,abc^4x^4 - 8\,ab)\,\mathrm{arccsc}\,(cx) - 2\,(3\,abc^2x^2 + 2\,ab +}{32\,x^4}$$

input
```
integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*arccsc(c*x)^2 - 8*a^2 + b^2
+ 2*(3*a*b*c^4*x^4 - 8*a*b)*arccsc(c*x) - 2*(3*a*b*c^2*x^2 + 2*a*b + (3*b^
2*c^2*x^2 + 2*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x^4
```

3.23.6 Sympy [F]

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))^2}{x^5}\,dx$$

input
```
integrate((a+b*acsc(c*x))**2/x**5,x)
```

output
```
Integral((a + b*acsc(c*x))**2/x**5, x)
```

3.23.7 Maxima [F]

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)^2}{x^5}\,dx$$

input
```
integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
-1/16*a*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c^2
*x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) -
 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/16*(4
*(2*(c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*c^2*log(c
)^2 - 16*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 3
2*c^2*integrate(1/4*x^2*log(x)/(c^2*x^7 - x^5), x)*log(c) - 16*c^2*integra
te(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 16*c^2*integrate(1/4*
x^2*log(x)^2/(c^2*x^7 - x^5), x) + 4*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c
^2*x^7 - x^5), x) - (2*c^4*log(c*x + 1) + 2*c^4*log(c*x - 1) - 4*c^4*log(x
) + (2*c^2*x^2 + 1)/x^4)*log(c)^2 + 16*integrate(1/4*log(c^2*x^2)/(c^2*x^7
 - x^5), x)*log(c) - 32*integrate(1/4*log(x)/(c^2*x^7 - x^5), x)*log(c) +
8*integrate(1/4*sqrt(c*x + 1)*sqrt(c*x - 1)*arctan(1/(sqrt(c*x + 1)*sqrt(c
*x - 1)))/(c^2*x^7 - x^5), x) + 16*integrate(1/4*log(c^2*x^2)*log(x)/(c^2*
x^7 - x^5), x) - 16*integrate(1/4*log(x)^2/(c^2*x^7 - x^5), x) - 4*integra
te(1/4*log(c^2*x^2)/(c^2*x^7 - x^5), x))*x^4 + 4*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))^2 - log(c^2*x^2)^2)*b^2/x^4 - 1/4*a^2/x^4
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(116) = 232$.

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.27

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx =$$
$$-\frac{1}{256}\left(64\,b^2c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^2+128\,abc^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)+128\,b^2c^3\left(\frac{1}{c^2x^2}-1\right)\mathrm{a}\right.$$

input
```
integrate((a+b*arccsc(c*x))^2/x^5,x, algorithm="giac")
```

output
```
-1/256*(64*b^2*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^2 + 128*a*b*c^3*(1/
(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 128*b^2*c^3*(1/(c^2*x^2) - 1)*arcsin(1/
(c*x))^2 - 8*b^2*c^3*(1/(c^2*x^2) - 1)^2 + 256*a*b*c^3*(1/(c^2*x^2) - 1)*a
rcsin(1/(c*x)) + 40*b^2*c^3*arcsin(1/(c*x))^2 - 40*b^2*c^3*(1/(c^2*x^2) -
1) + 80*a*b*c^3*arcsin(1/(c*x)) - 32*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcs
in(1/(c*x))/x - 17*b^2*c^3 - 32*a*b*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 80*b^
2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 80*a*b*c^2*sqrt(-1/(c^2*x
^2) + 1)/x + 64*a^2/(c*x^4))*c
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\csc^{-1}(cx))^2}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

input
```
int((a + b*asin(1/(c*x)))^2/x^5,x)
```

output
```
int((a + b*asin(1/(c*x)))^2/x^5, x)
```

3.24 $\int x^3 (a+b\csc^{-1}(cx))^3 \, dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 207

$$\int x^3 (a+b\csc^{-1}(cx))^3 \, dx = \frac{b^3\sqrt{1-\frac{1}{c^2x^2}}x}{4c^3} + \frac{b^2x^2(a+b\csc^{-1}(cx))}{4c^2} + \frac{ib(a+b\csc^{-1}(cx))^2}{2c^4} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x(a+b\csc^{-1}(cx))^2}{2c^3} + \frac{b\sqrt{1-\frac{1}{c^2x^2}}x^3(a+b\csc^{-1}(cx))^2}{4c} + \frac{1}{4}x^4(a+b\csc^{-1}(cx))^3 - \frac{b^2(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^4} + \frac{ib^3\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^4}$$

output
```
1/4*b^2*x^2*(a+b*arccsc(c*x))/c^2+1/2*I*b*(a+b*arccsc(c*x))^2/c^4+1/4*x^4*
(a+b*arccsc(c*x))^3-b^2*(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))
^2)/c^4+1/2*I*b^3*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/c^4+1/4*b^3*x*(
1-1/c^2/x^2)^(1/2)/c^3+1/2*b*x*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c^3
+1/4*b*x^3*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

3.24.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.38

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx$$
$$=\frac{2a^2bc\sqrt{1-\frac{1}{c^2x^2}}x+b^3c\sqrt{1-\frac{1}{c^2x^2}}x+ab^2c^2x^2+a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3+a^3c^4x^4+b^2\left(3ac^4x^4+b\left(2i+2c\sqrt{1\right.\right.}{}$$

input
```
Integrate[x^3*(a + b*ArcCsc[c*x])^3,x]
```

output
```
(2*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*b^2
*c^2*x^2 + a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 + b^2*(3*a*c^
4*x^4 + b*(2*I + 2*c*Sqrt[1 - 1/(c^2*x^2)]*x + c^3*Sqrt[1 - 1/(c^2*x^2)]*x
^3))*ArcCsc[c*x]^2 + b^3*c^4*x^4*ArcCsc[c*x]^3 + b*ArcCsc[c*x]*(c*x*(b^2*c
*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2)) - 4*b^2*Lo
g[1 - E^((2*I)*ArcCsc[c*x])]) - 4*a*b^2*Log[1/(c*x)] + (2*I)*b^3*PolyLog[2
, E^((2*I)*ArcCsc[c*x])])/(4*c^4)
```

3.24.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {5746, 4910, 3042, 4674, 3042, 4254, 24, 4672, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx$$

$$\Big\downarrow \text{5746}$$

$$-\frac{\int c^5\sqrt{1-\frac{1}{c^2x^2}}x^5\left(a+b\csc^{-1}(cx)\right)^3d\csc^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{4910}$$

$$-\frac{\frac{3}{4}b\int c^4x^4\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{4}c^4x^4\left(a+b\csc^{-1}(cx)\right)^3}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3}{4}b\int\left(a+b\csc^{-1}(cx)\right)^2\csc\left(\csc^{-1}(cx)\right)^4d\csc^{-1}(cx)-\frac{1}{4}c^4x^4\left(a+b\csc^{-1}(cx)\right)^3}{c^4}$$

$\Big\downarrow$ 4674

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\int c^2x^2\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)+\frac{1}{3}b^2\int c^2x^2d\csc^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\csc^{-1}(cx)\right)-\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x}}\right.}{c^4}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\csc^{-1}(cx)\right)^2\csc\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)+\frac{1}{3}b^2\int\csc\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\csc^{-}\right.\right.}{c^4}$$

$\Big\downarrow$ 4254

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\csc^{-1}(cx)\right)^2\csc\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{3}b^2\int 1d\left(c\sqrt{1-\frac{1}{c^2x^2}}x\right)-\frac{1}{3}bc^2x^2\left(a+b\csc^{-1}(cx)\right)-\frac{1}{3}\right.}{c^4}$$

$\Big\downarrow$ 24

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\csc^{-1}(cx)\right)^2\csc\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\csc^{-1}(cx)\right)-\frac{1}{3}c^3x^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-}\right.\right.}{c^4}$$

$\Big\downarrow$ 4672

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\left(2b\int c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)-\frac{1}{3}bc^2x^2\left(a+b\csc^{-1}(\right.\right.}{c^4}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\left(2b\int-\left(\left(a+b\csc^{-1}(cx)\right)\tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)-\frac{1}{3}bc^2x^2\left(a\right.\right.}{c^4}$$

$\Big\downarrow$ 25

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\left(-2b\int\left(a+b\csc^{-1}(cx)\right)\tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)-\frac{1}{3}bc^2x^2\left(a+\right.\right.}{c^4}$$

$\Big\downarrow$ 4200

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b}-2i\int-\frac{e^{2i\csc^{-1}(cx)}\left(a\right.}{1-e^{2i\csc}}\right.\right.\right.\right.}{c}$$

3.24. $\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx$

$\downarrow$ 25

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(2i\int\frac{e^{2i\csc^{-1}(cx)}\left(a+b\csc^{-1}(cx)\right)}{1-e^{2i\csc^{-1}(cx)}}d\csc^{-1}(\cdots\right.\right.\right.}{c^4}$$

$\downarrow$ 2620

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\,\mathrm{cs}\cdots\right.\right.\right.\right.}{\cdots}$$

$\downarrow$ 2715

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\,\mathrm{cs}\cdots\right.\right.\right.\right.}{\cdots}$$

$\downarrow$ 2838

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\,\mathrm{cs}\cdots\right.\right.\right.\right.}{\cdots}$$

input
```
Int[x^3*(a + b*ArcCsc[c*x])^3,x]
```

output
```
-((-1/4*(c^4*x^4*(a + b*ArcCsc[c*x])^3) + (3*b*(-1/3*(b^2*c*Sqrt[1 - 1/(c^
2*x^2)]*x) - (b*c^2*x^2*(a + b*ArcCsc[c*x]))/3 - (c^3*Sqrt[1 - 1/(c^2*x^2)
]*x^3*(a + b*ArcCsc[c*x])^2)/3 + (2*(-(c*Sqrt[1 - 1/(c^2*x^2)]*x*(a + b*Ar
cCsc[c*x])^2) - 2*b*(((I/2)*(a + b*ArcCsc[c*x])^2)/b + (2*I)*((I/2)*(a + b
*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])] + (b*PolyLog[2, E^((2*I)*ArcC
sc[c*x])])/4))))/3))/4)/c^4)
```

3.24.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.24. $\int x^3(a+b\csc^{-1}(cx))^3\,dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.24.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arccsc}(cx)^3c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arccsc}(cx)^2}{2}+\frac{c^2x^2\operatorname{arccsc}(cx)}{4}+\cdots\right.}{\cdots}$
default	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{\operatorname{arccsc}(cx)^3c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arccsc}(cx)^2}{2}+\frac{c^2x^2\operatorname{arccsc}(cx)}{4}+\cdots\right.}{\cdots}$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{\operatorname{arccsc}(cx)^3c^4x^4}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3x^3}{4}+\frac{\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}{2}+\frac{i\operatorname{arccsc}(cx)^2}{2}+\frac{c^2x^2\operatorname{arccsc}(cx)}{4}+\cdots\right.}{\cdots}$

input
```
int(x^3*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*arccsc(c*x)^3*c^4*x^4+1/4*arccsc(c*x)^2*((
c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/2*arccsc(c*x)^2*((c^2*x^2-1)/c^2/x^2)^
(1/2)*c*x+1/2*I*arccsc(c*x)^2+1/4*c^2*x^2*arccsc(c*x)+1/4*x*c*((c^2*x^2-1)
/c^2/x^2)^(1/2)-1/4*I-arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))-arccsc(c
*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))
+I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(1/4*arccsc(c*x)^2*c^4*x
^4+1/6*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*x^3+1/12*c^2*x^2+1/3*ar
ccsc(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)-1/3*ln(1/c/x))+3*a^2*b*(1/4*c^4*
x^4*arccsc(c*x)+1/12*(c^2*x^2-1)*(c^2*x^2+2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/c
/x))
```

3.24.5 Fricas [F]

$$\int x^3(a+b\csc^{-1}(cx))^3\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arccsc(c*x)^3 + 3*a*b^2*x^3*arccsc(c*x)^2 + 3*a^2*b*x^3*a
rccsc(c*x) + a^3*x^3, x)
```

3.24.6 Sympy [F]

$$\int x^3(a+b\csc^{-1}(cx))^3\,dx=\int x^3(a+b\,\mathrm{acsc}\,(cx))^3\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))**3,x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))**3, x)
```

3.24.7 Maxima [F]

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arccsc(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arccsc(c*x) + (c^2*
x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*a^2*b + 1/
16*(4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*x^4*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 16*integrate(3/16*(16*c^2*x^5*a
rctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 16*x^3*arctan2(1, sqrt(c
*x + 1)*sqrt(c*x - 1))*log(c)^2 + 16*(c^2*x^5*arctan2(1, sqrt(c*x + 1)*sqr
t(c*x - 1)) - x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2 - (4*x
^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x^3*log(c^2*x^2)^2)*sqrt(c*
x + 1)*sqrt(c*x - 1) - 4*((4*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*l
og(c) + c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5 - (4*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1))*log(c) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)))*x^3 + 4*(c^2*x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - x^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x))*log(c^2*x^2) + 32*(c^2*x^5*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c) - x^3*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1))*log(c))*log(x))/(c^2*x^2 - 1), x))*b^3 + 1/4*(2*c^4*x^4*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*c^2*x^2*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)) + (c^2*x^2 + 2*log(x^2))*sqrt(c*x + 1)*sqrt(c*x - 1) - 4*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*a*b^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c
^4)
```

3.24.8 Giac [F]

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^3*x^3, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int x^3\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

input
```
int(x^3*(a + b*asin(1/(c*x)))^3,x)
```

output
```
int(x^3*(a + b*asin(1/(c*x)))^3, x)
```

3.25 $\int x^2 (a + b \csc^{-1}(cx))^3 \, dx$

3.25.1 Optimal result

Integrand size = 14, antiderivative size = 220

$$\begin{aligned}\int x^2 (a + b \csc^{-1}(cx))^3 \, dx = {} & \frac{b^2 x (a + b \csc^{-1}(cx))}{c^2} + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}\, x^2 (a + b \csc^{-1}(cx))^2}{2c} \\ & + \frac{1}{3} x^3 (a + b \csc^{-1}(cx))^3 \\ & + \frac{b (a + b \csc^{-1}(cx))^2 \operatorname{arctanh}\left(e^{i \csc^{-1}(cx)}\right)}{c^3} \\ & + \frac{b^3 \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c^3} \\ & - \frac{i b^2 (a + b \csc^{-1}(cx)) \operatorname{PolyLog}\left(2, -e^{i \csc^{-1}(cx)}\right)}{c^3} \\ & + \frac{i b^2 (a + b \csc^{-1}(cx)) \operatorname{PolyLog}\left(2, e^{i \csc^{-1}(cx)}\right)}{c^3} \\ & + \frac{b^3 \operatorname{PolyLog}\left(3, -e^{i \csc^{-1}(cx)}\right)}{c^3} - \frac{b^3 \operatorname{PolyLog}\left(3, e^{i \csc^{-1}(cx)}\right)}{c^3}\end{aligned}$$

output
```
b^2*x*(a+b*arccsc(c*x))/c^2+1/3*x^3*(a+b*arccsc(c*x))^3+b*(a+b*arccsc(c*x)
)^2*arctanh(I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+b^3*arctanh((1-1/c^2/x^2)^(1/2)
)/c^3-I*b^2*(a+b*arccsc(c*x))*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c^3+I*
b^2*(a+b*arccsc(c*x))*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c^3+b^3*polylog
(3,-I/c/x-(1-1/c^2/x^2)^(1/2))/c^3-b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2)
)/c^3+1/2*b*x^2*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/c
```

3.25.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 580 vs. 2(220) = 440.

Time = 7.39 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.64

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx$$

$$=\frac{a^3x^3}{3}+\frac{a^2bx^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+a^2bx^3\csc^{-1}(cx)+\frac{a^2b\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{2c^3}$$

$$+\frac{ab^2\left(-8i\,\mathrm{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)+2c^3x^3\left(2+4\csc^{-1}(cx)^2-2\cos\left(2\csc^{-1}(cx)\right)-\frac{3\csc^{-1}(cx)\log\left(1-e^{i\csc^{-1}}\right.}{cx}\right.\right.}{}$$

$$+\frac{b^3\left(24\csc^{-1}(cx)\cot\left(\frac{1}{2}\csc^{-1}(cx)\right)+4\csc^{-1}(cx)^3\cot\left(\frac{1}{2}\csc^{-1}(cx)\right)+6\csc^{-1}(cx)^2\csc^2\left(\frac{1}{2}\csc^{-1}(cx)\right)+\right.}{}$$

input
```
Integrate[x^2*(a + b*ArcCsc[c*x])^3,x]
```

output
```
(a^3*x^3)/3 + (a^2*b*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + a^2*b*x^3
*ArcCsc[c*x] + (a^2*b*Log[x*(1 + Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])])/(2*c^3)
 + (a*b^2*((-8*I)*PolyLog[2, -E^(I*ArcCsc[c*x])] + 2*c^3*x^3*(2 + 4*ArcCsc
[c*x]^2 - 2*Cos[2*ArcCsc[c*x]] - (3*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])]
)/(c*x) + (3*ArcCsc[c*x]*Log[1 + E^(I*ArcCsc[c*x])])/(c*x) + ((4*I)*PolyLo
g[2, E^(I*ArcCsc[c*x])])/(c^3*x^3) + 2*ArcCsc[c*x]*Sin[2*ArcCsc[c*x]] + Ar
cCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])]*Sin[3*ArcCsc[c*x]] - ArcCsc[c*x]*Log[
1 + E^(I*ArcCsc[c*x])]*Sin[3*ArcCsc[c*x]])))/(8*c^3) + (b^3*(24*ArcCsc[c*x
]*Cot[ArcCsc[c*x]/2] + 4*ArcCsc[c*x]^3*Cot[ArcCsc[c*x]/2] + 6*ArcCsc[c*x]^
2*Csc[ArcCsc[c*x]/2]^2 + (ArcCsc[c*x]^3*Csc[ArcCsc[c*x]/2]^4)/(c*x) - 24*A
rcCsc[c*x]^2*Log[1 - E^(I*ArcCsc[c*x])] + 24*ArcCsc[c*x]^2*Log[1 + E^(I*Ar
cCsc[c*x])] - 48*Log[Tan[ArcCsc[c*x]/2]] - (48*I)*ArcCsc[c*x]*PolyLog[2, -
E^(I*ArcCsc[c*x])] + (48*I)*ArcCsc[c*x]*PolyLog[2, E^(I*ArcCsc[c*x])] + 48
*PolyLog[3, -E^(I*ArcCsc[c*x])] - 48*PolyLog[3, E^(I*ArcCsc[c*x])] - 6*Arc
Csc[c*x]^2*Sec[ArcCsc[c*x]/2]^2 + 16*c^3*x^3*ArcCsc[c*x]^3*Sin[ArcCsc[c*x]
/2]^4 + 24*ArcCsc[c*x]*Tan[ArcCsc[c*x]/2] + 4*ArcCsc[c*x]^3*Tan[ArcCsc[c*x
]/2]))/(48*c^3)
```

3.25.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5746, 4910, 3042, 4674, 3042, 4257, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\csc^{-1}(cx)\right)^3 dx$$

$$\Big\downarrow \text{5746}$$

$$-\frac{\int c^4\sqrt{1-\frac{1}{c^2x^2}}x^4\left(a+b\csc^{-1}(cx)\right)^3 d\csc^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{4910}$$

$$-\frac{b\int c^3x^3\left(a+b\csc^{-1}(cx)\right)^2 d\csc^{-1}(cx)-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^3}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \left(a+b\csc^{-1}(cx)\right)^2 \csc\left(\csc^{-1}(cx)\right)^3 d\csc^{-1}(cx)-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^3}{c^3}$$

$$\Big\downarrow \text{4674}$$

$$-\frac{b\left(\frac{1}{2}\int cx\left(a+b\csc^{-1}(cx)\right)^2 d\csc^{-1}(cx)+b^2\int cxd\csc^{-1}(cx)-\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-bcx\left(a+b\mathrm{cs}\right.\right.}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\frac{1}{2}\int \left(a+b\csc^{-1}(cx)\right)^2 \csc\left(\csc^{-1}(cx)\right) d\csc^{-1}(cx)+b^2\int \csc\left(\csc^{-1}(cx)\right) d\csc^{-1}(cx)-\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\right.\right.}{c^3}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{b\left(\frac{1}{2}\int \left(a+b\csc^{-1}(cx)\right)^2 \csc\left(\csc^{-1}(cx)\right) d\csc^{-1}(cx)-\frac{1}{2}c^2x^2\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-bcx\left(a+b\csc^{-1}(cx\right.\right.}{c^3}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^3+b\left(\frac{1}{2}\left(-2b\int \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{i\csc^{-1}(cx)}\right)d\csc^{-1}(cx)+2b\int \left(a+b\csc^{-1}(c\right.\right.\right.}{}$$

$\Big\downarrow$ 3011

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^3+b\left(\frac{1}{2}\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)-ib\int\text{PolyLog}\left(2,-e^{i\csc^{-1}(c\ldots\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 2720

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^3+b\left(\frac{1}{2}\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)-b\int e^{-i\csc^{-1}(cx)}\,\text{PolyLog}\left(2\ldots\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 7143

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\csc^{-1}(cx)\right)^3+b\left(\frac{1}{2}\left(-2\text{arctanh}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2+2b\left(i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(\ldots\right.\right.\right.\right.}{}$$

input
```
Int[x^2*(a + b*ArcCsc[c*x])^3,x]
```

output
```
-((-1/3*(c^3*x^3*(a + b*ArcCsc[c*x])^3) + b*(-(b*c*x*(a + b*ArcCsc[c*x]))
- (c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(a + b*ArcCsc[c*x])^2)/2 - b^2*ArcTanh[Sq
rt[1 - 1/(c^2*x^2)]] + (-2*(a + b*ArcCsc[c*x])^2*ArcTanh[E^(I*ArcCsc[c*x])
] + 2*b*(I*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x])] - b*PolyLog[
3, -E^(I*ArcCsc[c*x])]) - 2*b*(I*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*ArcCs
c[c*x])] - b*PolyLog[3, E^(I*ArcCsc[c*x])]))/2))/c^3)
```

3.25.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.25. $\int x^2 (a + b \csc^{-1}(cx))^3 \, dx$

3.25.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\dfrac{\frac{c^3x^3a^3}{3}+b^3\left(\frac{\operatorname{arccsc}(cx)\left(2c^2x^2\operatorname{arccsc}(cx)^2+3\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2}+i\operatorname{arccsc}(cx)\right.}{}$
default	$\dfrac{\frac{c^3x^3a^3}{3}+b^3\left(\frac{\operatorname{arccsc}(cx)\left(2c^2x^2\operatorname{arccsc}(cx)^2+3\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2}+i\operatorname{arccsc}(cx)\right.}{}$
parts	$\frac{a^3x^3}{3}+\dfrac{b^3\left(\frac{\operatorname{arccsc}(cx)\left(2c^2x^2\operatorname{arccsc}(cx)^2+3\operatorname{arccsc}(cx)cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\right)cx}{6}-\frac{\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)}{2}+i\operatorname{arccsc}(cx)\right.}{}$

input
```
int(x^2*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/3*c^3*x^3*a^3+b^3*(1/6*arccsc(c*x)*(2*c^2*x^2*arccsc(c*x)^2+3*arc
csc(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+6)*c*x-1/2*arccsc(c*x)^2*ln(1-I/c
/x-(1-1/c^2/x^2)^(1/2))+I*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))
-polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^
2/x^2)^(1/2))-I*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+polylog(
3,-I/c/x-(1-1/c^2/x^2)^(1/2))+2*arctanh(I/c/x+(1-1/c^2/x^2)^(1/2)))+3*a*b^
2*(1/3*(c^2*x^2*arccsc(c*x)^2+arccsc(c*x)*c*x*((c^2*x^2-1)/c^2/x^2)^(1/2)+
1)*c*x-1/3*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+1/3*I*polylog(2,I/c
/x+(1-1/c^2/x^2)^(1/2))+1/3*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/
3*I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a^2*b*(1/3*c^3*x^3*arccsc(c*x
)+1/6*(c^2*x^2-1)^(1/2)*(c*x*(c^2*x^2-1)^(1/2)+ln(c*x+(c^2*x^2-1)^(1/2)))/
((c^2*x^2-1)/c^2/x^2)^(1/2)/c/x))
```

3.25.5 Fricas [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int(b\operatorname{arccsc}(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arccsc(c*x)^3 + 3*a*b^2*x^2*arccsc(c*x)^2 + 3*a^2*b*x^2*a
rccsc(c*x) + a^3*x^2, x)
```

3.25.6 Sympy [F]

$$\int x^2 (a + b\csc^{-1}(cx))^3 \, dx = \int x^2 (a + b \operatorname{acsc}(cx))^3 \, dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))**3,x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))**3, x)
```

3.25.7 Maxima [F]

$$\int x^2 (a + b\csc^{-1}(cx))^3 \, dx = \int (b \operatorname{arccsc}(cx) + a)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 1/4*b^3*x^3*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 1/2*a*b^2*c^2*(2*(c^2*x
^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*log(c)^2 - 12*b^3
*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^2 -
1), x)*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqr
t(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(
1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*l
og(c) + 12*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(
c) - 24*a*b^2*c^2*integrate(1/4*x^4*log(x)/(c^2*x^2 - 1), x)*log(c) + 1/3*
a^3*x^3 + 12*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x -
1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*b^3*c^2*integrate(1/4*x^4*
arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^2 - 1), x) + 12*a*
b^2*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x
^2 - 1), x) + 4*b^3*c^2*integrate(1/4*x^4*arctan(1/(sqrt(c*x + 1)*sqrt(c*x
 - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x) - 3*a*b^2*c^2*integrate(1/4*x^4*log
(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^4*log(c^2*x^2
)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*c^2*integrate(1/4*x^4*log(x)^2/(c^2*
x^2 - 1), x) + 3/2*a*b^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*l
og(c)^2 + 12*b^3*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*integrate(1/4*x^2*arctan(1/(sqrt(c...
```

3.25.8 Giac [F]

$$\int x^2\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int(b\operatorname{arccsc}(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^3*x^2, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\csc^{-1}(cx)\right)^3 \, dx = \int x^2 \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3 \, dx$$

input
```
int(x^2*(a + b*asin(1/(c*x)))^3,x)
```

output
```
int(x^2*(a + b*asin(1/(c*x)))^3, x)
```

3.26 $\int x(a+b\csc^{-1}(cx))^3\,dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 126

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\frac{3ib\left(a+b\csc^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\csc^{-1}(cx)\right)^3-\frac{3b^2\left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{c^2}+\frac{3ib^3\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2c^2}$$

output
```
3/2*I*b*(a+b*arccsc(c*x))^2/c^2+1/2*x^2*(a+b*arccsc(c*x))^3-3*b^2*(a+b*arc
csc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*I*b^3*polylog(2,(I/c
/x+(1-1/c^2/x^2)^(1/2))^2)/c^2+3/2*b*x*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(
1/2)/c
```

3.26.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.44

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx$$
$$=\frac{3b^2\left(ac^2x^2+b\left(i+c\sqrt{1-\frac{1}{c^2x^2}}x\right)\right)\csc^{-1}(cx)^2+b^3c^2x^2\csc^{-1}(cx)^3+3b\csc^{-1}(cx)\left(acx\left(2b\sqrt{1-\frac{1}{c^2x^2}}+ac\right.\right.}{}$$

input
```
Integrate[x*(a + b*ArcCsc[c*x])^3,x]
```

output
```
(3*b^2*(a*c^2*x^2 + b*(I + c*Sqrt[1 - 1/(c^2*x^2)]*x))*ArcCsc[c*x]^2 + b^3
*c^2*x^2*ArcCsc[c*x]^3 + 3*b*ArcCsc[c*x]*(a*c*x*(2*b*Sqrt[1 - 1/(c^2*x^2)]
 + a*c*x) - 2*b^2*Log[1 - E^((2*I)*ArcCsc[c*x])]) + a*(a*c*x*(3*b*Sqrt[1 -
 1/(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + (3*I)*b^3*PolyLog[2, E^((2*
I)*ArcCsc[c*x])])/(2*c^2)
```

3.26.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5746, 4910, 3042, 4672, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx$$

$$\Big\downarrow \text{ 5746}$$

$$-\frac{\int c^3\sqrt{1-\frac{1}{c^2x^2}}x^3\left(a+b\csc^{-1}(cx)\right)^3d\csc^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{ 4910}$$

$$-\frac{\frac{3}{2}b\int c^2x^2\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^3}{c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{3}{2}b\int \left(a+b\csc^{-1}(cx)\right)^2\csc\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^3}{c^2}$$

$\Big\downarrow$ 4672

$$-\frac{\frac{3}{2}b\left(2b\int c\sqrt{1-\frac{1}{c^2x^2}}x\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)}{c^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{3}{2}b\left(2b\int -\left(\left(a+b\csc^{-1}(cx)\right)\tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)-\frac{1}{2}c^2x^2\left(a+\right.}{c^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{3}{2}b\left(-2b\int\left(a+b\csc^{-1}(cx)\right)\tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)-\frac{1}{2}c^2x^2\left(a+b\,\mathrm{c}\right.}{c^2}$$

$\Big\downarrow$ 4200

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{2}b\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2b}-2i\int-\frac{e^{2i\csc^{-1}(cx)}\left(a+b\right.}{1-e^{2i\csc^{-1}}}\right.\right.}{c^2}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{2}b\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(2i\int\frac{e^{2i\csc^{-1}(cx)}\left(a+b\csc^{-1}(cx)\right)}{1-e^{2i\csc^{-1}(cx)}}d\csc^{-1}(cx)\right.\right.}{c^2}$$

$\Big\downarrow$ 2620

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{2}b\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-}\right.\right.\right.\right.}{c^2}$$

$\Big\downarrow$ 2715

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{2}b\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-}\right.\right.\right.\right.}{c^2}$$

$\Big\downarrow$ 2838

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\csc^{-1}(cx)\right)^3+\frac{3}{2}b\left(-cx\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-2b\left(2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-}\right.\right.\right.\right.}{c^2}$$

3.26. $\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx$

input
```
Int[x*(a + b*ArcCsc[c*x])^3,x]
```

output
```
-((-1/2*(c^2*x^2*(a + b*ArcCsc[c*x])^3) + (3*b*(-(c*Sqrt[1 - 1/(c^2*x^2)]*
x*(a + b*ArcCsc[c*x])^2) - 2*b*(((I/2)*(a + b*ArcCsc[c*x])^2)/b + (2*I)*((
I/2)*(a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*ArcCsc[c*x])] + (b*PolyLog[2, E^
((2*I)*ArcCsc[c*x])])/4))))/2)/c^2)
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.26.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(142) = 284.

Time = 1.59 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{c^2x^2a^3}{2}+b^3\left(\frac{\operatorname{arccsc}(cx)^2\left(c^2x^2\operatorname{arccsc}(cx)+3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}-3\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-3\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}\right.\right.$
default	$\frac{c^2x^2a^3}{2}+b^3\left(\frac{\operatorname{arccsc}(cx)^2\left(c^2x^2\operatorname{arccsc}(cx)+3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}-3\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-3\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}\right.\right.$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{\operatorname{arccsc}(cx)^2\left(c^2x^2\operatorname{arccsc}(cx)+3xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-3i\right)}{2}-3\operatorname{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)-3\operatorname{arccsc}(cx)\ln\left(1+\frac{i}{cx}\right.\right.}{c^2}$

input
```
int(x*(a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(1/2*c^2*x^2*a^3+b^3*(1/2*arccsc(c*x)^2*(c^2*x^2*arccsc(c*x)+3*x*c*(
(c^2*x^2-1)/c^2/x^2)^(1/2)-3*I)-3*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/
2))-3*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+3*I*arccsc(c*x)^2+3*I*po
lylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))+3*I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2
)))+3*a*b^2*(1/2*c^2*x^2*arccsc(c*x)^2+arccsc(c*x)*c*x*((c^2*x^2-1)/c^2/x^
2)^(1/2)-ln(1/c/x))+3*a^2*b*(1/2*c^2*x^2*arccsc(c*x)+1/2/((c^2*x^2-1)/c^2/
x^2)^(1/2)/c/x*(c^2*x^2-1)))
```

3.26.5 Fricas [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int(b\operatorname{arccsc}(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arccsc(c*x)^3 + 3*a*b^2*x*arccsc(c*x)^2 + 3*a^2*b*x*arccsc(
c*x) + a^3*x, x)
```

3.26.6 Sympy [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int x(a+b\operatorname{acsc}(cx))^3\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))**3,x)
```

output
```
Integral(x*(a + b*acsc(c*x))**3, x)
```

3.26.7 Maxima [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arccsc(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsc(c*x) + x*sqrt(
-1/(c^2*x^2) + 1)/c)*a^2*b + 3*(x*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x)/c + l
og(x)/c^2)*a*b^2 + 1/8*(4*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 -
3*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 - 8*integrate
(3/8*(8*c^2*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 8*x*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 + 8*(c^2*x^3*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)) - x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(
x)^2 - (4*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 - x*log(c^2*x^2)^2)*
sqrt(c*x + 1)*sqrt(c*x - 1) - 4*((2*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1))*log(c) + c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 - (2*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c) + arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)))*x + 2*(c^2*x^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - x*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x))*log(c^2*x^2) + 16*(c^2*x^3*ar
ctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c) - x*arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))*log(c))*log(x))/(c^2*x^2 - 1), x))*b^3
```

3.26.8 Giac [F]

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^3*x, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\csc^{-1}(cx)\right)^3\,dx=\int x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

input
```
int(x*(a + b*asin(1/(c*x)))^3,x)
```

output
```
int(x*(a + b*asin(1/(c*x)))^3, x)
```

3.27 $\int (a+b\csc^{-1}(cx))^3\,dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 144

$$\int (a+b\csc^{-1}(cx))^3\,dx = x(a+b\csc^{-1}(cx))^3+\frac{6b(a+b\csc^{-1}(cx))^2\,\text{arctanh}\left(e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6ib^2(a+b\csc^{-1}(cx))\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)}{c} + \frac{6ib^2(a+b\csc^{-1}(cx))\,\text{PolyLog}\left(2,e^{i\csc^{-1}(cx)}\right)}{c} + \frac{6b^3\,\text{PolyLog}\left(3,-e^{i\csc^{-1}(cx)}\right)}{c} - \frac{6b^3\,\text{PolyLog}\left(3,e^{i\csc^{-1}(cx)}\right)}{c}$$

output
```
x*(a+b*arccsc(c*x))^3+6*b*(a+b*arccsc(c*x))^2*arctanh(I/c/x+(1-1/c^2/x^2)^
(1/2))/c-6*I*b^2*(a+b*arccsc(c*x))*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))/c
+6*I*b^2*(a+b*arccsc(c*x))*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))/c+6*b^3*po
lylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2))/c-6*b^3*polylog(3,I/c/x+(1-1/c^2/x^2)^
(1/2))/c
```

3.27.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.84

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx$$
$$=\frac{a^3cx+3a^2bcx\csc^{-1}(cx)+3ab^2cx\csc^{-1}(cx)^2+b^3cx\csc^{-1}(cx)^3-6ab^2\csc^{-1}(cx)\log\left(1-e^{i\csc^{-1}(cx)}\right)-3b}{}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^3,x]
```

output
```
(a^3*c*x + 3*a^2*b*c*x*ArcCsc[c*x] + 3*a*b^2*c*x*ArcCsc[c*x]^2 + b^3*c*x*A
rcCsc[c*x]^3 - 6*a*b^2*ArcCsc[c*x]*Log[1 - E^(I*ArcCsc[c*x])] - 3*b^3*ArcC
sc[c*x]^2*Log[1 - E^(I*ArcCsc[c*x])] + 6*a*b^2*ArcCsc[c*x]*Log[1 + E^(I*Ar
cCsc[c*x])] + 3*b^3*ArcCsc[c*x]^2*Log[1 + E^(I*ArcCsc[c*x])] + 3*a^2*b*Log
[c*(1 + Sqrt[1 - 1/(c^2*x^2)])*x] - (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[
2, -E^(I*ArcCsc[c*x])] + (6*I)*b^2*(a + b*ArcCsc[c*x])*PolyLog[2, E^(I*Arc
Csc[c*x])] + 6*b^3*PolyLog[3, -E^(I*ArcCsc[c*x])] - 6*b^3*PolyLog[3, E^(I*
ArcCsc[c*x])])/c
```

3.27.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5740, 4910, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx$$
$$\Big\downarrow \text{5740}$$
$$-\frac{\int c^2\sqrt{1-\frac{1}{c^2x^2}}x^2\left(a+b\csc^{-1}(cx)\right)^3d\csc^{-1}(cx)}{c}$$
$$\Big\downarrow \text{4910}$$
$$-\frac{3b\int cx\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-cx\left(a+b\csc^{-1}(cx)\right)^3}{c}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{3b\int \left(a+b\csc^{-1}(cx)\right)^2 \csc\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx) - cx\left(a+b\csc^{-1}(cx)\right)^3}{c}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{-cx\left(a+b\csc^{-1}(cx)\right)^3 + 3b\left(-2b\int \left(a+b\csc^{-1}(cx)\right)\log\left(1-e^{i\csc^{-1}(cx)}\right)d\csc^{-1}(cx) + 2b\int \left(a+b\csc^{-1}(cx)\right)\text{lo}\right.}{c}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{-cx\left(a+b\csc^{-1}(cx)\right)^3 + 3b\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right) - ib\int \text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)d\right.\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{-cx\left(a+b\csc^{-1}(cx)\right)^3 + 3b\left(2b\left(i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right) - b\int e^{-i\csc^{-1}(cx)}\,\text{PolyLog}\left(2,-e^{i}\right.\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{-cx\left(a+b\csc^{-1}(cx)\right)^3 + 3b\left(-2\text{arctanh}\left(e^{i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2 + 2b\left(i\,\text{PolyLog}\left(2,-e^{i\csc^{-1}(cx)}\right)\left(a+b\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcCsc[c*x])^3,x]
```

output
```
-((-(c*x*(a + b*ArcCsc[c*x])^3) + 3*b*(-2*(a + b*ArcCsc[c*x])^2*ArcTanh[E^
(I*ArcCsc[c*x])] + 2*b*(I*(a + b*ArcCsc[c*x])*PolyLog[2, -E^(I*ArcCsc[c*x]
)] - b*PolyLog[3, -E^(I*ArcCsc[c*x])]) - 2*b*(I*(a + b*ArcCsc[c*x])*PolyLo
g[2, E^(I*ArcCsc[c*x])] - b*PolyLog[3, E^(I*ArcCsc[c*x])])))/c)
```

3.27.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 5740
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   Su
bst[Int[(a + b*x)^n*Csc[x]*Cot[x], x], x, ArcCsc[c*x]], x] /; FreeQ[{a, b,
c, n}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.27. $\int \left(a+b\csc^{-1}(cx)\right)^3 \, dx$

3.27.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.62

method	result
derivativedivides	$cx\,a^3+b^3\left(\operatorname{arccsc}(cx)^3 cx-3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+6i\operatorname{arccsc}(cx)\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-6\operatorname{polylog}\left(3,\right.\right.$
default	$cx\,a^3+b^3\left(\operatorname{arccsc}(cx)^3 cx-3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+6i\operatorname{arccsc}(cx)\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-6\operatorname{polylog}\left(3,\right.\right.$
parts	$a^3x+\frac{b^3\left(\operatorname{arccsc}(cx)^3 cx-3\operatorname{arccsc}(cx)^2\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+6i\operatorname{arccsc}(cx)\operatorname{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-6\operatorname{polylog}\left(\right.\right.}{}$

input
```
int((a+b*arccsc(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^3+b^3*(arccsc(c*x)^3*c*x-3*arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^
2)^(1/2))+6*I*arccsc(c*x)*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-6*polylog(3
,I/c/x+(1-1/c^2/x^2)^(1/2))+3*arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2)
)-6*I*arccsc(c*x)*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))+6*polylog(3,-I/c/x
-(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(arccsc(c*x)^2*c*x-2*arccsc(c*x)*ln(1-I/c/x
-(1-1/c^2/x^2)^(1/2))+2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-2*I*di
log(1+I/c/x+(1-1/c^2/x^2)^(1/2))+2*I*dilog(1-I/c/x-(1-1/c^2/x^2)^(1/2)))+3
*a^2*b*(arccsc(c*x)*c*x+ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))))
```

3.27.5 Fricas [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int \left(b\operatorname{arccsc}(cx)+a\right)^3\,dx$$

input
```
integrate((a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x) +
 a^3, x)
```

3.27.6 Sympy [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int \left(a+b\,\mathrm{acsc}\,(cx)\right)^3\,dx$$

input
```
integrate((a+b*acsc(c*x))**3,x)
```

output
```
Integral((a + b*acsc(c*x))**3, x)
```

3.27.7 Maxima [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

output
```
-3/2*a*b^2*c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*log(c)^2 -
12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*
x^2 - 1), x)*log(c)^2 + b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 -
3/4*b^3*x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*b^3*
c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)
/(c^2*x^2 - 1), x)*log(c) - 24*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*
x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*a*b^2*c^2*inte
grate(1/4*x^2*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c) - 24*a*b^2*c^2*integra
te(1/4*x^2*log(x)/(c^2*x^2 - 1), x)*log(c) + 12*b^3*c^2*integrate(1/4*x^2*
arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^2 - 1),
 x) - 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
*log(x)^2/(c^2*x^2 - 1), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqr
t(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^2 - 1), x) + 12*b^3*c^2*integrate(1/4*
x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)
 - 3*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)^2/(c^2*x^2 - 1), x) + 12*a*b
^2*c^2*integrate(1/4*x^2*log(c^2*x^2)*log(x)/(c^2*x^2 - 1), x) - 12*a*b^2*
c^2*integrate(1/4*x^2*log(x)^2/(c^2*x^2 - 1), x) - 3/2*a*b^2*(log(c*x + 1)
/c - log(c*x - 1)/c)*log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x +
1)*sqrt(c*x - 1)))/(c^2*x^2 - 1), x)*log(c)^2 - 12*b^3*integrate(1/4*arcta
n(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^2 - 1), x)*log(c...
```

3.27.8 Giac [F]

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arccsc(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^3, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\csc^{-1}(cx)\right)^3\,dx=\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

input
```
int((a + b*asin(1/(c*x)))^3,x)
```

output
```
int((a + b*asin(1/(c*x)))^3, x)
```

3.28 $\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx$

3.28.1 Optimal result

Integrand size = 14, antiderivative size = 124

$$\begin{aligned}\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx &= \frac{i(a+b\csc^{-1}(cx))^4}{4b}-(a+b\csc^{-1}(cx))^3\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\&\quad+\frac{3}{2}ib(a+b\csc^{-1}(cx))^2\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\\&\quad-\frac{3}{2}b^2(a+b\csc^{-1}(cx))\operatorname{PolyLog}\left(3,e^{2i\csc^{-1}(cx)}\right)\\&\quad-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

output
```
1/4*I*(a+b*arccsc(c*x))^4/b-(a+b*arccsc(c*x))^3*ln(1-(I/c/x+(1-1/c^2/x^2)^
(1/2))^2)+3/2*I*b*(a+b*arccsc(c*x))^2*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2)
)^2)-3/2*b^2*(a+b*arccsc(c*x))*polylog(3,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-3/
4*I*b^3*polylog(4,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.95

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx = a^3\log(cx)+\frac{3}{2}ia^2b\Big(\csc^{-1}(cx)\left(\csc^{-1}(cx)+2i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\Big)+\frac{1}{8}iab^2\Big(\pi^3-8\csc^{-1}(cx)^3+24i\csc^{-1}(cx)^2\log\left(1-e^{-2i\csc^{-1}(cx)}\right)-24\csc^{-1}(cx)\,\text{PolyLog}\left(2,e^{-2i\csc^{-1}(cx)}\right)+12i\,\text{PolyLog}\left(3,e^{-2i\csc^{-1}(cx)}\right)\Big)+\frac{1}{64}ib^3\Big(\pi^4-16\csc^{-1}(cx)^4+64i\csc^{-1}(cx)^3\log\left(1-e^{-2i\csc^{-1}(cx)}\right)-96\csc^{-1}(cx)^2\,\text{PolyLog}\left(2,e^{-2i\csc^{-1}(cx)}\right)+96i\csc^{-1}(cx)\,\text{PolyLog}\left(3,e^{-2i\csc^{-1}(cx)}\right)+48\,\text{PolyLog}\left(4,e^{-2i\csc^{-1}(cx)}\right)\Big)$$

input
```
Integrate[(a + b*ArcCsc[c*x])^3/x,x]
```

output
```
a^3*Log[c*x] + ((3*I)/2)*a^2*b*(ArcCsc[c*x]*(ArcCsc[c*x] + (2*I)*Log[1 - E
^((2*I)*ArcCsc[c*x])]) + PolyLog[2, E^((2*I)*ArcCsc[c*x])]) + (I/8)*a*b^2*
(Pi^3 - 8*ArcCsc[c*x]^3 + (24*I)*ArcCsc[c*x]^2*Log[1 - E^((-2*I)*ArcCsc[c*
x])] - 24*ArcCsc[c*x]*PolyLog[2, E^((-2*I)*ArcCsc[c*x])] + (12*I)*PolyLog[
3, E^((-2*I)*ArcCsc[c*x])]) + (I/64)*b^3*(Pi^4 - 16*ArcCsc[c*x]^4 + (64*I)
*ArcCsc[c*x]^3*Log[1 - E^((-2*I)*ArcCsc[c*x])] - 96*ArcCsc[c*x]^2*PolyLog[
2, E^((-2*I)*ArcCsc[c*x])] + (96*I)*ArcCsc[c*x]*PolyLog[3, E^((-2*I)*ArcCs
c[c*x])] + 48*PolyLog[4, E^((-2*I)*ArcCsc[c*x])])
```

3.28.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5746, 3042, 25, 4200, 25, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\,dx$$

$$\Big\downarrow \text{5746}$$

$$-\int c\sqrt{1-\frac{1}{c^2x^2}}\,x\left(a+b\csc^{-1}(cx)\right)^3 d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\int -\left(a+b\csc^{-1}(cx)\right)^3\tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)$$

$$\Big\downarrow \text{25}$$

$$\int \tan\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)\left(a+b\csc^{-1}(cx)\right)^3 d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4200}$$

$$\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}-2i\int -\frac{e^{2i\csc^{-1}(cx)}\left(a+b\csc^{-1}(cx)\right)^3}{1-e^{2i\csc^{-1}(cx)}}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{25}$$

$$2i\int \frac{e^{2i\csc^{-1}(cx)}\left(a+b\csc^{-1}(cx)\right)^3}{1-e^{2i\csc^{-1}(cx)}}d\csc^{-1}(cx)+\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^3-\frac{3}{2}ib\int \left(a+b\csc^{-1}(cx)\right)^2\log\left(1-e^{2i\csc^{-1}(cx)}\right)d\csc^{-1}(cx)\right)+\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^3-\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-ib\int (a+\right.\right.$$
$$\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}$$

$$\Big\downarrow \text{7163}$$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^3-\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-ib\left(\frac{1}{2}ib\int\right.\right.\right.$$
$$\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}$$

$$\Big\downarrow \text{2720}$$

3.28. $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x}\,dx$

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^3-\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-ib\left(\frac{1}{4}b\int e\right.\right.\right.$$
$$\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}$$

$\downarrow$ 7143

$$2i\left(\frac{1}{2}i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^3-\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\left(a+b\csc^{-1}(cx)\right)^2-ib\left(\frac{1}{4}b\,\text{Pol}\right.\right.\right.$$
$$\frac{i\left(a+b\csc^{-1}(cx)\right)^4}{4b}$$

input
```
Int[(a + b*ArcCsc[c*x])^3/x,x]
```

output
```
((I/4)*(a + b*ArcCsc[c*x])^4)/b + (2*I)*((I/2)*(a + b*ArcCsc[c*x])^3*Log[1
 - E^((2*I)*ArcCsc[c*x])] - ((3*I)/2)*b*((I/2)*(a + b*ArcCsc[c*x])^2*PolyL
og[2, E^((2*I)*ArcCsc[c*x])] - I*b*((-1/2*I)*(a + b*ArcCsc[c*x])*PolyLog[3
, E^((2*I)*ArcCsc[c*x])] + (b*PolyLog[4, E^((2*I)*ArcCsc[c*x])])/4)))
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.28. $\int \frac{\left(a+b \csc^{-1}(cx)\right)^3}{x}\, dx$

3.28.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(169) = 338$.

Time = 1.20 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.90

method	result
parts	$a^3 \ln(x) + b^3\left(\frac{i \operatorname{arccsc}(cx)^4}{4} - \operatorname{arccsc}(cx)^3 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 3i \operatorname{arccsc}(cx)^2 \operatorname{polyl}\right.$
derivativedivides	$a^3 \ln(cx) + b^3\left(\frac{i \operatorname{arccsc}(cx)^4}{4} - \operatorname{arccsc}(cx)^3 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 3i \operatorname{arccsc}(cx)^2 \operatorname{poly}\right.$
default	$a^3 \ln(cx) + b^3\left(\frac{i \operatorname{arccsc}(cx)^4}{4} - \operatorname{arccsc}(cx)^3 \ln\left(1 - \frac{i}{cx} - \sqrt{1 - \frac{1}{c^2x^2}}\right) + 3i \operatorname{arccsc}(cx)^2 \operatorname{poly}\right.$

input
```
int((a+b*arccsc(c*x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(1/4*I*arccsc(c*x)^4-arccsc(c*x)^3*ln(1-I/c/x-(1-1/c^2/x^2)^
(1/2))+3*I*arccsc(c*x)^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-6*arccsc(c*x
)*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))-6*I*polylog(4,I/c/x+(1-1/c^2/x^2)^(
1/2))-arccsc(c*x)^3*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+3*I*arccsc(c*x)^2*poly
log(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-6*arccsc(c*x)*polylog(3,-I/c/x-(1-1/c^2/
x^2)^(1/2))-6*I*polylog(4,-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a*b^2*(1/3*I*arcc
sc(c*x)^3-arccsc(c*x)^2*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+2*I*arccsc(c*x)*po
lylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))-2*polylog(3,I/c/x+(1-1/c^2/x^2)^(1/2))-
arccsc(c*x)^2*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+2*I*arccsc(c*x)*polylog(2,-I
/c/x-(1-1/c^2/x^2)^(1/2))-2*polylog(3,-I/c/x-(1-1/c^2/x^2)^(1/2)))+3*a^2*b
*(1/2*I*arccsc(c*x)^2-arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*polylo
g(2,I/c/x+(1-1/c^2/x^2)^(1/2))-arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))
+I*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2)))
```

3.28.5 Fricas [F]

$$\int \frac{(a + b \csc^{-1}(cx))^3}{x} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arccsc(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x)
+ a^3)/x, x)
```

3.28.6 Sympy [F]

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx = \int \frac{(a+b\operatorname{acsc}(cx))^3}{x}\,dx$$

input
```
integrate((a+b*acsc(c*x))**3/x,x)
```

output
```
Integral((a + b*acsc(c*x))**3/x, x)
```

3.28.7 Maxima [F]

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))^3/x,x, algorithm="maxima")
```

output
```
-3/2*a*b^2*c^2*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*log(c)^2 - 12*b^3*c^2
*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^3 - x),
x)*log(c)^2 + 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*
x - 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) - 24*b^3*c^2*integrate(1/4*
x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^3 - x), x)*log(c
) + 12*a*b^2*c^2*integrate(1/4*x^2*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) -
 24*a*b^2*c^2*integrate(1/4*x^2*log(x)/(c^2*x^3 - x), x)*log(c) + b^3*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3*log(x) - 3/4*b^3*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2*log(x) + 24*b^3*c^2*integrate(1/4*x^2*
arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^3 - x),
 x) - 12*b^3*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))
*log(x)^2/(c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*arctan(1/(sqr
t(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^3 - x), x) - 3*a*b^2*c^2*integrate(1/4
*x^2*log(c^2*x^2)^2/(c^2*x^3 - x), x) + 12*a*b^2*c^2*integrate(1/4*x^2*log
(c^2*x^2)*log(x)/(c^2*x^3 - x), x) - 12*a*b^2*c^2*integrate(1/4*x^2*log(x)
^2/(c^2*x^3 - x), x) + 12*a^2*b*c^2*integrate(1/4*x^2*arctan(1/(sqrt(c*x +
 1)*sqrt(c*x - 1)))/(c^2*x^3 - x), x) + 3/2*a*b^2*(log(c*x + 1) + log(c*x
- 1) - 2*log(x))*log(c)^2 + 12*b^3*integrate(1/4*arctan(1/(sqrt(c*x + 1)*s
qrt(c*x - 1)))/(c^2*x^3 - x), x)*log(c)^2 - 12*b^3*integrate(1/4*arctan(1/
(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^3 - x), x)*log(c) + ...
```

3.28.8 Giac [F]

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)^3}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^3/x, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x}\,dx = \int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*asin(1/(c*x)))^3/x,x)
```

output
```
int((a + b*asin(1/(c*x)))^3/x, x)
```

3.29 $\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx$

3.29.1 Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx = 6b^3c\sqrt{1-\frac{1}{c^2x^2}}+\frac{6b^2(a+b\csc^{-1}(cx))}{x} - 3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2-\frac{(a+b\csc^{-1}(cx))^3}{x}$$

output
```
6*b^2*(a+b*arccsc(c*x))/x-(a+b*arccsc(c*x))^3/x+6*b^3*c*(1-1/c^2/x^2)^(1/2
)-3*b*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.69

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx = -\frac{a^3-6ab^2+3a^2bc\sqrt{1-\frac{1}{c^2x^2}}x-6b^3c\sqrt{1-\frac{1}{c^2x^2}}x+3b\left(a^2-2b^2+2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\csc^{-1}(cx)+3b^2\left(a\right.}{x}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^3/x^2,x]
```

output
```
-((a^3 - 6*a*b^2 + 3*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 - 1/
(c^2*x^2)]*x + 3*b*(a^2 - 2*b^2 + 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[
c*x] + 3*b^2*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCsc[c*x]^2 + b^3*ArcCsc[
c*x]^3)/x)
```

3.29.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5746, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{5746}$$

$$-c\int \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^3d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-c\int \left(a+b\csc^{-1}(cx)\right)^3\sin\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3777}$$

$$-c\left(3b\int -\frac{\left(a+b\csc^{-1}(cx)\right)^2}{cx}d\csc^{-1}(cx)+\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}\right)$$

$$\Big\downarrow \text{25}$$

$$-c\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}-3b\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{cx}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}-3b\int \left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3777}$$

$$-c\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}-3b\left(2b\int \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)\right)$$

$$\Big\downarrow \; 3042$$

$$-c\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}-3b\left(2b\int\left(a+b\csc^{-1}(cx)\right)\sin\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(c\right.\right.\right.$$

$$\Big\downarrow \; 3777$$

$$-c\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}-3b\left(2b\left(b\int-\frac{1}{cx}d\csc^{-1}(cx)+\frac{a+b\csc^{-1}(cx)}{cx}\right)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)\right)$$

$$\Big\downarrow \; 25$$

$$-c\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}-3b\left(2b\left(\frac{a+b\csc^{-1}(cx)}{cx}-b\int\frac{1}{cx}d\csc^{-1}(cx)\right)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)\right)$$

$$\Big\downarrow \; 3042$$

$$-c\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}-3b\left(2b\left(\frac{a+b\csc^{-1}(cx)}{cx}-b\int\sin\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)\right)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(c\right.\right.\right.$$

$$\Big\downarrow \; 3118$$

$$-c\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{cx}-3b\left(2b\left(\frac{a+b\csc^{-1}(cx)}{cx}+b\sqrt{1-\frac{1}{c^2x^2}}\right)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\right)\right)$$

input
```
Int[(a + b*ArcCsc[c*x])^3/x^2,x]
```

output
```
-(c*((a + b*ArcCsc[c*x])^3/(c*x) - 3*b*(-(Sqrt[1 - 1/(c^2*x^2)]*(a + b*Arc
Csc[c*x])^2) + 2*b*(b*Sqrt[1 - 1/(c^2*x^2)] + (a + b*ArcCsc[c*x])/(c*x))))
)
```

3.29. $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^2}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(76) = 152$.

Time = 0.89 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.46

method	result
parts	$-\frac{a^3}{x}+b^3c\left(-\frac{\operatorname{arccsc}(cx)^3}{cx}-3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arccsc}(cx)}{cx}\right)+3a\,b^2c\Big(-$
derivativedivides	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{cx}-3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arccsc}(cx)}{cx}\right)+3a\,b^2\Big(-\right.$
default	$c\left(-\frac{a^3}{cx}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{cx}-3\operatorname{arccsc}(cx)^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}+6\sqrt{\frac{c^2x^2-1}{c^2x^2}}+\frac{6\operatorname{arccsc}(cx)}{cx}\right)+3a\,b^2\Big(-\right.$

input
```
int((a+b*arccsc(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x+b^3*c*(-1/c/x*arccsc(c*x)^3-3*arccsc(c*x)^2*((c^2*x^2-1)/c^2/x^2)^(
1/2)+6*((c^2*x^2-1)/c^2/x^2)^(1/2)+6/c/x*arccsc(c*x))+3*a*b^2*c*(-1/c/x*ar
ccsc(c*x)^2+2/c/x-2*arccsc(c*x)*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a^2*b*c*(-1
/c/x*arccsc(c*x)-1/((c^2*x^2-1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x^2-1))
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx =$$
$$-\frac{b^3\operatorname{arccsc}(cx)^3 + 3\,ab^2\operatorname{arccsc}(cx)^2 + a^3 - 6\,ab^2 + 3\,(a^2b - 2\,b^3)\operatorname{arccsc}(cx) + 3\,\big(b^3\operatorname{arccsc}(cx)^2 + 2\,ab^2\,a}{x}$$

input
```
integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
-(b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + a^3 - 6*a*b^2 + 3*(a^2*b - 2
*b^3)*arccsc(c*x) + 3*(b^3*arccsc(c*x)^2 + 2*a*b^2*arccsc(c*x) + a^2*b - 2
*b^3)*sqrt(c^2*x^2 - 1))/x
```

3.29.6 Sympy [F]

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx = \int \frac{(a+b\operatorname{acsc}(cx))^3}{x^2}\,dx$$

input
```
integrate((a+b*acsc(c*x))**3/x**2,x)
```

output
```
Integral((a + b*acsc(c*x))**3/x**2, x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.84

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx = -\frac{b^3\operatorname{arccsc}(cx)^3}{x} - 3\left(c\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\operatorname{arccsc}(cx)}{x}\right)a^2b - 6\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx) - \frac{1}{x}\right)ab^2 - 3\left(c\sqrt{-\frac{1}{c^2x^2}+1}\operatorname{arccsc}(cx)^2 - 2\,c\sqrt{-\frac{1}{c^2x^2}+1} - \frac{2\operatorname{arccsc}(cx)}{x}\right)b^3 - \frac{3\,ab^2\operatorname{arccsc}(cx)^2}{x} - \frac{a^3}{x}$$

input
```
integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
-b^3*arccsc(c*x)^3/x - 3*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*a^2*b
- 6*(c*sqrt(-1/(c^2*x^2) + 1)*arccsc(c*x) - 1/x)*a*b^2 - 3*(c*sqrt(-1/(c^2
*x^2) + 1)*arccsc(c*x)^2 - 2*c*sqrt(-1/(c^2*x^2) + 1) - 2*arccsc(c*x)/x)*b
^3 - 3*a*b^2*arccsc(c*x)^2/x - a^3/x
```

3.29.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(76) = 152.

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.44

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx = -\left(3\,b^3\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right)^2 + 6\,ab^2\sqrt{-\frac{1}{c^2x^2}+1}\arcsin\left(\frac{1}{cx}\right) + \frac{b^3\arcsin\left(\frac{1}{cx}\right)^3}{cx} + 3\,a^2b\sqrt{-\frac{1}{c^2x^2}+ \cdots\right.$$

input
```
integrate((a+b*arccsc(c*x))^3/x^2,x, algorithm="giac")
```

output
```
-(3*b^3*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))^2 + 6*a*b^2*sqrt(-1/(c^2*x^
2) + 1)*arcsin(1/(c*x)) + b^3*arcsin(1/(c*x))^3/(c*x) + 3*a^2*b*sqrt(-1/(c
^2*x^2) + 1) - 6*b^3*sqrt(-1/(c^2*x^2) + 1) + 3*a*b^2*arcsin(1/(c*x))^2/(c
*x) + 3*a^2*b*arcsin(1/(c*x))/(c*x) - 6*b^3*arcsin(1/(c*x))/(c*x) + a^3/(c
*x) - 6*a*b^2/(c*x))*c
```

3.29.9 Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.94

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^2}\,dx = \frac{b^3\left(6\,\mathrm{asin}\left(\frac{1}{c\,x}\right)-\mathrm{asin}\left(\frac{1}{c\,x}\right)^3\right)}{x}-\frac{a^3}{x}-3\,a^2\,b\,c\left(\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)}{c\,x}\right)-b^3\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\left(3\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-6\right)-3\,a\,b^2\,c\left(2\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\sqrt{1-\frac{1}{c^2\,x^2}}+\frac{\mathrm{asin}\left(\frac{1}{c\,x}\right)^2-2}{c\,x}\right)$$

input
```
int((a + b*asin(1/(c*x)))^3/x^2,x)
```

output
```
(b^3*(6*asin(1/(c*x)) - asin(1/(c*x))^3))/x - a^3/x - 3*a^2*b*c*((1 - 1/(c
^2*x^2))^(1/2) + asin(1/(c*x))/(c*x)) - b^3*c*(1 - 1/(c^2*x^2))^(1/2)*(3*a
sin(1/(c*x))^2 - 6) - 3*a*b^2*c*(2*asin(1/(c*x))*(1 - 1/(c^2*x^2))^(1/2) +
 (asin(1/(c*x))^2 - 2)/(c*x))
```

3.30 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx$

3.30.1 Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx = \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2\csc^{-1}(cx) + \frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{4x^2} - \frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{4x} + \frac{1}{4}c^2\left(a+b\csc^{-1}(cx)\right)^3 - \frac{\left(a+b\csc^{-1}(cx)\right)^3}{2x^2}$$

output
```
-3/8*b^3*c^2*arccsc(c*x)+3/4*b^2*(a+b*arccsc(c*x))/x^2+1/4*c^2*(a+b*arccsc
(c*x))^3-1/2*(a+b*arccsc(c*x))^3/x^2+3/8*b^3*c*(1-1/c^2/x^2)^(1/2)/x-3/4*b
*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

3.30.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.49

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx = \frac{-4a^3+6ab^2-6a^2bc\sqrt{1-\frac{1}{c^2x^2}}x+3b^3c\sqrt{1-\frac{1}{c^2x^2}}x+6b\left(-2a^2+b^2-2abc\sqrt{1-\frac{1}{c^2x^2}}x\right)\csc^{-1}(cx)-6b^2}{8x}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^3/x^3,x]
```

output
```
(-4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 3*b^3*c*Sqrt[1 - 1
/(c^2*x^2)]*x + 6*b*(-2*a^2 + b^2 - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)*ArcCs
c[c*x] - 6*b^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x + a*(2 - c^2*x^2))*ArcCsc[c*x]
^2 + 2*b^3*(-2 + c^2*x^2)*ArcCsc[c*x]^3 - 3*b*(-2*a^2 + b^2)*c^2*x^2*ArcSi
n[1/(c*x)])/(8*x^2)
```

3.30.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5746, 4904, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b \csc^{-1}(cx)\right)^3}{x^3} \, dx$$

$$\Big\downarrow \text{5746}$$

$$-c^2 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b \csc^{-1}(cx)\right)^3}{cx}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4904}$$

$$-c^2 \left(\frac{\left(a+b \csc^{-1}(cx)\right)^3}{2c^2x^2}-\frac{3}{2}b\int \frac{\left(a+b \csc^{-1}(cx)\right)^2}{c^2x^2}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^2 \left(\frac{\left(a+b \csc^{-1}(cx)\right)^3}{2c^2x^2}-\frac{3}{2}b\int \left(a+b \csc^{-1}(cx)\right)^2 \sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3792}$$

$$-c^2 \left(\frac{\left(a+b \csc^{-1}(cx)\right)^3}{2c^2x^2}-\frac{3}{2}b\left(\frac{1}{2}\int \left(a+b \csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{2}b^2\int \frac{1}{c^2x^2}d\csc^{-1}(cx)+\frac{b\left(a+b \csc^{-1}(cx)\right)}{2c^2x^2}\right.\right.$$

$$\Big\downarrow \text{17}$$

$$-c^2\left(\frac{(a+b\csc^{-1}(cx))^3}{2c^2x^2}-\frac{3}{2}b\left(-\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2cx}+\frac{b(a+b\csc^{-1}(cx))}{2c^2x^2}+\right.\right.$$

$\big\downarrow$ 3042

$$-c^2\left(\frac{(a+b\csc^{-1}(cx))^3}{2c^2x^2}-\frac{3}{2}b\left(-\frac{1}{2}b^2\int\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2cx}+\frac{b(a+b\,\mathrm{cs}}{2c^2}\right.\right.$$

$\big\downarrow$ 3115

$$-c^2\left(\frac{(a+b\csc^{-1}(cx))^3}{2c^2x^2}-\frac{3}{2}b\left(-\frac{1}{2}b^2\left(\frac{1}{2}\int 1d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}\right)-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2cx}+\frac{b(a+}{}\right.\right.$$

$\big\downarrow$ 24

$$-c^2\left(\frac{(a+b\csc^{-1}(cx))^3}{2c^2x^2}-\frac{3}{2}b\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2cx}+\frac{b(a+b\csc^{-1}(cx))}{2c^2x^2}+\frac{(a+b\csc^{-1}(cx))^3}{6b}-\frac{1}{2}b^2\right.\right.$$

input
```
Int[(a + b*ArcCsc[c*x])^3/x^3,x]
```

output
```
-(c^2*((a + b*ArcCsc[c*x])^3/(2*c^2*x^2) - (3*b*(-1/2*(b^2*(-1/2*Sqrt[1 -
1/(c^2*x^2)]/(c*x) + ArcCsc[c*x]/2)) + (b*(a + b*ArcCsc[c*x]))/(2*c^2*x^2)
 - (Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/(2*c*x) + (a + b*ArcCsc[c
*x])^3/(6*b)))/2))
```

3.30.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.30.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(109) = 218$.

Time = 1.07 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.54

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^3}{2c^2x^2}-\frac{3\operatorname{arccsc}(cx)^2\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}-\frac{3(c^2x^2-1)\operatorname{arccsc}(cx)}{4c^2x^2}\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^3}{2c^2x^2}-\frac{3\operatorname{arccsc}(cx)^2\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}-\frac{3(c^2x^2-1)\operatorname{arccsc}(cx)}{4c^2x^2}\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(\frac{(c^2x^2-1)\operatorname{arccsc}(cx)^3}{2c^2x^2}-\frac{3\operatorname{arccsc}(cx)^2\left(\operatorname{arccsc}(cx)cx+\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{4cx}-\frac{3(c^2x^2-1)\operatorname{arccsc}(cx)}{4c^2x^2}+\frac{3}{\ldots}\right.$

input
```
int((a+b*arccsc(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^3/c^2/x^2+b^3*(1/2*(c^2*x^2-1)/c^2/x^2*arccsc(c*x)^3-3/4*arccs
c(c*x)^2*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x-3/4*(c^2*x^2-1)
/c^2/x^2*arccsc(c*x)+3/8/c/x*((c^2*x^2-1)/c^2/x^2)^(1/2)+3/8*arccsc(c*x)+1
/2*arccsc(c*x)^3)+3*a*b^2*(1/2*(c^2*x^2-1)/c^2/x^2*arccsc(c*x)^2-1/2*arccs
c(c*x)*(arccsc(c*x)*c*x+((c^2*x^2-1)/c^2/x^2)^(1/2))/c/x+1/4*arccsc(c*x)^2
+1/4/c^2/x^2)+3*a^2*b*(-1/2/c^2/x^2*arccsc(c*x)-1/4*(c^2*x^2-1)^(1/2)*(-ar
ctan(1/(c^2*x^2-1)^(1/2))*c^2*x^2+(c^2*x^2-1)^(1/2))/((c^2*x^2-1)/c^2/x^2)
^(1/2)/c^3/x^3))
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^3}\,dx = \frac{2\,(b^3c^2x^2-2\,b^3)\,\mathrm{arccsc}\,(cx)^3-4\,a^3+6\,ab^2+6\,(ab^2c^2x^2-2\,ab^2)\,\mathrm{arccsc}\,(cx)^2+3\,((2\,a^2b-b^3)c^2x^2-4\,a^2b}{8\,x^2}$$

input
```
integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="fricas")
```

output
```
1/8*(2*(b^3*c^2*x^2 - 2*b^3)*arccsc(c*x)^3 - 4*a^3 + 6*a*b^2 + 6*(a*b^2*c^
2*x^2 - 2*a*b^2)*arccsc(c*x)^2 + 3*((2*a^2*b - b^3)*c^2*x^2 - 4*a^2*b + 2*
b^3)*arccsc(c*x) - 3*(2*b^3*arccsc(c*x)^2 + 4*a*b^2*arccsc(c*x) + 2*a^2*b
- b^3)*sqrt(c^2*x^2 - 1))/x^2
```

3.30.6 Sympy [F]

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))^3}{x^3}\,dx$$

input
```
integrate((a+b*acsc(c*x))**3/x**3,x)
```

output
```
Integral((a + b*acsc(c*x))**3/x**3, x)
```

3.30.7 Maxima [F]

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(b\operatorname{arccsc}\left(cx\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="maxima")
```

output
```
3/4*a^2*b*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) -
 c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) - 1/2*a^3/
x^2 - 1/8*(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(a*b^2*c^2*(log(c*x +
 1) + log(c*x - 1) - 2*log(x))*log(c)^2 + 16*b^3*c^2*integrate(1/8*x^2*arc
tan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^5 - x^3), x)*log(c)^2 - 16*b^3
*c^2*integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2
)/(c^2*x^5 - x^3), x)*log(c) + 32*b^3*c^2*integrate(1/8*x^2*arctan(1/(sqrt
(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*a*b^2*c^2
*integrate(1/8*x^2*log(c^2*x^2)/(c^2*x^5 - x^3), x)*log(c) + 32*a*b^2*c^2*
integrate(1/8*x^2*log(x)/(c^2*x^5 - x^3), x)*log(c) - 16*b^3*c^2*integrate
(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)*log(x)/(c^2*
x^5 - x^3), x) + 16*b^3*c^2*integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt
(c*x - 1)))*log(x)^2/(c^2*x^5 - x^3), x) - 16*a*b^2*c^2*integrate(1/8*x^2*
arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))^2/(c^2*x^5 - x^3), x) + 8*b^3*c^2*
integrate(1/8*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^
2*x^5 - x^3), x) + 4*a*b^2*c^2*integrate(1/8*x^2*log(c^2*x^2)^2/(c^2*x^5 -
 x^3), x) - 16*a*b^2*c^2*integrate(1/8*x^2*log(c^2*x^2)*log(x)/(c^2*x^5 -
x^3), x) + 16*a*b^2*c^2*integrate(1/8*x^2*log(x)^2/(c^2*x^5 - x^3), x) - (
c^2*log(c*x + 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*log(c...
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(109) = 218$.

Time = 0.32 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.42

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^3}\,dx=$$

$$-\frac{1}{8}\left(4\,b^3c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^3+12\,ab^2c\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)^2+2\,b^3c\arcsin\left(\frac{1}{cx}\right)^3+12\,a^2bc\right.$$

input
```
integrate((a+b*arccsc(c*x))^3/x^3,x, algorithm="giac")
```

output
```
-1/8*(4*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^3 + 12*a*b^2*c*(1/(c^2*x^2
) - 1)*arcsin(1/(c*x))^2 + 2*b^3*c*arcsin(1/(c*x))^3 + 12*a^2*b*c*(1/(c^2*
x^2) - 1)*arcsin(1/(c*x)) - 6*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 6*
a*b^2*c*arcsin(1/(c*x))^2 + 4*a^3*c*(1/(c^2*x^2) - 1) - 6*a*b^2*c*(1/(c^2*
x^2) - 1) + 6*a^2*b*c*arcsin(1/(c*x)) - 3*b^3*c*arcsin(1/(c*x)) + 6*b^3*sq
rt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))^2/x - 3*a*b^2*c + 12*a*b^2*sqrt(-1/(c
^2*x^2) + 1)*arcsin(1/(c*x))/x + 6*a^2*b*sqrt(-1/(c^2*x^2) + 1)/x - 3*b^3*
sqrt(-1/(c^2*x^2) + 1)/x)*c
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*asin(1/(c*x)))^3/x^3,x)
```

output
```
int((a + b*asin(1/(c*x)))^3/x^3, x)
```

3.31 $\int \frac{(a+b\csc^{-1}(cx))^3}{x^4}\,dx$

3.31.1 Optimal result

Integrand size = 14, antiderivative size = 170

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^4}\,dx = \frac{14}{9}b^3c^3\sqrt{1-\frac{1}{c^2x^2}} - \frac{2}{27}b^3c^3\left(1-\frac{1}{c^2x^2}\right)^{3/2} + \frac{2b^2(a+b\csc^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a+b\csc^{-1}(cx))}{3x} - \frac{2}{3}bc^3\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2 - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,(a+b\csc^{-1}(cx))^2}{3x^2} - \frac{(a+b\csc^{-1}(cx))^3}{3x^3}$$

output
```
-2/27*b^3*c^3*(1-1/c^2/x^2)^(3/2)+2/9*b^2*(a+b*arccsc(c*x))/x^3+4/3*b^2*c^
2*(a+b*arccsc(c*x))/x-1/3*(a+b*arccsc(c*x))^3/x^3+14/9*b^3*c^3*(1-1/c^2/x^
2)^(1/2)-2/3*b*c^3*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)-1/3*b*c*(a+b*ar
ccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^2
```

3.31.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^4}\,dx = \frac{-9a^3 - 9a^2bc\sqrt{1-\frac{1}{c^2x^2}}\,x(1+2c^2x^2) + 6ab^2(1+6c^2x^2) + 2b^3c\sqrt{1-\frac{1}{c^2x^2}}\,x(1+20c^2x^2) + 3b\left(-9a^2 - 6ab\right.}{}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^3/x^4,x]
```

output
```
(-9*a^3 - 9*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 6*a*b^2*(1 +
 6*c^2*x^2) + 2*b^3*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 20*c^2*x^2) + 3*b*(-9*a
^2 - 6*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2) + 2*b^2*(1 + 6*c^2*x^
2))*ArcCsc[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(1 + 2*c^2*x^2)
)*ArcCsc[c*x]^2 - 9*b^3*ArcCsc[c*x]^3)/(27*x^3)
```

3.31.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {5746, 4904, 3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b \csc^{-1}(cx)\right)^3}{x^4} \, dx$$

$$\Big\downarrow \text{5746}$$

$$-c^3 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b \csc^{-1}(cx)\right)^3}{c^2x^2}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4904}$$

$$-c^3\left(\frac{\left(a+b \csc^{-1}(cx)\right)^3}{3c^3x^3}-b\int \frac{\left(a+b \csc^{-1}(cx)\right)^2}{c^3x^3}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^3\left(\frac{\left(a+b \csc^{-1}(cx)\right)^3}{3c^3x^3}-b\int \left(a+b \csc^{-1}(cx)\right)^2 \sin\left(\csc^{-1}(cx)\right)^3d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3792}$$

$$-c^3\left(\frac{\left(a+b \csc^{-1}(cx)\right)^3}{3c^3x^3}-b\left(\frac{2}{3}\int \frac{\left(a+b \csc^{-1}(cx)\right)^2}{cx}d\csc^{-1}(cx)-\frac{2}{9}b^2\int \frac{1}{c^3x^3}d\csc^{-1}(cx)+\frac{2b\left(a+b \csc^{-1}(cx)\right)}{9c^3x^3}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-c^3\Bigg(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3c^3x^3}-b\Bigg(\frac{2}{3}\int \left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\frac{2}{9}b^2\int \sin\left(\csc^{-1}(cx)\right)^3d\csc^{-1}$$

$$\Big\downarrow \text{ 3113}$$

$$-c^3\Bigg(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3c^3x^3}-b\Bigg(\frac{2}{3}\int \left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)+\frac{2}{9}b^2\int \frac{1}{c^2x^2}d\sqrt{1-\frac{1}{c^2x^2}}+\frac{2b}{}$$

$$\Big\downarrow \text{ 2009}$$

$$-c^3\Bigg(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3c^3x^3}-b\Bigg(\frac{2}{3}\int \left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)+\frac{2b\left(a+b\csc^{-1}(cx)\right)}{9c^3x^3}-\frac{\sqrt{1-}}{}$$

$$\Big\downarrow \text{ 3777}$$

$$-c^3\Bigg(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3c^3x^3}-b\Bigg(\frac{2}{3}\Bigg(2b\int \sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)d\csc^{-1}(cx)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\Bigg)$$

$$\Big\downarrow \text{ 3042}$$

$$-c^3\Bigg(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3c^3x^3}-b\Bigg(\frac{2}{3}\Bigg(2b\int \left(a+b\csc^{-1}(cx)\right)\sin\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)d\csc^{-1}(cx)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc\right.$$

$$\Big\downarrow \text{ 3777}$$

$$-c^3\Bigg(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3c^3x^3}-b\Bigg(\frac{2}{3}\Bigg(2b\left(b\int -\frac{1}{cx}d\csc^{-1}(cx)+\frac{a+b\csc^{-1}(cx)}{cx}\right)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\Bigg)$$

$$\Big\downarrow \text{ 25}$$

$$-c^3\Bigg(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{3c^3x^3}-b\Bigg(\frac{2}{3}\Bigg(2b\left(\frac{a+b\csc^{-1}(cx)}{cx}-b\int \frac{1}{cx}d\csc^{-1}(cx)\right)-\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2\Bigg)+$$

$$\Big\downarrow \text{ 3042}$$

3.31. $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^4}\,dx$

$$-c^3\left(\frac{(a+b\csc^{-1}(cx))^3}{3c^3x^3}-b\left(\frac{2}{3}\left(2b\left(\frac{a+b\csc^{-1}(cx)}{cx}-b\int\sin\left(\csc^{-1}(cx)\right)d\csc^{-1}(cx)\right)-\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc$$

$$\Big\downarrow 3118$$

$$-c^3\left(\frac{(a+b\csc^{-1}(cx))^3}{3c^3x^3}-b\left(\frac{2b(a+b\csc^{-1}(cx))}{9c^3x^3}-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{3c^2x^2}+\frac{2}{3}\left(2b\left(\frac{a+b\csc^{-1}(cx)}{cx}+b\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcCsc[c*x])^3/x^4,x]
```

output
```
-(c^3*((a + b*ArcCsc[c*x])^3/(3*c^3*x^3) - b*((2*b^2*(Sqrt[1 - 1/(c^2*x^2)
] - (1 - 1/(c^2*x^2))^(3/2)/3))/9 + (2*b*(a + b*ArcCsc[c*x]))/(9*c^3*x^3)
- (Sqrt[1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/(3*c^2*x^2) + (2*(-(Sqrt[1
 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2) + 2*b*(b*Sqrt[1 - 1/(c^2*x^2)] + (a
 + b*ArcCsc[c*x])/(c*x))))/3)))
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(148) = 296$.

Time = 1.51 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.76

method	result
derivativedivides	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\frac{2\operatorname{arccs}}{9c^3}\right.\right.$
default	$c^3\left(-\frac{a^3}{3c^3x^3}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\frac{2\operatorname{arccs}}{9c^3}\right.\right.$
parts	$-\frac{a^3}{3x^3}+b^3c^3\left(-\frac{\operatorname{arccsc}(cx)^3}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)^2(2c^2x^2+1)\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3c^2x^2}+\frac{4\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{3}+\frac{4\operatorname{arccsc}(cx)}{3cx}+\frac{2\operatorname{arccsc}(cx}{9c^3x^3}\right.$

input
```
int((a+b*arccsc(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arccsc(c*x)^3-1/3*arccsc(c*x)^2*(2
*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+4/3*((c^2*x^2-1)/c^2/x^2)^
(1/2)+4/3/c/x*arccsc(c*x)+2/9/c^3/x^3*arccsc(c*x)+2/27*(2*c^2*x^2+1)/c^2/x
^2*((c^2*x^2-1)/c^2/x^2)^(1/2))+3*a*b^2*(-1/3/c^3/x^3*arccsc(c*x)^2-2/9*ar
ccsc(c*x)*(2*c^2*x^2+1)/c^2/x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)+2/27/c^3/x^3+4
/9/c/x)+3*a^2*b*(-1/3/c^3/x^3*arccsc(c*x)-1/9*(c^2*x^2-1)*(2*c^2*x^2+1)/((
c^2*x^2-1)/c^2/x^2)^(1/2)/c^4/x^4))
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^4}\, dx$$
$$= \frac{36\, ab^2c^2x^2 - 9\, b^3 \operatorname{arccsc}(cx)^3 - 27\, ab^2 \operatorname{arccsc}(cx)^2 - 9\, a^3 + 6\, ab^2 + 3\,(12\, b^3c^2x^2 - 9\, a^2b + 2\, b^3) \operatorname{arccsc}(cx)}{}$$

input
```
integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="fricas")
```

output
```
1/27*(36*a*b^2*c^2*x^2 - 9*b^3*arccsc(c*x)^3 - 27*a*b^2*arccsc(c*x)^2 - 9*
a^3 + 6*a*b^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b + 2*b^3)*arccsc(c*x) - (2*(9*a
^2*b - 20*b^3)*c^2*x^2 + 9*a^2*b - 2*b^3 + 9*(2*b^3*c^2*x^2 + b^3)*arccsc(
c*x)^2 + 18*(2*a*b^2*c^2*x^2 + a*b^2)*arccsc(c*x))*sqrt(c^2*x^2 - 1))/x^3
```

3.31.6 Sympy [F]

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^4}\, dx = \int \frac{(a + b \operatorname{acsc}(cx))^3}{x^4}\, dx$$

input
```
integrate((a+b*acsc(c*x))**3/x**4,x)
```

output
```
Integral((a + b*acsc(c*x))**3/x**4, x)
```

3.31.7 Maxima [F]

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^4}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="maxima")
```

output
```
1/3*a^2*b*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c
 - 3*arccsc(c*x)/x^3) - a*b^2*arccsc(c*x)^2/x^3 + 1/12*(12*x^3*integrate(-
1/4*(12*c^2*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 - 12*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c)^2 + 12*(c^2*x^2*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)
^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*(4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
)^2 - log(c^2*x^2)^2) - 4*((3*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c) - c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^2 - 3*arctan2(1, s
qrt(c*x + 1)*sqrt(c*x - 1))*log(c) + 3*(c^2*x^2*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x) + arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1)))*log(c^2*x^2) + 24*(c^2*x^2*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1))*log(c) - arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
*log(c))*log(x))/(c^2*x^6 - x^4), x) - 4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
 - 1))^3 + 3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2)*b^3/x
^3 - 1/3*a^3/x^3 - 2/9*(6*c^5*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
- 3*c^3*x^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - (6*c^3*x^2 + c)*sqrt
(c*x + 1)*sqrt(c*x - 1) - 3*c*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*a*b
^2/(sqrt(c*x + 1)*sqrt(c*x - 1)*c*x^3)
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(148) = 296$.

Time = 0.31 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.52

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^4}\,dx$$
$$= \frac{1}{27}\left(9\,b^3c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right)^2 + 18\,ab^2c^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}\arcsin\left(\frac{1}{cx}\right) - 27\,b^3c^2\sqrt{-\frac{1}{c^2x^2}+1}\,\mathrm{ar}\right.$$

input
```
integrate((a+b*arccsc(c*x))^3/x^4,x, algorithm="giac")
```

output
```
1/27*(9*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))^2 + 18*a*b^2*c^2*
(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x)) - 27*b^3*c^2*sqrt(-1/(c^2*x^2) +
1)*arcsin(1/(c*x))^2 - 9*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))^3/x + 9*a
^2*b*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 2*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2) - 5
4*a*b^2*c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x)) - 27*a*b^2*c*(1/(c^2*x^
2) - 1)*arcsin(1/(c*x))^2/x - 9*b^3*c*arcsin(1/(c*x))^3/x - 27*a^2*b*c^2*s
qrt(-1/(c^2*x^2) + 1) + 42*b^3*c^2*sqrt(-1/(c^2*x^2) + 1) - 27*a^2*b*c*(1/
(c^2*x^2) - 1)*arcsin(1/(c*x))/x + 6*b^3*c*(1/(c^2*x^2) - 1)*arcsin(1/(c*x
))/x - 27*a*b^2*c*arcsin(1/(c*x))^2/x + 6*a*b^2*c*(1/(c^2*x^2) - 1)/x - 27
*a^2*b*c*arcsin(1/(c*x))/x + 42*b^3*c*arcsin(1/(c*x))/x + 42*a*b^2*c/x - 9
*a^3/(c*x^3))*c
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \csc^{-1}(cx))^3}{x^4} \, dx = \int \frac{\left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)^3}{x^4} \, dx$$

input
```
int((a + b*asin(1/(c*x)))^3/x^4,x)
```

output
```
int((a + b*asin(1/(c*x)))^3/x^4, x)
```

3.32 $\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^5}\,dx$

3.32.1 Optimal result

Integrand size = 14, antiderivative size = 208

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^5}\,dx = \frac{3b^3c\sqrt{1-\frac{1}{c^2x^2}}}{128x^3}+\frac{45b^3c^3\sqrt{1-\frac{1}{c^2x^2}}}{256x}-\frac{45}{256}b^3c^4\csc^{-1}(cx)+\frac{3b^2\left(a+b\csc^{-1}(cx)\right)}{32x^4}+\frac{9b^2c^2\left(a+b\csc^{-1}(cx)\right)}{32x^2}-\frac{3bc\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{16x^3}-\frac{9bc^3\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^2}{32x}+\frac{3}{32}c^4\left(a+b\csc^{-1}(cx)\right)^3-\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4x^4}$$

output
```
-45/256*b^3*c^4*arccsc(c*x)+3/32*b^2*(a+b*arccsc(c*x))/x^4+9/32*b^2*c^2*(a
+b*arccsc(c*x))/x^2+3/32*c^4*(a+b*arccsc(c*x))^3-1/4*(a+b*arccsc(c*x))^3/x
^4+3/128*b^3*c*(1-1/c^2/x^2)^(1/2)/x^3+45/256*b^3*c^3*(1-1/c^2/x^2)^(1/2)/
x-3/16*b*c*(a+b*arccsc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x^3-9/32*b*c^3*(a+b*arc
csc(c*x))^2*(1-1/c^2/x^2)^(1/2)/x
```

3.32.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^5}\,dx$$
$$= \frac{-64a^3+24ab^2-48a^2bc\sqrt{1-\frac{1}{c^2x^2}}x+6b^3c\sqrt{1-\frac{1}{c^2x^2}}x+72ab^2c^2x^2-72a^2bc^3\sqrt{1-\frac{1}{c^2x^2}}x^3+45b^3c^3\sqrt{1-}}{}$$

input
```
Integrate[(a + b*ArcCsc[c*x])^3/x^5,x]
```

output
```
(-64*a^3 + 24*a*b^2 - 48*a^2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1
- 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 - 1/(c^2*x^2)]*x
^3 + 45*b^3*c^3*Sqrt[1 - 1/(c^2*x^2)]*x^3 + 24*b*(-8*a^2 + b^2*(1 + 3*c^2*
x^2) - 2*a*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2))*ArcCsc[c*x] - 24*b
^2*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(2 + 3*c^2*x^2) + a*(8 - 3*c^4*x^4))*ArcCs
c[c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsc[c*x]^3 + 9*b*(8*a^2 - 5*b^2)*c^4*
x^4*ArcSin[1/(c*x)])/(256*x^4)
```

3.32.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5746, 4904, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^5}\,dx$$

$$\Big\downarrow \text{5746}$$

$$-c^4\int \frac{\sqrt{1-\frac{1}{c^2x^2}}\left(a+b\csc^{-1}(cx)\right)^3}{c^3x^3}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4904}$$

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\int \frac{\left(a+b\csc^{-1}(cx)\right)^2}{c^4x^4}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\int\left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)^4d\csc^{-1}(cx)\right)$$

$\downarrow$ 3792

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\frac{\left(a+b\csc^{-1}(cx)\right)^2}{c^2x^2}d\csc^{-1}(cx)-\frac{1}{8}b^2\int\frac{1}{c^4x^4}d\csc^{-1}(cx)+\frac{b(a+b\csc^{-1}(cx)}{8c^4x^4}\right.\right.$$

$\downarrow$ 3042

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{8}b^2\int\sin\left(\csc^{-1}(cx)\right)^4d\,cs\right.\right.$$

$\downarrow$ 3115

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\int\frac{1}{c^2x^2}d\csc^{-1}(cx)\right.\right.\right.$$

$\downarrow$ 3042

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\int\sin\left(\csc^{-1}(cx)\right)^2\right.\right.\right.$$

$\downarrow$ 3115

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{8}b^2\left(\frac{3}{4}\left(\frac{1}{2}\int 1d\csc^{-1}(cx)\right.\right.\right.\right.$$

$\downarrow$ 24

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\csc^{-1}(cx)\right)^2\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)+\frac{b\left(a+b\csc^{-1}(cx)\right)}{8c^4x^4}-\frac{\sqrt{1-}}{}\right.\right.$$

$\downarrow$ 3792

$$-c^4\left(\frac{\left(a+b\csc^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\csc^{-1}(cx)+\frac{b(a+b\csc^{-1}(}{2c^2x^2}\right.\right.\right.$$

3.32. $\int\frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^5}\,dx$

$\Big\downarrow$ 17

$$-c^4\left(\frac{(a+b\csc^{-1}(cx))^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2cx}+\frac{b(a+b\csc^{-1}(cx)}{2c^2x^2}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$-c^4\left(\frac{(a+b\csc^{-1}(cx))^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\int\sin\left(\csc^{-1}(cx)\right)^2d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2cx}+\frac{b(a+}{}\right.\right.\right.$$

$\Big\downarrow$ 3115

$$-c^4\left(\frac{(a+b\csc^{-1}(cx))^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\left(\frac{1}{2}\int 1d\csc^{-1}(cx)-\frac{\sqrt{1-\frac{1}{c^2x^2}}}{2cx}\right)-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2cx}+\frac{b}{}\right.\right.\right.$$

$\Big\downarrow$ 24

$$-c^4\left(\frac{(a+b\csc^{-1}(cx))^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{\sqrt{1-\frac{1}{c^2x^2}}(a+b\csc^{-1}(cx))^2}{2cx}+\frac{b(a+b\csc^{-1}(cx))}{2c^2x^2}+\frac{(a+b\csc^{-1}(cx))^3}{6b}-\right.\right.\right.$$

input
```
Int[(a + b*ArcCsc[c*x])^3/x^5,x]
```

output
```
-(c^4*((a + b*ArcCsc[c*x])^3/(4*c^4*x^4) - (3*b*(-1/8*(b^2*(-1/4*Sqrt[1 -
1/(c^2*x^2)]/(c^3*x^3) + (3*(-1/2*Sqrt[1 - 1/(c^2*x^2)]/(c*x) + ArcCsc[c*x
]/2))/4)) + (b*(a + b*ArcCsc[c*x]))/(8*c^4*x^4) - (Sqrt[1 - 1/(c^2*x^2)]*(
a + b*ArcCsc[c*x])^2)/(4*c^3*x^3) + (3*(-1/2*(b^2*(-1/2*Sqrt[1 - 1/(c^2*x^
2)]/(c*x) + ArcCsc[c*x]/2)) + (b*(a + b*ArcCsc[c*x]))/(2*c^2*x^2) - (Sqrt[
1 - 1/(c^2*x^2)]*(a + b*ArcCsc[c*x])^2)/(2*c*x) + (a + b*ArcCsc[c*x])^3/(6
*b)))/4))/4))
```

3.32. $\int\frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx$

3.32.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.32. $\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx$

3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(182) = 364$.

Time = 1.66 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.30

method	result
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\operatorname{arccsc}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arccsc}(cx)^2\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arccsc}(cx)}{32c^4x^4}\right.$
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arccsc}(cx)^2\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arccsc}}{32c^4x}\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\operatorname{arccsc}(cx)^3}{4c^4x^4}+\frac{3\operatorname{arccsc}(cx)^2\left(3c^3x^3\operatorname{arccsc}(cx)-3c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}-2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\right)}{32c^3x^3}+\frac{3\operatorname{arccsc}}{32c^4x}\right.\right.$

input
```
int((a+b*arccsc(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4+b^3*c^4*(-1/4/c^4/x^4*arccsc(c*x)^3+3/32*arccsc(c*x)^2*(3*c^3
*x^3*arccsc(c*x)-3*c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c^2/
x^2)^(1/2))/c^3/x^3+3/32*arccsc(c*x)/c^4/x^4+3/256*(3*c^2*x^2+2)/c^3/x^3*(
(c^2*x^2-1)/c^2/x^2)^(1/2)+27/256*arccsc(c*x)-9/32*(c^2*x^2-1)/c^2/x^2*arc
csc(c*x)+9/64/c/x*((c^2*x^2-1)/c^2/x^2)^(1/2)-3/16*arccsc(c*x)^3)+3*a*b^2*
c^4*(-1/4/c^4/x^4*arccsc(c*x)^2+1/16*arccsc(c*x)*(3*c^3*x^3*arccsc(c*x)-3*
c^2*x^2*((c^2*x^2-1)/c^2/x^2)^(1/2)-2*((c^2*x^2-1)/c^2/x^2)^(1/2))/c^3/x^3
-3/32*arccsc(c*x)^2+1/128*(3*c^2*x^2+2)^2/c^4/x^4)-3/4*a^2*b/x^4*arccsc(c*
x)+9/32*a^2*b*c^3*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*arctan(1
/(c^2*x^2-1)^(1/2))-9/32*a^2*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
^3-3/16*a^2*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^5
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\csc^{-1}(cx))^3}{x^5}\,dx$$
$$=\frac{72\,ab^2c^2x^2+8\,(3\,b^3c^4x^4-8\,b^3)\operatorname{arccsc}(cx)^3-64\,a^3+24\,ab^2+24\,(3\,ab^2c^4x^4-8\,ab^2)\operatorname{arccsc}(cx)^2+3\,(3\,(8}{}$$

input
```
integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="fricas")
```

output
```
1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*arccsc(c*x)^3 - 64*a^3
 + 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8*a*b^2)*arccsc(c*x)^2 + 3*(3*(8*a^2*b
 - 5*b^3)*c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b + 8*b^3)*arccsc(c*x) - 3*(3*
(8*a^2*b - 5*b^3)*c^2*x^2 + 16*a^2*b - 2*b^3 + 8*(3*b^3*c^2*x^2 + 2*b^3)*a
rccsc(c*x)^2 + 16*(3*a*b^2*c^2*x^2 + 2*a*b^2)*arccsc(c*x))*sqrt(c^2*x^2 -
1))/x^4
```

3.32.6 Sympy [F]

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^5}\,dx=\int \frac{\left(a+b\operatorname{acsc}\left(cx\right)\right)^3}{x^5}\,dx$$

input
```
integrate((a+b*acsc(c*x))**3/x**5,x)
```

output
```
Integral((a + b*acsc(c*x))**3/x**5, x)
```

3.32.7 Maxima [F]

$$\int \frac{\left(a+b\csc^{-1}(cx)\right)^3}{x^5}\,dx=\int \frac{\left(b\operatorname{arccsc}\left(cx\right)+a\right)^3}{x^5}\,dx$$

input
```
integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="maxima")
```

output
```
-3/32*a^2*b*((3*c^5*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)) + (3*c^8*x^3*(-1/(c
^2*x^2) + 1)^(3/2) + 5*c^6*x*sqrt(-1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2)
 - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c + 8*arccsc(c*x)/x^4) - 1/4*a
^3/x^4 - 1/16*(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^3 - 3*b^3*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(c^2*x^2)^2 + 12*(2*(c^2*log(c*x +
 1) + c^2*log(c*x - 1) - 2*c^2*log(x) + 1/x^2)*a*b^2*c^2*log(c)^2 + 64*b^3
*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*x^7 -
 x^5), x)*log(c)^2 - 64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)
*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x)*log(c) + 128*b^3*c^2*int
egrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)/(c^2*x^7 -
x^5), x)*log(c) - 64*a*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2)/(c^2*x^7 -
x^5), x)*log(c) + 128*a*b^2*c^2*integrate(1/16*x^2*log(x)/(c^2*x^7 - x^5),
 x)*log(c) - 64*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*
x - 1)))*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 64*b^3*c^2*integrate(1/
16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1)))*log(x)^2/(c^2*x^7 - x^5), x
) - 64*a*b^2*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x + 1)*sqrt(c*x - 1))
)^2/(c^2*x^7 - x^5), x) + 16*b^3*c^2*integrate(1/16*x^2*arctan(1/(sqrt(c*x
 + 1)*sqrt(c*x - 1)))*log(c^2*x^2)/(c^2*x^7 - x^5), x) + 16*a*b^2*c^2*inte
grate(1/16*x^2*log(c^2*x^2)^2/(c^2*x^7 - x^5), x) - 64*a*b^2*c^2*integrate
(1/16*x^2*log(c^2*x^2)*log(x)/(c^2*x^7 - x^5), x) + 64*a*b^2*c^2*integr...
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(182) = 364$.

Time = 0.32 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.77

$$\int \frac{(a + b \csc^{-1}(cx))^3}{x^5} \, dx =$$

$$-\frac{1}{256}\left(64\,b^3c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^3+192\,ab^2c^3\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)^2+128\,b^3c^3\left(\frac{1}{c^2x^2}-1\right.\right.$$

input
```
integrate((a+b*arccsc(c*x))^3/x^5,x, algorithm="giac")
```

output
```
-1/256*(64*b^3*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^3 + 192*a*b^2*c^3*(
1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))^2 + 128*b^3*c^3*(1/(c^2*x^2) - 1)*arcsi
n(1/(c*x))^3 + 192*a^2*b*c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) - 24*b^3*
c^3*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x)) + 384*a*b^2*c^3*(1/(c^2*x^2) - 1)*
arcsin(1/(c*x))^2 + 40*b^3*c^3*arcsin(1/(c*x))^3 - 24*a*b^2*c^3*(1/(c^2*x^
2) - 1)^2 + 384*a^2*b*c^3*(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) - 120*b^3*c^3*
(1/(c^2*x^2) - 1)*arcsin(1/(c*x)) + 120*a*b^2*c^3*arcsin(1/(c*x))^2 - 48*b
^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))^2/x - 120*a*b^2*c^3*(1/(c^
2*x^2) - 1) + 120*a^2*b*c^3*arcsin(1/(c*x)) - 51*b^3*c^3*arcsin(1/(c*x)) -
 96*a*b^2*c^2*(-1/(c^2*x^2) + 1)^(3/2)*arcsin(1/(c*x))/x + 120*b^3*c^2*sqr
t(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))^2/x - 51*a*b^2*c^3 - 48*a^2*b*c^2*(-1/
(c^2*x^2) + 1)^(3/2)/x + 6*b^3*c^2*(-1/(c^2*x^2) + 1)^(3/2)/x + 240*a*b^2*
c^2*sqrt(-1/(c^2*x^2) + 1)*arcsin(1/(c*x))/x + 120*a^2*b*c^2*sqrt(-1/(c^2*
x^2) + 1)/x - 51*b^3*c^2*sqrt(-1/(c^2*x^2) + 1)/x + 64*a^3/(c*x^4))*c
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\csc^{-1}(cx))^3}{x^5}\,dx = \int \frac{\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

input
```
int((a + b*asin(1/(c*x)))^3/x^5,x)
```

output
```
int((a + b*asin(1/(c*x)))^3/x^5, x)
```

3.33 $\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \text{Int}\left(\frac{x}{a+b\csc^{-1}(cx)},x\right)$$

output
```
Unintegrable(x/(a+b*arccsc(c*x)),x)
```

3.33.2 Mathematica [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$

input
```
Integrate[x/(a + b*ArcCsc[c*x]),x]
```

output
```
Integrate[x/(a + b*ArcCsc[c*x]), x]
```

3.33.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{x}{a+b\csc^{-1}(cx)}dx$$

input
```
Int[x/(a + b*ArcCsc[c*x]),x]
```

output
```
$Aborted
```

3.33.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.33.4 Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\,\operatorname{arccsc}\left(cx\right)}dx$$

input
```
int(x/(a+b*arccsc(c*x)),x)
```

output
```
int(x/(a+b*arccsc(c*x)),x)
```

3.33.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{b\,\text{arccsc}\,(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral(x/(b*arccsc(c*x) + a), x)
```

3.33.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{a+b\,\text{acsc}\,(cx)}\,dx$$

input
```
integrate(x/(a+b*acsc(c*x)),x)
```

output
```
Integral(x/(a + b*acsc(c*x)), x)
```

3.33.7 Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx = \int \frac{x}{b\,\text{arccsc}\,(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/(b*arccsc(c*x) + a), x)
```

3.33.8 Giac [N/A]

Not integrable

Time = 33.77 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx=\int \frac{x}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate(x/(b*arccsc(c*x) + a), x)
```

3.33.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a+b\csc^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int(x/(a + b*asin(1/(c*x))),x)
```

output
```
int(x/(a + b*asin(1/(c*x))), x)
```

3.34 $\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\text{Int}\left(\frac{1}{a+b\csc^{-1}(cx)},x\right)$$

output
```
Unintegrable(1/(a+b*arccsc(c*x)),x)
```

3.34.2 Mathematica [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx=\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])^(-1),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])^(-1), x]
```

3.34.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \frac{1}{a+b\csc^{-1}(cx)}dx$$

input
```
Int[(a + b*ArcCsc[c*x])^(-1),x]
```

output
```
$Aborted
```

3.34.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.34.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\,\operatorname{arccsc}(cx)}dx$$

input
```
int(1/(a+b*arccsc(c*x)),x)
```

output
```
int(1/(a+b*arccsc(c*x)),x)
```

3.34.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx = \int \frac{1}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccsc(c*x) + a), x)
```

3.34.6 Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx = \int \frac{1}{a+b\,\mathrm{acsc}\,(cx)}\,dx$$

input
```
integrate(1/(a+b*acsc(c*x)),x)
```

output
```
Integral(1/(a + b*acsc(c*x)), x)
```

3.34.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx = \int \frac{1}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccsc(c*x) + a), x)
```

3.34.8 Giac [N/A]

Not integrable

Time = 11.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx = \int \frac{1}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arccsc(c*x) + a), x)
```

3.34.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a+b\csc^{-1}(cx)}\,dx = \int \frac{1}{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int(1/(a + b*asin(1/(c*x))),x)
```

output
```
int(1/(a + b*asin(1/(c*x))), x)
```

3.35 $\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$

3.35.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\csc^{-1}(cx)\right)},x\right)$$

output
```
Unintegrable(1/x/(a+b*arccsc(c*x)),x)
```

3.35.2 Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx = \int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcCsc[c*x])),x]
```

output
```
Integrate[1/(x*(a + b*ArcCsc[c*x])), x]
```

3.35.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}dx$$

input
```
Int[1/(x*(a + b*ArcCsc[c*x])),x]
```

output
```
$Aborted
```

3.35.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.35.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\,\operatorname{arccsc}\left(cx\right)\right)}dx$$

input
```
int(1/x/(a+b*arccsc(c*x)),x)
```

output
```
int(1/x/(a+b*arccsc(c*x)),x)
```

3.35.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arccsc}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*arccsc(c*x) + a*x), x)
```

3.35.6 Sympy [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\,\mathrm{acsc}\,(cx)\right)}\,dx$$

input
```
integrate(1/x/(a+b*acsc(c*x)),x)
```

output
```
Integral(1/(x*(a + b*acsc(c*x))), x)
```

3.35.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arccsc}\,(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccsc(c*x) + a)*x), x)
```

3.35.8 Giac [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arccsc}\left(cx\right)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arccsc(c*x) + a)*x), x)
```

3.35.9 Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x*(a + b*asin(1/(c*x)))),x)
```

output
```
int(1/(x*(a + b*asin(1/(c*x)))), x)
```

3.36 $\int \frac{1}{x^2(a+b\csc^{-1}(cx))}\,dx$

3.36.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b} - \frac{c\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}$$

output
```
-c*Ci(a/b+arccsc(c*x))*cos(a/b)/b-c*Si(a/b+arccsc(c*x))*sin(a/b)/b
```

3.36.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c\left(\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)+\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right)}{b}$$

input
```
Integrate[1/(x^2*(a + b*ArcCsc[c*x])),x]
```

output
```
-((c*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] + Sin[a/b]*SinIntegral[a/b +
 ArcCsc[c*x]]))/b)
```

3.36.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5746, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+b\csc^{-1}(cx))} \, dx$$

$$\Big\downarrow \text{5746}$$

$$-c\int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-c\int \frac{\sin\left(\csc^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3784}$$

$$-c\left(\cos\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)+\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c\left(\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)+\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\csc^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3780}$$

$$-c\left(\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a}{b}+\csc^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)+\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}\right)$$

$$\Big\downarrow \text{3783}$$

$$-c\left(\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}+\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^2*(a + b*ArcCsc[c*x])),x]
```

output
```
-(c*((Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]])/b + (Sin[a/b]*SinIntegral[a
/b + ArcCsc[c*x]])/b))
```

3.36.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.36.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$c\left(-\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}\right)$	48
default	$c\left(-\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{b}\right)$	48

input
```
int(1/x^2/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c*(-Si(a/b+arccsc(c*x))*sin(a/b)/b-Ci(a/b+arccsc(c*x))*cos(a/b)/b)
```

3.36.5 Fricas [F]

$$\int \frac{1}{x^2 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*arccsc(c*x) + a*x^2), x)
```

3.36.6 Sympy [F]

$$\int \frac{1}{x^2 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{x^2 (a + b \operatorname{acsc}(cx))} \, dx$$

input
```
integrate(1/x**2/(a+b*acsc(c*x)),x)
```

output
```
Integral(1/(x**2*(a + b*acsc(c*x))), x)
```

3.36.7 Maxima [F]

$$\int \frac{1}{x^2 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccsc(c*x) + a)*x^2), x)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx = -c\left(\frac{\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{a}{b}+\arcsin\left(\frac{1}{cx}\right)\right)}{b}+\frac{\sin\left(\frac{a}{b}\right)\mathrm{Si}\left(\frac{a}{b}+\arcsin\left(\frac{1}{cx}\right)\right)}{b}\right)$$

input
```
integrate(1/x^2/(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
-c*(cos(a/b)*cos_integral(a/b + arcsin(1/(c*x)))/b + sin(a/b)*sin_integral
(a/b + arcsin(1/(c*x)))/b)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\csc^{-1}(cx)\right)}\,dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x^2*(a + b*asin(1/(c*x)))),x)
```

output
```
int(1/(x^2*(a + b*asin(1/(c*x)))), x)
```

3.37 $\int \frac{1}{x^3(a+b\csc^{-1}(cx))}\,dx$

3.37.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx=\frac{c^2\,\text{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{2b}$$

output
```
-1/2*c^2*cos(2*a/b)*Si(2*a/b+2*arccsc(c*x))/b+1/2*c^2*Ci(2*a/b+2*arccsc(c*
x))*sin(2*a/b)/b
```

3.37.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx=-\frac{c^2\left(-\text{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\sin\left(\frac{2a}{b}\right)+\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)\right)}{2b}$$

input
```
Integrate[1/(x^3*(a + b*ArcCsc[c*x])),x]
```

output
```
-1/2*(c^2*(-(CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b]) + Cos[(2*a
)/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]]))/b
```

3.37.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5746, 4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+b \csc^{-1}(cx))} \, dx$$

$$\Big\downarrow \text{5746}$$

$$-c^2 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{cx(a+b\csc^{-1}(cx))}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4906}$$

$$-c^2 \int \frac{\sin\left(2\csc^{-1}(cx)\right)}{2\left(a+b\csc^{-1}(cx)\right)}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}c^2 \int \frac{\sin\left(2\csc^{-1}(cx)\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}c^2 \int \frac{\sin\left(2\csc^{-1}(cx)\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)$$

$$\Big\downarrow \text{3784}$$

$$-\frac{1}{2}c^2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\cos\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}c^2\left(\cos\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\csc^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3780}$$

$$-\frac{1}{2}c^2\left(\frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{b}-\sin\left(\frac{2a}{b}\right)\int \frac{\sin\left(\frac{2a}{b}+2\csc^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\csc^{-1}(cx)}d\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{3783}$$

$$-\frac{1}{2}c^2\left(\frac{\cos\left(\frac{2a}{b}\right)\mathrm{Si}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{b}-\frac{\sin\left(\frac{2a}{b}\right)\mathrm{CosIntegral}\left(\frac{2a}{b}+2\csc^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^3*(a + b*ArcCsc[c*x])),x]
```

output
```
-1/2*(c^2*(-((CosIntegral[(2*a)/b + 2*ArcCsc[c*x]]*Sin[(2*a)/b])/b) + (Cos
[(2*a)/b]*SinIntegral[(2*a)/b + 2*ArcCsc[c*x]])/b))
```

3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.37.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$c^2\left(-\frac{\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arccsc}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arccsc}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58
default	$c^2\left(-\frac{\operatorname{Si}\left(\frac{2a}{b}+2\operatorname{arccsc}(cx)\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\operatorname{Ci}\left(\frac{2a}{b}+2\operatorname{arccsc}(cx)\right)\sin\left(\frac{2a}{b}\right)}{2b}\right)$	58

input
```
int(1/x^3/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*Si(2*a/b+2*arccsc(c*x))*cos(2*a/b)/b+1/2*Ci(2*a/b+2*arccsc(c*x))
*sin(2*a/b)/b)
```

3.37.5 Fricas [F]

$$\int \frac{1}{x^3\left(a+b\csc^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arccsc}(cx)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^3*arccsc(c*x) + a*x^3), x)
```

3.37.6 Sympy [F]

$$\int \frac{1}{x^3 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{x^3 (a + b \operatorname{acsc}(cx))} \, dx$$

input
```
integrate(1/x**3/(a+b*acsc(c*x)),x)
```

output
```
Integral(1/(x**3*(a + b*acsc(c*x))), x)
```

3.37.7 Maxima [F]

$$\int \frac{1}{x^3 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{(b \operatorname{arccsc}(cx) + a) x^3} \, dx$$

input
```
integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccsc(c*x) + a)*x^3), x)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3 (a + b \csc^{-1}(cx))} \, dx = \frac{1}{2} \left(\frac{2 c \cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\frac{2a}{b} + 2 \arcsin\left(\frac{1}{cx}\right)\right) \sin\left(\frac{a}{b}\right)}{b} - \frac{2 c \cos\left(\frac{a}{b}\right)^2 \operatorname{Si}\left(\frac{2a}{b} + 2 \arcsin\left(\frac{1}{cx}\right)\right)}{b} + \frac{c \operatorname{Si}\left(\frac{2a}{b} + 2 \arcsin\right.}{b}$$

input
```
integrate(1/x^3/(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/2*(2*c*cos(a/b)*cos_integral(2*a/b + 2*arcsin(1/(c*x)))*sin(a/b)/b - 2*c
*cos(a/b)^2*sin_integral(2*a/b + 2*arcsin(1/(c*x)))/b + c*sin_integral(2*a
/b + 2*arcsin(1/(c*x)))/b)*c
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{x^3 \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)} \, dx$$

input
```
int(1/(x^3*(a + b*asin(1/(c*x)))),x)
```

output
```
int(1/(x^3*(a + b*asin(1/(c*x)))), x)
```

3.38 $\int \frac{1}{x^4(a+b\csc^{-1}(cx))}\,dx$

3.38.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c^3\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b} + \frac{c^3\cos\left(\frac{3a}{b}\right)\operatorname{CosIntegral}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b} - \frac{c^3\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b} + \frac{c^3\sin\left(\frac{3a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b}$$

output
```
-1/4*c^3*Ci(a/b+arccsc(c*x))*cos(a/b)/b+1/4*c^3*Ci(3*a/b+3*arccsc(c*x))*co
s(3*a/b)/b-1/4*c^3*Si(a/b+arccsc(c*x))*sin(a/b)/b+1/4*c^3*Si(3*a/b+3*arccs
c(c*x))*sin(3*a/b)/b
```

3.38.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(a+b\csc^{-1}(cx)\right)}\,dx = -\frac{c^3\left(\cos\left(\frac{a}{b}\right)\operatorname{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)-\cos\left(\frac{3a}{b}\right)\operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\csc^{-1}(cx)\right)\right)+\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\csc\right.\right.}{4b}$$

input
```
Integrate[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

output
```
-1/4*(c^3*(Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]] - Cos[(3*a)/b]*CosInteg
ral[3*(a/b + ArcCsc[c*x])] + Sin[a/b]*SinIntegral[a/b + ArcCsc[c*x]] - Sin
[(3*a)/b]*SinIntegral[3*(a/b + ArcCsc[c*x])]))/b
```

3.38.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5746, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+b\csc^{-1}(cx))} \, dx$$

$$\Big\downarrow \text{5746}$$

$$-c^3 \int \frac{\sqrt{1-\frac{1}{c^2x^2}}}{c^2x^2 (a+b\csc^{-1}(cx))} d\csc^{-1}(cx)$$

$$\Big\downarrow \text{4906}$$

$$-c^3 \int \left(\frac{\sqrt{1-\frac{1}{c^2x^2}}}{4 (a+b\csc^{-1}(cx))} - \frac{\cos (3\csc^{-1}(cx))}{4 (a+b\csc^{-1}(cx))}\right) d\csc^{-1}(cx)$$

$$\Big\downarrow \text{2009}$$

$$-c^3 \left(\frac{\cos\left(\frac{a}{b}\right) \text{CosIntegral}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b} - \frac{\cos\left(\frac{3a}{b}\right) \text{CosIntegral}\left(\frac{3a}{b}+3\csc^{-1}(cx)\right)}{4b} + \frac{\sin\left(\frac{a}{b}\right) \text{Si}\left(\frac{a}{b}+\csc^{-1}(cx)\right)}{4b}\right.$$

input
```
Int[1/(x^4*(a + b*ArcCsc[c*x])),x]
```

output
```
-(c^3*((Cos[a/b]*CosIntegral[a/b + ArcCsc[c*x]])/(4*b) - (Cos[(3*a)/b]*Cos
Integral[(3*a)/b + 3*ArcCsc[c*x]])/(4*b) + (Sin[a/b]*SinIntegral[a/b + Arc
Csc[c*x]])/(4*b) - (Sin[(3*a)/b]*SinIntegral[(3*a)/b + 3*ArcCsc[c*x]])/(4*
b)))
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5746
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[-
(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcC
sc[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n
, 0] || LtQ[m, -1])
```

3.38.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
derivativedivides	$c^3\left(-\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arccsc}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arccsc}(\right.}{4b}\right.$
default	$c^3\left(-\frac{\mathrm{Si}\left(\frac{a}{b}+\mathrm{arccsc}(cx)\right)\sin\left(\frac{a}{b}\right)}{4b}-\frac{\mathrm{Ci}\left(\frac{a}{b}+\mathrm{arccsc}(cx)\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\mathrm{Si}\left(\frac{3a}{b}+3\,\mathrm{arccsc}(cx)\right)\sin\left(\frac{3a}{b}\right)}{4b}+\frac{\mathrm{Ci}\left(\frac{3a}{b}+3\,\mathrm{arccsc}(\right.}{4b}\right.$

input
```
int(1/x^4/(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/4*Si(a/b+arccsc(c*x))*sin(a/b)/b-1/4*Ci(a/b+arccsc(c*x))*cos(a/b)/
b+1/4*Si(3*a/b+3*arccsc(c*x))*sin(3*a/b)/b+1/4*Ci(3*a/b+3*arccsc(c*x))*cos
(3*a/b)/b)
```

3.38.5 Fricas [F]

$$\int \frac{1}{x^4 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^4*arccsc(c*x) + a*x^4), x)
```

3.38.6 Sympy [F]

$$\int \frac{1}{x^4 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{x^4 (a + b \operatorname{acsc}(cx))} \, dx$$

input
```
integrate(1/x**4/(a+b*acsc(c*x)),x)
```

output
```
Integral(1/(x**4*(a + b*acsc(c*x))), x)
```

3.38.7 Maxima [F]

$$\int \frac{1}{x^4 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{(b \operatorname{arccsc}(cx) + a)x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccsc(c*x) + a)*x^4), x)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^4 (a + b \csc^{-1}(cx))} \, dx$$
$$= \frac{1}{4} \left(\frac{4 c^2 \cos\left(\frac{a}{b}\right)^3 \operatorname{Ci}\left(\frac{3a}{b} + 3 \arcsin\left(\frac{1}{cx}\right)\right)}{b} + \frac{4 c^2 \cos\left(\frac{a}{b}\right)^2 \sin\left(\frac{a}{b}\right) \operatorname{Si}\left(\frac{3a}{b} + 3 \arcsin\left(\frac{1}{cx}\right)\right)}{b} - \frac{3 c^2 \cos\left(\frac{a}{b}\right) \operatorname{Ci}\left(\right.}{} \right.$$

input
```
integrate(1/x^4/(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/4*(4*c^2*cos(a/b)^3*cos_integral(3*a/b + 3*arcsin(1/(c*x)))/b + 4*c^2*co
s(a/b)^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(1/(c*x)))/b - 3*c^2*cos(a/
b)*cos_integral(3*a/b + 3*arcsin(1/(c*x)))/b - c^2*cos(a/b)*cos_integral(a
/b + arcsin(1/(c*x)))/b - c^2*sin(a/b)*sin_integral(3*a/b + 3*arcsin(1/(c*
x)))/b - c^2*sin(a/b)*sin_integral(a/b + arcsin(1/(c*x)))/b)*c
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b \csc^{-1}(cx))} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{asin}\left(\frac{1}{c x}\right)\right)} \, dx$$

input
```
int(1/(x^4*(a + b*asin(1/(c*x)))),x)
```

output
```
int(1/(x^4*(a + b*asin(1/(c*x)))), x)
```

3.39 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx$

3.39.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx = \operatorname{Int}\left((dx)^m \left(a+b\csc^{-1}(cx)\right)^3, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

3.39.2 Mathematica [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^3, x]
```

3.39.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx$$

$$\Big\downarrow \text{ 5772}$$

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcCsc[c*x])^3,x]
```

output
```
$Aborted
```

3.39.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.39.4 Maple [N/A] (verified)

Not integrable

Time = 0.86 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\ \mathrm{arccsc}\left(cx\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

output
```
int((d*x)^m*(a+b*arccsc(c*x))^3,x)
```

3.39.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 dx = \int (b\,\mathrm{arccsc}\,(cx)+a)^3(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccsc(c*x)^3 + 3*a*b^2*arccsc(c*x)^2 + 3*a^2*b*arccsc(c*x)
+ a^3)*(d*x)^m, x)
```

3.39.6 Sympy [N/A]

Not integrable

Time = 21.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 dx = \int (dx)^m\left(a+b\,\mathrm{acsc}\,(cx)\right)^3 dx$$

input
```
integrate((d*x)**m*(a+b*acsc(c*x))**3,x)
```

output
```
Integral((d*x)**m*(a + b*acsc(c*x))**3, x)
```

3.39.7 Maxima [N/A]

Not integrable

Time = 15.47 (sec) , antiderivative size = 1279, normalized size of antiderivative = 79.94

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 dx = \int (b\,\mathrm{arccsc}\,(cx)+a)^3(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/4*(4*b^3*d^m*x*x^m*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1))^3 - 3*b^3*d^m*x*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
 1))*log(c^2*x^2)^2 - 4*(m + 1)*integrate(-3/4*((a*b^2*d^m*m + a*b^2*d^m -
 (a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*log(c^2*x^2)^2 + 4*((b^3*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*d^m*m - ((b^3*arctan2(1, sqrt(
c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*d^m*m + (b^3*arctan2(1, sqrt(c*x + 1)
*sqrt(c*x - 1)) + a*b^2)*c^2*d^m)*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt
(c*x - 1)) + a*b^2)*d^m)*x^m*log(x)^2 + 8*((b^3*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)) + a*b^2)*d^m*m*log(c) - ((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + a*b^2)*c^2*d^m*m*log(c) + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c
*x - 1)) + a*b^2)*c^2*d^m*log(c))*x^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt
(c*x - 1)) + a*b^2)*d^m*log(c))*x^m*log(x) + (4*b^3*d^m*x^m*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1))^2 - b^3*d^m*x^m*log(c^2*x^2)^2)*sqrt(c*x + 1)*sq
rt(c*x - 1) - 4*((a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) - (b^3*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1)) + a*b^2)*log(c)^2)*d^m*m + (((b^3*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1)) + a*b^2)*c^2*log(c)^2 - (a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1))^2 + a^2*b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*c^2)*d^m*m +
((b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a*b^2)*c^2*log(c)^2 - (a*b
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + a^2*b*arctan2(1, sqrt(c*...
```

3.39.8 Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx=\int (b\,\mathrm{arccsc}\,(cx)+a)^3(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^3*(d*x)^m, x)
```

3.39.9 Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^3 \, dx=\int {\left(d\,x\right)}^m\,{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}^3 \,d x$$

input
```
int((d*x)^m*(a + b*asin(1/(c*x)))^3,x)
```

output
```
int((d*x)^m*(a + b*asin(1/(c*x)))^3, x)
```

3.40 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$

3.40.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \text{Int}\left((dx)^m \left(a+b\csc^{-1}(cx)\right)^2,x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

3.40.2 Mathematica [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcCsc[c*x])^2, x]
```

3.40.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 dx$$

$$\Big\downarrow \text{ 5772}$$

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcCsc[c*x])^2,x]
```

output
```
$Aborted
```

3.40.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.40.4 Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\,\mathrm{arccsc}\left(cx\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

output
```
int((d*x)^m*(a+b*arccsc(c*x))^2,x)
```

3.40.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (b\operatorname{arccsc}(cx)+a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2)*(d*x)^m, x)
```

3.40.6 Sympy [N/A]

Not integrable

Time = 9.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\operatorname{acsc}(cx)\right)^2 \, dx$$

input
```
integrate((d*x)**m*(a+b*acsc(c*x))**2,x)
```

output
```
Integral((d*x)**m*(a + b*acsc(c*x))**2, x)
```

3.40.7 Maxima [N/A]

Not integrable

Time = 6.87 (sec) , antiderivative size = 551, normalized size of antiderivative = 34.44

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (b\operatorname{arccsc}(cx)+a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

output
```
(d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/4*(4*b^2*d^m*x*x^m*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1))^2 - b^2*d^m*x*x^m*log(c^2*x^2)^2 + 4*(m + 1)*integrate(
(2*sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*d^m*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c
*x - 1)) + (b^2*d^m*m + b^2*d^m - (b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*l
og(x)^2 + 2*(b^2*d^m*m*log(c) + b^2*d^m*log(c) - (b^2*c^2*d^m*m*log(c) + b
^2*c^2*d^m*log(c))*x^2)*x^m*log(x) + ((b^2*log(c)^2 - 2*a*b*arctan2(1, sqr
t(c*x + 1)*sqrt(c*x - 1)))*d^m*m - ((b^2*c^2*log(c)^2 - 2*a*b*c^2*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1)))*d^m*m + (b^2*c^2*log(c)^2 - 2*a*b*c^2*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*d^m)*x^2 + (b^2*log(c)^2 - 2*a*b*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*d^m)*x^m - ((b^2*d^m*m + b^2*d^m - (
b^2*c^2*d^m*m + b^2*c^2*d^m)*x^2)*x^m*log(x) + (b^2*d^m*m*log(c) - (b^2*c^
2*d^m*m*log(c) + (b^2*c^2*log(c) + b^2*c^2)*d^m)*x^2 + (b^2*log(c) + b^2)*
d^m)*x^m)*log(c^2*x^2))/((c^2*m + c^2)*x^2 - m - 1), x))/(m + 1)
```

3.40.8 Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arccsc}(cx) + a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)^2*(d*x)^m, x)
```

3.40.9 Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)^2 \, dx = \int (d\,x)^m \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int((d*x)^m*(a + b*asin(1/(c*x)))^2,x)
```

output
```
int((d*x)^m*(a + b*asin(1/(c*x)))^2, x)
```

3.41 $\int (dx)^m \left(a+b\csc^{-1}(cx)\right)\,dx$

3.41.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{(dx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*arccsc(c*x))/d/(1+m)+b*(d*x)^m*hypergeom([1/2, -1/2*m],[1
-1/2*m],1/c^2/x^2)/c/m/(1+m)
```

3.41.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{(dx)^m\left((1+m)x\left(a+b\csc^{-1}(cx)\right)+\frac{b\sqrt{1-c^2x^2}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{c\sqrt{1-\frac{1}{c^2x^2}}}\right)}{(1+m)^2}$$

input
```
Integrate[(d*x)^m*(a + b*ArcCsc[c*x]),x]
```

output
```
((d*x)^m*((1 + m)*x*(a + b*ArcCsc[c*x]) + (b*Sqrt[1 - c^2*x^2]*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(c*Sqrt[1 - 1/(c^2*x^2)])))/(1
 + m)^2
```

3.41.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5744, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 5744}$$

$$\frac{bd\int \frac{(dx)^{m-1}}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{c(m+1)}+\frac{(dx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{d(m+1)}$$

$$\Big\downarrow \text{ 862}$$

$$\frac{(dx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{d(m+1)}-\frac{b\left(\frac{1}{x}\right)^m (dx)^m\int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c(m+1)}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(dx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{d(m+1)}+\frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

input
```
Int[(d*x)^m*(a + b*ArcCsc[c*x]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometri
c2F1[1/2, -1/2*m, 1 - m/2, 1/(c^2*x^2)])/(c*m*(1 + m))
```

3.41.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 5744
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcCsc[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.41.4 Maple [F]

$$\int (dx)^m (a + b\,\mathrm{arccsc}\,(cx))\,dx$$

input
```
int((d*x)^m*(a+b*arccsc(c*x)),x)
```

output
```
int((d*x)^m*(a+b*arccsc(c*x)),x)
```

3.41.5 Fricas [F]

$$\int (dx)^m \left(a + b\csc^{-1}(cx)\right)\,dx = \int (b\,\mathrm{arccsc}\,(cx) + a)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)*(d*x)^m, x)
```

3.41.6 Sympy [F]

$$\int (dx)^m \left(a + b\csc^{-1}(cx)\right)\,dx = \int (dx)^m\,(a + b\,\mathrm{acsc}\,(cx))\,dx$$

input
```
integrate((d*x)**m*(a+b*acsc(c*x)),x)
```

output
```
Integral((d*x)**m*(a + b*acsc(c*x)), x)
```

3.41.7 Maxima [F]

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
(d^m*x*x^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + (c^2*d^m*m + c^2*d^m)
*integrate(-sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(c^2*m - (c^4*m + c^4)*x^2 + c
^2), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

3.41.8 Giac [F]

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)\,dx=\int (b\,\mathrm{arccsc}\,(cx)+a)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*(d*x)^m, x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\csc^{-1}(cx)\right)\,dx=\int (d\,x)^m\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d*x)^m*(a + b*asin(1/(c*x))),x)
```

output
```
int((d*x)^m*(a + b*asin(1/(c*x))), x)
```

3.42 $\int \frac{(dx)^m}{a+b\csc^{-1}(cx)} dx$

3.42.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)} dx = \text{Int}\left(\frac{(dx)^m}{a+b\csc^{-1}(cx)}, x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arccsc(c*x)),x)
```

3.42.2 Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)} dx = \int \frac{(dx)^m}{a+b\csc^{-1}(cx)} dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcCsc[c*x]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcCsc[c*x]), x]
```

3.42.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx$$
$$\Big\downarrow \ 5772$$
$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcCsc[c*x]),x]
```

output
```
$Aborted
```

3.42.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.42.4 Maple [N/A] (verified)

Not integrable

Time = 3.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\,\mathrm{arccsc}\,(cx)}dx$$

input
```
int((d*x)^m/(a+b*arccsc(c*x)),x)
```

output
```
int((d*x)^m/(a+b*arccsc(c*x)),x)
```

3.42.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arccsc(c*x) + a), x)
```

3.42.6 Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\,\mathrm{acsc}\,(cx)}\,dx$$

input
```
integrate((d*x)**m/(a+b*acsc(c*x)),x)
```

output
```
Integral((d*x)**m/(a + b*acsc(c*x)), x)
```

3.42.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arccsc(c*x) + a), x)
```

3.42.8 Giac [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\mathrm{arccsc}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arccsc(c*x) + a), x)
```

3.42.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a+b\csc^{-1}(cx)}\,dx = \int \frac{(d\,x)^m}{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int((d*x)^m/(a + b*asin(1/(c*x))),x)
```

output
```
int((d*x)^m/(a + b*asin(1/(c*x))), x)
```

3.43 $\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$

3.43.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx=\text{Int}\left(\frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

3.43.2 Mathematica [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\csc^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcCsc[c*x])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcCsc[c*x])^2, x]
```

3.43.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2} \, dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcCsc[c*x])^2,x]
```

output
```
$Aborted
```

3.43.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.43.4 Maple [N/A] (verified)

Not integrable

Time = 2.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\,\operatorname{arccsc}(cx))^2}dx$$

input
```
int((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

output
```
int((d*x)^m/(a+b*arccsc(c*x))^2,x)
```

3.43.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\mathrm{arccsc}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arccsc(c*x)^2 + 2*a*b*arccsc(c*x) + a^2), x)
```

3.43.6 Sympy [N/A]

Not integrable

Time = 5.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(a+b\,\mathrm{acsc}\,(cx))^2}\,dx$$

input
```
integrate((d*x)**m/(a+b*acsc(c*x))**2,x)
```

output
```
Integral((d*x)**m/(a + b*acsc(c*x))**2, x)
```

3.43.7 Maxima [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 684, normalized size of antiderivative = 42.75

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\mathrm{arccsc}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="maxima")
```

output
```
(4*sqrt(c*x + 1)*sqrt(c*x - 1)*(b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
+ a)*d^m*x*x^m - (4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + b^3*lo
g(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*log(x)^2 + 8*a
*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*log(c) + b
^3*log(x))*log(c^2*x^2))*integrate(4*((b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
 - 1)) + a)*d^m*m - ((b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*c^2*d
^m*m + 2*(b*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*c^2*d^m)*x^2 + (b
*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a)*d^m)*sqrt(c*x + 1)*sqrt(c*x
- 1)*x^m/(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 4*b^3*log(c)^2
 + 8*a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(b^3*c^2*
log(c)^2 + (b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*a*b^2*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + a^2*b)*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*
log(c^2*x^2)^2 - 4*(b^3*c^2*x^2 - b^3)*log(x)^2 + 4*(b^3*c^2*x^2*log(c) -
b^3*log(c) + (b^3*c^2*x^2 - b^3)*log(x))*log(c^2*x^2) - 8*(b^3*c^2*x^2*log
(c) - b^3*log(c))*log(x)), x))/(4*b^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1))^2 + b^3*log(c^2*x^2)^2 + 4*b^3*log(c)^2 + 8*b^3*log(c)*log(x) + 4*b^3*
log(x)^2 + 8*a*b^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 4*a^2*b - 4*(
b^3*log(c) + b^3*log(x))*log(c^2*x^2))
```

3.43.8 Giac [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\operatorname{arccsc}(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsc(c*x))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arccsc(c*x) + a)^2, x)
```

3.43.9 Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{(a+b\csc^{-1}(cx))^2}\,dx = \int \frac{(d\,x)^m}{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int((d*x)^m/(a + b*asin(1/(c*x)))^2,x)
```

output
```
int((d*x)^m/(a + b*asin(1/(c*x)))^2, x)
```

3.44 $\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) dx$

3.44.1 Optimal result

Integrand size = 16, antiderivative size = 167

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) dx = \frac{be(9c^2d^2+e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1-\frac{1}{c^2x^2}}\,x^3}{12c}-\frac{bd^4\csc^{-1}(cx)}{4e}+\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bd(2c^2d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{2c^3}$$

output
```
-1/4*b*d^4*arccsc(c*x)/e+1/4*(e*x+d)^4*(a+b*arccsc(c*x))/e+1/2*b*d*(2*c^2*
d^2+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3+1/6*b*e*(9*c^2*d^2+e^2)*x*(1-1/c
^2/x^2)^(1/2)/c^3+1/2*b*d*e^2*x^2*(1-1/c^2/x^2)^(1/2)/c+1/12*b*e^3*x^3*(1-
1/c^2/x^2)^(1/2)/c
```

3.44.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) \, dx$$
$$= \frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)+be\sqrt{1-\frac{1}{c^2x^2}}x(2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2e}{12c^3}$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcCsc[c*x]),x]
```

output
```
(3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*Sqrt[1 - 1/(c
^2*x^2)]*x*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 +
 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcCsc[c*x] + 6*b*d*(2*c^2*d^2 + e^2)*
Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(12*c^3)
```

3.44.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5750, 1892, 1803, 540, 25, 2338, 27, 2338, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 5750$$

$$\frac{b\int \frac{(d+ex)^4}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{4ce}+\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 1892$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^4x^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{4ce}+\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \; 1803$$

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^4x^4}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{4ce}$$

$\Big\downarrow$ 540

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(-\frac{1}{3}\int -\frac{\left(\frac{3d^4}{x^3}+\frac{12ed^3}{x^2}+12e^3d+\frac{2e^2\left(9d^2+\frac{e^2}{c^2}\right)}{x}\right)x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}$$

$\Big\downarrow$ 25

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\int \frac{\left(\frac{3d^4}{x^3}+\frac{12ed^3}{x^2}+12e^3d+\frac{2e^2\left(9d^2+\frac{e^2}{c^2}\right)}{x}\right)x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}$$

$\Big\downarrow$ 2338

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(-\frac{1}{2}\int -\frac{2\left(\frac{3d^4}{x^2}+\frac{6e\left(2d^2+\frac{e^2}{c^2}\right)d}{x}+2e^2\left(9d^2+\frac{e^2}{c^2}\right)\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}$$

$\Big\downarrow$ 27

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(\int \frac{\left(\frac{3d^4}{x^2}+\frac{6e\left(2d^2+\frac{e^2}{c^2}\right)d}{x}+2e^2\left(9d^2+\frac{e^2}{c^2}\right)\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}$$

$\Big\downarrow$ 2338

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(-\int -\frac{3d\left(\frac{d^3}{x}+2e\left(2d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}$$

$\Big\downarrow$ 27

3.44. $\int (d+ex)^3\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\int\frac{\left(\frac{d^3}{x}+2e\left(2d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e^4x^3\sqrt{1-\frac{1}{c^2x^2}}\right)}{4ce}$$

$$\Big\downarrow \text{538}$$

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(d^3\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-}{4ce}$$

$$\Big\downarrow \text{223}$$

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(2e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+cd^3\arcsin\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e}{4ce}$$

$$\Big\downarrow \text{243}$$

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+cd^3\arcsin\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{3}e}{4ce}$$

$$\Big\downarrow \text{73}$$

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(cd^3\arcsin\left(\frac{1}{cx}\right)-2c^2e\left(\frac{e^2}{c^2}+2d^2\right)\int\frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\right.\right.}{4ce}$$

$$\Big\downarrow \text{221}$$

$$\frac{(d+ex)^4\left(a+b\csc^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(cd^3\arcsin\left(\frac{1}{cx}\right)-2e\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\left(\frac{e^2}{c^2}+2d^2\right)\right)-2e^2x\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{e^2}{c^2}+9d^2\right)-6de^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right.\right.}{4ce}$$

input
```
Int[(d + e*x)^3*(a + b*ArcCsc[c*x]),x]
```

3.44. $\int (d+ex)^3\left(a+b\csc^{-1}(cx)\right)\,dx$

output
```
((d + e*x)^4*(a + b*ArcCsc[c*x]))/(4*e) - (b*(-1/3*(e^4*Sqrt[1 - 1/(c^2*x^
2)]*x^3) + (-2*e^2*(9*d^2 + e^2/c^2)*Sqrt[1 - 1/(c^2*x^2)]*x - 6*d*e^3*Sqr
t[1 - 1/(c^2*x^2)]*x^2 + 3*d*(c*d^3*ArcSin[1/(c*x)] - 2*e*(2*d^2 + e^2/c^2
)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]]))/3))/(4*c*e)
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.44.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(147) = 294$.

Time = 0.63 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.40

method	result
parts	$\frac{a(ex+d)^4}{4e}+\frac{b\,e^3\operatorname{arccsc}(cx)x^4}{4}+b\,e^2\operatorname{arccsc}(cx)\,x^3d+\frac{3be\operatorname{arccsc}(cx)x^2d^2}{2}+b\operatorname{arccsc}(cx)\,x\,d^3+\frac{b\,c}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{bc\operatorname{arccsc}(cx)d^4}{4e}+b\operatorname{arccsc}(cx)d^3cx+\frac{3bce\operatorname{arccsc}(cx)d^2x^2}{2}+bc\,e^2\operatorname{arccsc}(cx)\,d\,x^3+\frac{bc\,e^3\operatorname{arccsc}(cx)x^4}{4}-\frac{b\sqrt{c^2x^2-1}}{4c}}{}$
default	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{bc\operatorname{arccsc}(cx)d^4}{4e}+b\operatorname{arccsc}(cx)d^3cx+\frac{3bce\operatorname{arccsc}(cx)d^2x^2}{2}+bc\,e^2\operatorname{arccsc}(cx)\,d\,x^3+\frac{bc\,e^3\operatorname{arccsc}(cx)x^4}{4}-\frac{b\sqrt{c^2x^2-1}}{4c}}{}$

input
```
int((e*x+d)^3*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x+d)^4/e+1/4*b*e^3*arccsc(c*x)*x^4+b*e^2*arccsc(c*x)*x^3*d+3/2*b*
e*arccsc(c*x)*x^2*d^2+b*arccsc(c*x)*x*d^3+1/4*b*d^4*arccsc(c*x)/e+1/12*b/c
^3*e^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x-1/4*b/c/e*(c^2*x^2-1)^(1/
2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^4*arctan(1/(c^2*x^2-1)^(1/2))+1/2*b/c^3
*e^2*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+b/c^2*(c^2*x^2-1)^(1/2)/((c
^2*x^2-1)/c^2/x^2)^(1/2)/x*d^3*ln(c*x+(c^2*x^2-1)^(1/2))+3/2*b/c^3*e*(c^2*
x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2+1/2*b/c^4*e^2*(c^2*x^2-1)^(1/2)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))+1/6*b/c^5*e^3*(c^
2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.74

$$\int (d+ex)^3\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^4e^3x^4+12\,ac^4de^2x^3+18\,ac^4d^2ex^2+12\,ac^4d^3x+3\left(bc^4e^3x^4+4\,bc^4de^2x^3+6\,bc^4d^2ex^2+4\,bc^4d^3x-4\,b\right.}{}$$

input
```
integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*e^3*x^4 + 12*a*c^4*d*e^2*x^3 + 18*a*c^4*d^2*e*x^2 + 12*a*c^4
*d^3*x + 3*(b*c^4*e^3*x^4 + 4*b*c^4*d*e^2*x^3 + 6*b*c^4*d^2*e*x^2 + 4*b*c^
4*d^3*x - 4*b*c^4*d^3 - 6*b*c^4*d^2*e - 4*b*c^4*d*e^2 - b*c^4*e^3)*arccsc(
c*x) - 6*(4*b*c^4*d^3 + 6*b*c^4*d^2*e + 4*b*c^4*d*e^2 + b*c^4*e^3)*arctan(
-c*x + sqrt(c^2*x^2 - 1)) - 6*(2*b*c^3*d^3 + b*c*d*e^2)*log(-c*x + sqrt(c^
2*x^2 - 1)) + (b*c^2*e^3*x^2 + 6*b*c^2*d*e^2*x + 18*b*c^2*d^2*e + 2*b*e^3)
*sqrt(c^2*x^2 - 1))/c^4
```

3.44.6 Sympy [A] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.17

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) dx = ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + bd^3x\operatorname{acsc}(cx) + \frac{3bd^2ex^2\operatorname{acsc}(cx)}{2} + bde^2x^3\operatorname{acsc}(cx) + \frac{be^3x^4\operatorname{acsc}(cx)}{4} + \frac{bd^3\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{3bd^2e\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c} + \frac{bde^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c} + \frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}} + \frac{ix}{2c\sqrt{-c^2x^2+1}} - \frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{c} + \frac{be^3\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c} + \frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c} + \frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

input
```
integrate((e*x+d)**3*(a+b*acsc(c*x)),x)
```

output
```
a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + b*d**3*x*ac
sc(c*x) + 3*b*d**2*e*x**2*acsc(c*x)/2 + b*d*e**2*x**3*acsc(c*x) + b*e**3*x
**4*acsc(c*x)/4 + b*d**3*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*a
sin(c*x), True))/c + 3*b*d**2*e*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2
*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2*c) + b*d*e**2*Piecewise(
(x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (
-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*as
in(c*x)/(2*c**2), True))/c + b*e**3*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3
*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c
**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.61

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right) dx$$
$$= \frac{1}{4} ae^3x^4 + ade^2x^3 + \frac{3}{2} ad^2ex^2 + \frac{3}{2} \left(x^2 \operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c} \right) bd^2e + \frac{1}{4} \left(4x^3 \operatorname{arccsc}(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c} \right) bde^2 + \frac{1}{12} \left(3x^4 \operatorname{arccsc}(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3} \right) be^3 + ad^3x + \frac{\left(2cx \operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^3}{2c}$$

input
```
integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arccsc(c*x) + x*s
qrt(-1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^
2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1
)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arcc
sc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/
c^3)*b*e^3 + a*d^3*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1)
 + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^3/c
```

3.44.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. 2(147) = 294.

Time = 2.48 (sec) , antiderivative size = 1130, normalized size of antiderivative = 6.77

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/192*(3*b*e^3*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*
e^3*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 24*b*d*e^2*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3*arcsin(1/(c*x))/c + 24*a*d*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^3/c + 2*b*e^3*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 72*b*d^2*e*x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 72*a*d^2*e*x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2/c + 24*b*d*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/
c^2 + 96*b*d^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 12*b*e^3
*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 96*a*d^3*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c + 12*a*e^3*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^
3 + 144*b*d^2*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 72*b*d*e^2*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 72*a*d*e^2*x*(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^3 + 192*b*d^3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - 192*b*d^3
*log(1/(abs(c)*abs(x)))/c^2 + 18*b*e^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4
+ 144*b*d^2*e*arcsin(1/(c*x))/c^3 + 144*a*d^2*e/c^3 + 96*b*d*e^2*log(sqrt(
-1/(c^2*x^2) + 1) + 1)/c^4 - 96*b*d*e^2*log(1/(abs(c)*abs(x)))/c^4 + 18*b*
e^3*arcsin(1/(c*x))/c^5 + 96*b*d^3*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^
2) + 1) + 1)) + 18*a*e^3/c^5 + 96*a*d^3/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1
)) - 144*b*d^2*e/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b*d*e^2*arcsin(
1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*a*d*e^2/(c^5*x*(sqrt(-1
/(c^2*x^2) + 1) + 1)) - 18*b*e^3/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +...
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^3\,dx$$

input
```
int((a + b*asin(1/(c*x)))*(d + e*x)^3,x)
```

output
```
int((a + b*asin(1/(c*x)))*(d + e*x)^3, x)
```

3.45 $\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$

3.45.1 Optimal result

Integrand size = 16, antiderivative size = 123

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{bde\sqrt{1-\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x^2}{6c} - \frac{bd^3\csc^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2+e^2)\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{6c^3}$$

output
```
-1/3*b*d^3*arccsc(c*x)/e+1/3*(e*x+d)^3*(a+b*arccsc(c*x))/e+1/6*b*(6*c^2*d^
2+e^2)*arctanh((1-1/c^2/x^2)^(1/2))/c^3+b*d*e*x*(1-1/c^2/x^2)^(1/2)/c+1/6*
b*e^2*x^2*(1-1/c^2/x^2)^(1/2)/c
```

3.45.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{c^2x\left(be\sqrt{1-\frac{1}{c^2x^2}}(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\csc^{-1}(cx)+b(6c^2d^2+}{6c^3}$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(c^2*x*(b*e*Sqrt[1 - 1/(c^2*x^2)]*(6*d + e*x) + 2*a*c*(3*d^2 + 3*d*e*x + e
^2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCsc[c*x] + b*(6*c^2*d^
2 + e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3)
```

3.45.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5750, 1892, 1803, 540, 25, 2338, 25, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right) dx$$

$$\Big\downarrow \text{5750}$$

$$\frac{b\int \frac{(d+ex)^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{3ce}+\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{1892}$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^3x}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{3ce}+\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{1803}$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^3x^3}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{3ce}$$

$$\Big\downarrow \text{540}$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(-\frac{1}{2}\int -\frac{\left(\frac{2d^3}{x^2}+6e^2d+\frac{e\left(6d^2+\frac{e^2}{c^2}\right)}{x}\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

$$\Big\downarrow \text{25}$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\int\frac{\left(\frac{2d^3}{x^2}+6e^2d+\frac{e\left(6d^2+\frac{e^2}{c^2}\right)}{x}\right)x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

$$\Big\downarrow \ 2338$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(-\int-\frac{\left(\frac{2d^3}{x}+e\left(6d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

$$\Big\downarrow \ 25$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(\int\frac{\left(\frac{2d^3}{x}+e\left(6d^2+\frac{e^2}{c^2}\right)\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

$$\Big\downarrow \ 538$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(2d^3\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

$$\Big\downarrow \ 223$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

$$\Big\downarrow \ 243$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(\frac{1}{2}e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

$$\Big\downarrow \ 73$$

3.45. $\int(d+ex)^2\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(-c^2e\left(\frac{e^2}{c^2}+6d^2\right)\int\frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}+2cd^3\arcsin\left(\frac{1}{cx}\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

$$\Big\downarrow 221$$

$$\frac{(d+ex)^3\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(2cd^3\arcsin\left(\frac{1}{cx}\right)-e\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\left(\frac{e^2}{c^2}+6d^2\right)-6de^2x\sqrt{1-\frac{1}{c^2x^2}}\right)-\frac{1}{2}e^3x^2\sqrt{1-\frac{1}{c^2x^2}}\right)}{3ce}$$

input
```
Int[(d + e*x)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
((d + e*x)^3*(a + b*ArcCsc[c*x]))/(3*e) - (b*(-1/2*(e^3*Sqrt[1 - 1/(c^2*x^
2)]*x^2) + (-6*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x + 2*c*d^3*ArcSin[1/(c*x)] - e
*(6*d^2 + e^2/c^2)*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/2))/(3*c*e)
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.45. $\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right) dx$

3.45.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(109) = 218$.

Time = 0.57 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.47

method	result
parts	$\frac{a(ex+d)^3}{3e} + \frac{b e^2 \operatorname{arccsc}(cx) x^3}{3} + be \operatorname{arccsc}(cx) x^2 d + b \operatorname{arccsc}(cx) x d^2 + \frac{b d^3 \operatorname{arccsc}(cx)}{3e} - \frac{b\sqrt{c^2x^2-}}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e} + \frac{bc \operatorname{arccsc}(cx) d^3}{3e} + b \operatorname{arccsc}(cx) d^2 cx + bce \operatorname{arccsc}(cx) d x^2 + \frac{bc e^2 \operatorname{arccsc}(cx) x^3}{3} - \frac{b\sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e} + \frac{bc \operatorname{arccsc}(cx) d^3}{3e} + b \operatorname{arccsc}(cx) d^2 cx + bce \operatorname{arccsc}(cx) d x^2 + \frac{bc e^2 \operatorname{arccsc}(cx) x^3}{3} - \frac{b\sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{3e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}}{c}$

input
```
int((e*x+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(e*x+d)^3/e+1/3*b*e^2*arccsc(c*x)*x^3+b*e*arccsc(c*x)*x^2*d+b*arccsc
(c*x)*x*d^2+1/3*b*d^3*arccsc(c*x)/e-1/3*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-
1)/c^2/x^2)^(1/2)/x*d^3*arctan(1/(c^2*x^2-1)^(1/2))+1/6*b/c^3*e^2*(c^2*x^2
-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)+b/c^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x
^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))+b/c^3*e*(c^2*x^2-1)/((c^2*x^2-1)
/c^2/x^2)^(1/2)/x*d+1/6*b/c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^
(1/2)/x*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.70

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{2\,ac^3e^2x^3 + 6\,ac^3dex^2 + 6\,ac^3d^2x + 2\left(bc^3e^2x^3 + 3\,bc^3dex^2 + 3\,bc^3d^2x - 3\,bc^3d^2 - 3\,bc^3de - bc^3e^2\right)\operatorname{arccsc}(}{}$$

input
```
integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(b*c^3*e^2*x^3
+ 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)
*arccsc(c*x) - 4*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*arctan(-c*x + sqr
t(c^2*x^2 - 1)) - (6*b*c^2*d^2 + b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (b
*c*e^2*x + 6*b*c*d*e)*sqrt(c^2*x^2 - 1))/c^3
```

3.45.6 Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.85

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= ad^2x + adex^2 + \frac{ae^2x^3}{3} + bd^2x\operatorname{acsc}(cx) + bdex^2\operatorname{acsc}(cx) + \frac{be^2x^3\operatorname{acsc}(cx)}{3} + \frac{bd^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{bde\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{c} + \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x+d)**2*(a+b*acsc(c*x)),x)
```

output
```
a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + b*d**2*x*acsc(c*x) + b*d*e*x**2*ac
sc(c*x) + b*e**2*x**3*acsc(c*x)/3 + b*d**2*Piecewise((acosh(c*x), Abs(c**2
*x**2) > 1), (-I*asin(c*x), True))/c + b*d*e*Piecewise((sqrt(c**2*x**2 - 1
)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/c + b*e**2*Pie
cewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2)
> 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1))
 - I*asin(c*x)/(2*c**2), True))/(3*c)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.61

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right) dx$$
$$= \frac{1}{3}ae^2x^3 + adex^2 + \left(x^2 \operatorname{arccsc}(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bde + \frac{1}{12}\left(4x^3 \operatorname{arccsc}(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 + ad^2x + \frac{\left(2cx\operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2c}$$

input
```
integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)
*b*d*e + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*
x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*
x^2) + 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt
(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d^2/c
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(109) = 218.

Time = 2.19 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.89

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right) dx$$
$$= \frac{1}{24}\left(\frac{be^2x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3 \arcsin\left(\frac{1}{cx}\right)}{c} + \frac{ae^2x^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c} - \frac{24\,bdex^2\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{c}\right.$$

input
```
integrate((e*x+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/24*(b*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*e^2*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c - 24*b*d*e*x^2*(1/(c^2*x^2) - 1)*arcsi
n(1/(c*x))/c + b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 - 24*a*d*e*x^2
*(1/(c^2*x^2) - 1)/c + 12*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c
*x))/c + 12*a*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c + 3*b*e^2*x*(sqrt(-1/(c
^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 24*b*d*e*x*sqrt(-1/(c^2*x^2) + 1)/
c^2 + 3*a*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 24*b*d^2*log(sqrt(-1/(c
^2*x^2) + 1) + 1)/c^2 - 24*b*d^2*log(1/(abs(c)*abs(x)))/c^2 + 24*b*d*e*arc
sin(1/(c*x))/c^3 + 24*a*d*e/c^3 + 4*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/
c^4 - 4*b*e^2*log(1/(abs(c)*abs(x)))/c^4 + 12*b*d^2*arcsin(1/(c*x))/(c^3*x
*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*a*d^2/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) +
 1)) + 3*b*e^2*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3*a*
e^2/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b*e^2/(c^6*x^2*(sqrt(-1/(c^2*x^
2) + 1) + 1)^2) + b*e^2*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3) + a*e^2/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^2\,dx$$

input
```
int((a + b*asin(1/(c*x)))*(d + e*x)^2,x)
```

output
```
int((a + b*asin(1/(c*x)))*(d + e*x)^2, x)
```

3.46 $\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$

3.46.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}-\frac{bd^2\csc^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}+\frac{bd\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$$

output
```
-1/2*b*d^2*arccsc(c*x)/e+1/2*(e*x+d)^2*(a+b*arccsc(c*x))/e+b*d*arctanh((1-
1/c^2/x^2)^(1/2))/c+1/2*b*e*x*(1-1/c^2/x^2)^(1/2)/c
```

3.46.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx=adx+\frac{1}{2}aex^2+\frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+bdx\csc^{-1}(cx)+\frac{1}{2}bex^2\csc^{-1}(cx)+\frac{bd\sqrt{1-\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[(d + e*x)*(a + b*ArcCsc[c*x]),x]
```

output
```
a*d*x + (a*e*x^2)/2 + (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x
*ArcCsc[c*x] + (b*e*x^2*ArcCsc[c*x])/2 + (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*ArcT
anh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

3.46.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5750, 1892, 1730, 540, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)dx$$

$$\Big\downarrow \text{5750}$$

$$\frac{b\int \frac{(d+ex)^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{2ce}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{1892}$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^2}{\sqrt{1-\frac{1}{c^2x^2}}}dx}{2ce}+\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \text{1730}$$

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2ce}$$

$$\Big\downarrow \text{540}$$

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\left(e^2x\left(-\sqrt{1-\frac{1}{c^2x^2}}\right)-\int -\frac{d\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)}{2ce}$$

$$\Big\downarrow \text{25}$$

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\left(\int \frac{d\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\left(d\int\frac{\left(\frac{d}{x}+2e\right)x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

$\Big\downarrow$ 538

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(d\int\frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+2e\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

$\Big\downarrow$ 223

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(2e\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}+cd\arcsin\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

$\Big\downarrow$ 243

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(e\int\frac{x}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x^2}+cd\arcsin\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

$\Big\downarrow$ 73

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(cd\arcsin\left(\frac{1}{cx}\right)-2c^2e\int\frac{1}{c^2-c^2\sqrt{1-\frac{1}{c^2x^2}}}d\sqrt{1-\frac{1}{c^2x^2}}\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

$\Big\downarrow$ 221

$$\frac{(d+ex)^2\left(a+b\csc^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(cd\arcsin\left(\frac{1}{cx}\right)-2e\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)\right)-e^2x\sqrt{1-\frac{1}{c^2x^2}}\right)}{2ce}$$

input
```
Int[(d + e*x)*(a + b*ArcCsc[c*x]),x]
```

output
```
((d + e*x)^2*(a + b*ArcCsc[c*x]))/(2*e) - (b*(-(e^2*Sqrt[1 - 1/(c^2*x^2)]*
x) + d*(c*d*ArcSin[1/(c*x)] - 2*e*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])))/(2*c*e
)
```

3.46. $\int(d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1730
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.46.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

method	result	size
parts	$a\left(\frac{1}{2}e\,x^{2}+dx\right)+\frac{b\left(\frac{c\,\operatorname{arccsc}(cx)x^{2}e}{2}+\operatorname{arccsc}(cx)xcd+\frac{\sqrt{c^{2}x^{2}-1}\,\left(2dc\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)+e\sqrt{c^{2}x^{2}-1}\right)}{2c^{2}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}\right)}{c}$	110
derivativedivides	$\frac{\frac{a\left(dx\,c^{2}+\frac{1}{2}c^{2}e\,x^{2}\right)}{c}+\frac{b\left(\operatorname{arccsc}(cx)d\,c^{2}x+\frac{\operatorname{arccsc}(cx)e\,c^{2}x^{2}}{2}+\frac{\sqrt{c^{2}x^{2}-1}\,\left(2dc\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)+e\sqrt{c^{2}x^{2}-1}\right)}{2\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c}}{c}$	127
default	$\frac{\frac{a\left(dx\,c^{2}+\frac{1}{2}c^{2}e\,x^{2}\right)}{c}+\frac{b\left(\operatorname{arccsc}(cx)d\,c^{2}x+\frac{\operatorname{arccsc}(cx)e\,c^{2}x^{2}}{2}+\frac{\sqrt{c^{2}x^{2}-1}\,\left(2dc\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)+e\sqrt{c^{2}x^{2}-1}\right)}{2\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\right)}{c}}{c}$	127

input
```
int((e*x+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arccsc(c*x)*x^2*e+arccsc(c*x)*x*c*d+1/2/c^2/(
(c^2*x^2-1)/c^2/x^2)^(1/2)/x*(c^2*x^2-1)^(1/2)*(2*d*c*ln(c*x+(c^2*x^2-1)^(
1/2))+e*(c^2*x^2-1)^(1/2)))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.55

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{ac^2ex^2+2\,ac^2dx-2\,bcd\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}be+\left(bc^2ex^2+2\,bc^2dx-2\,bc^2d-bc^2e\right)\mathrm{arcc}}{2\,c^2}$$

input
```
integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/2*(a*c^2*e*x^2 + 2*a*c^2*d*x - 2*b*c*d*log(-c*x + sqrt(c^2*x^2 - 1)) + s
qrt(c^2*x^2 - 1)*b*e + (b*c^2*e*x^2 + 2*b*c^2*d*x - 2*b*c^2*d - b*c^2*e)*a
rccsc(c*x) - 2*(2*b*c^2*d + b*c^2*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/c^2
```

3.46.6 Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx = adx+\frac{aex^2}{2}+bdx\,\mathrm{acsc}\,(cx)+\frac{bex^2\,\mathrm{acsc}\,(cx)}{2} + \frac{bd\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{be\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$

input
```
integrate((e*x+d)*(a+b*acsc(c*x)),x)
```

output
```
a*d*x + a*e*x**2/2 + b*d*x*acsc(c*x) + b*e*x**2*acsc(c*x)/2 + b*d*Piecewis
e((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e*Piecewis
e((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c,
True))/(2*c)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{2}aex^2+\frac{1}{2}\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)be+adx$$
$$+\frac{\left(2\,cx\operatorname{arccsc}(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

input
```
integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1)/c)*b*e + a*d
*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(
-1/(c^2*x^2) + 1) + 1))*b*d/c
```

3.46.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(73) = 146.

Time = 0.44 (sec) , antiderivative size = 341, normalized size of antiderivative = 4.11

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{8}\left(\frac{bex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{aex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c}+\frac{4\,bdx\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\arcsin\left(\frac{1}{c\right.}}{c}\right.$$

input
```
integrate((e*x+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/8*(b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + a*e*x^2*(s
qrt(-1/(c^2*x^2) + 1) + 1)^2/c + 4*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcs
in(1/(c*x))/c + 4*a*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c + 2*b*e*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)/c^2 + 8*b*d*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - 8*b*
d*log(1/(abs(c)*abs(x)))/c^2 + 2*b*e*arcsin(1/(c*x))/c^3 + 2*a*e/c^3 + 4*b
*d*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 4*a*d/(c^3*x*(sq
rt(-1/(c^2*x^2) + 1) + 1)) - 2*b*e/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
b*e*arcsin(1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + a*e/(c^5*x^
2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2))*c
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\left(d+e\,x\right)\,dx$$

input
```
int((a + b*asin(1/(c*x)))*(d + e*x),x)
```

output
```
int((a + b*asin(1/(c*x)))*(d + e*x), x)
```

3.47 $\int (a + b\csc^{-1}(cx))\ dx$

3.47.1 Optimal result

Integrand size = 8, antiderivative size = 31

$$\int (a + b\csc^{-1}(cx))\ dx = ax + bx\csc^{-1}(cx) + \frac{b\operatorname{arctanh}\left(\sqrt{1 - \frac{1}{c^2x^2}}\right)}{c}$$

output
```
a*x+b*x*arccsc(c*x)+b*arctanh((1-1/c^2/x^2)^(1/2))/c
```

3.47.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int (a + b\csc^{-1}(cx))\ dx = ax + bx\csc^{-1}(cx) + \frac{b\sqrt{1 - \frac{1}{c^2x^2}}\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1 + c^2x^2}}$$

input
```
Integrate[a + b*ArcCsc[c*x],x]
```

output
```
a*x + b*x*ArcCsc[c*x] + (b*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[-1 +
 c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

3.47.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{b\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}+bx\csc^{-1}(cx)$$

input
```
Int[a + b*ArcCsc[c*x],x]
```

output
```
a*x + b*x*ArcCsc[c*x] + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/c
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.47.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result	size
default	$ax+bx\,\text{arccsc}\left(cx\right)+\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	37
parts	$ax+bx\,\text{arccsc}\left(cx\right)+\frac{b\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)}{c}$	37
derivativedivides	$\frac{acx+b\left(\text{arccsc}(cx)cx+\ln\left(cx+cx\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{c}$	40

input
```
int(a+b*arccsc(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arccsc(c*x)+b/c*ln(c*x+c*x*(1-1/c^2/x^2)^(1/2))
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{acx - 2\,bc\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + (bcx-bc)\,\mathrm{arccsc}\,(cx) - b\log\left(-cx+\sqrt{c^2x^2-1}\right)}{c}$$

input
```
integrate(a+b*arccsc(c*x),x, algorithm="fricas")
```

output
```
(a*c*x - 2*b*c*arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*x - b*c)*arccsc(c*x
) - b*log(-c*x + sqrt(c^2*x^2 - 1)))/c
```

3.47.6 Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx = ax + b\left(x\,\mathrm{acsc}\,(cx) + \frac{\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2| > 1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}}{c}\right)$$

input
```
integrate(a+b*acsc(c*x),x)
```

output
```
a*x + b*(x*acsc(c*x) + Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asi
n(c*x), True))/c)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=ax+\frac{\left(2\,cx\operatorname{arccsc}\left(cx\right)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)b}{2\,c}$$

input
```
integrate(a+b*arccsc(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt
(-1/(c^2*x^2) + 1) + 1))*b/c
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.00

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{2}\,bc\left(\frac{2\,x\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}\right)+ax$$

input
```
integrate(a+b*arccsc(c*x),x, algorithm="giac")
```

output
```
1/2*b*c*(2*x*arcsin(1/(c*x))/c + (log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-s
qrt(-1/(c^2*x^2) + 1) + 1))/c^2) + a*x
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \left(a+b\csc^{-1}(cx)\right)\,dx=a\,x+b\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)+\frac{b\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c}$$

input
```
int(a + b*asin(1/(c*x)),x)
```

output
```
a*x + b*x*asin(1/(c*x)) + (b*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c
```

3.48 $\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx$

3.48.1 Optimal result

Integrand size = 16, antiderivative size = 257

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{i\left(e-\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{i\left(e+\sqrt{-c^2d^2+e^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}$$

output
```
-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e+(a+b*arccsc(c*x))
*ln(1-I*(I/c/x+(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b*a
rccsc(c*x))*ln(1-I*(I/c/x+(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/
d)/e+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e-I*b*polylog(2,I*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(e-(-c^2*d^2+e^2)^(1/2))/c/d)/e-I*b*polylog(2,I*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(e+(-c^2*d^2+e^2)^(1/2))/c/d)/e
```

3.48.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.60

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \frac{a\log(d+ex)}{e} + \frac{b\left(i(\pi-2\csc^{-1}(cx))^2 + 32i\arcsin\left(\frac{\sqrt{1+\frac{e}{cd}}}{\sqrt{2}}\right)\arctan\left(\frac{(cd-e)\cot\left(\frac{1}{4}(\pi+2\csc^{-1}(cx))\right)}{\sqrt{-c^2d^2+e^2}}\right) - 4\left(\pi-2\csc^{-1}(cx)+\cdots\right.\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x),x]
```

output
```
(a*Log[d + e*x])/e + (b*(I*(Pi - 2*ArcCsc[c*x])^2 + (32*I)*ArcSin[Sqrt[1 +
 e/(c*d)]/Sqrt[2]]*ArcTan[((c*d - e)*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[-(c
^2*d^2) + e^2]] - 4*(Pi - 2*ArcCsc[c*x] + 4*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[
2]])*Log[1 + (I*(e - Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*ArcCsc[c*x]))] - 4
*(Pi - 2*ArcCsc[c*x] - 4*ArcSin[Sqrt[1 + e/(c*d)]/Sqrt[2]])*Log[1 + (I*(e
+ Sqrt[-(c^2*d^2) + e^2]))/(c*d*E^(I*ArcCsc[c*x]))] - 8*ArcCsc[c*x]*Log[1
- E^((2*I)*ArcCsc[c*x])] + 4*(Pi - 2*ArcCsc[c*x])*Log[e + d/x] + 8*ArcCsc[
c*x]*Log[e + d/x] + (8*I)*(PolyLog[2, (I*(-e + Sqrt[-(c^2*d^2) + e^2]))/(c
*d*E^(I*ArcCsc[c*x]))] + PolyLog[2, ((-I)*(e + Sqrt[-(c^2*d^2) + e^2]))/(c
*d*E^(I*ArcCsc[c*x]))]) + (4*I)*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCs
c[c*x])])))/(8*e)
```

3.48.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5748, 2998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx$$

$$\Big\downarrow \; 5748$$

$$\frac{b\int \frac{\log\left(1-\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{b\int \frac{\log\left(1-\frac{i\left(e+\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}-\frac{b\int \frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e}$$

$$\Big\downarrow \textbf{2998}$$

$$\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{i\left(\sqrt{e^2-c^2d^2}+e\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1-e^{2i\csc^{-1}(cx)}\right)(a+b\csc^{-1}(cx))}{e}-\frac{ib\,\text{PolyLog}\left(2,\frac{i\left(e-\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}-\frac{ib\,\text{PolyLog}\left(2,\frac{i\left(e+\sqrt{e^2-c^2d^2}\right)e^{i\csc^{-1}(cx)}}{cd}\right)}{e}+\frac{ib\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x),x]
```

output
```
((a + b*ArcCsc[c*x])*Log[1 - (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*ArcCsc[c
*x]))/(c*d)])/e + ((a + b*ArcCsc[c*x])*Log[1 - (I*(e + Sqrt[-(c^2*d^2) + e
^2])*E^(I*ArcCsc[c*x]))/(c*d)])/e - ((a + b*ArcCsc[c*x])*Log[1 - E^((2*I)*
ArcCsc[c*x])])/e - (I*b*PolyLog[2, (I*(e - Sqrt[-(c^2*d^2) + e^2])*E^(I*Ar
cCsc[c*x]))/(c*d)])/e - (I*b*PolyLog[2, (I*(e + Sqrt[-(c^2*d^2) + e^2])*E^
(I*ArcCsc[c*x]))/(c*d)])/e + ((I/2)*b*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/e
```

3.48. $\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx$

3.48.3.1 Defintions of rubi rules used

rule 2998
```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

rule 5748
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :> S
imp[(a + b*ArcCsc[c*x])*(Log[1 - I*(e - Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcCs
c[c*x])/(c*d))]/e), x] + (Simp[(a + b*ArcCsc[c*x])*(Log[1 - I*(e + Sqrt[(-c
^2)*d^2 + e^2])*(E^(I*ArcCsc[c*x])/(c*d))]/e), x] - Simp[(a + b*ArcCsc[c*x]
)*(Log[1 - E^(2*I*ArcCsc[c*x])]/e), x] + Simp[b/(c*e)   Int[Log[1 - I*(e -
Sqrt[(-c^2)*d^2 + e^2])*(E^(I*ArcCsc[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2
)]), x], x] + Simp[b/(c*e)   Int[Log[1 - I*(e + Sqrt[(-c^2)*d^2 + e^2])*(E^
(I*ArcCsc[c*x])/(c*d))]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] - Simp[b/(c*e)
  Int[Log[1 - E^(2*I*ArcCsc[c*x])]/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x]) /;
FreeQ[{a, b, c, d, e}, x]
```

3.48.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(316) = 632$.

Time = 3.60 (sec) , antiderivative size = 868, normalized size of antiderivative = 3.38

method	result
parts	$\frac{a\ln(ex+d)}{e}+\frac{b\left(-\frac{i\,\mathrm{dilog}\left(\frac{-cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-ie+\sqrt{c^2d^2-e^2}}{-ie+\sqrt{c^2d^2-e^2}}\right)c^3d^2}{e\left(c^2d^2-e^2\right)}-\frac{i\,\mathrm{dilog}\left(\frac{cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+ie+\sqrt{c^2d^2-e^2}}{ie+\sqrt{c^2d^2-e^2}}\right)c^3d^2}{e\left(c^2d^2-e^2\right)}\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{\mathrm{arccsc}(cx)\ln\left(\frac{cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+ie-\sqrt{c^2d^2-e^2}}{ie-\sqrt{c^2d^2-e^2}}\right)c^2d^2}{e\left(c^2d^2-e^2\right)}+\frac{\mathrm{arccsc}(cx)\ln\left(\cdots\right)}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{\mathrm{arccsc}(cx)\ln\left(\frac{cd\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)+ie-\sqrt{c^2d^2-e^2}}{ie-\sqrt{c^2d^2-e^2}}\right)c^2d^2}{e\left(c^2d^2-e^2\right)}+\frac{\mathrm{arccsc}(cx)\ln\left(\cdots\right)}{\cdots}\right.}{\cdots}$

input
```
int((a+b*arccsc(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e+b/c*(-I/e/(c^2*d^2-e^2)*dilog((-c*d*(I/c/x+(1-1/c^2/x^2)^(1/
2))-I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2)))*c^3*d^2-I/e/(c^2*
d^2-e^2)*dilog((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(
I*e+(c^2*d^2-e^2)^(1/2)))*c^3*d^2-c/e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)
^(1/2))-I*c/e*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))+I*c/e*dilog(1+I/c/x+(1-1/c^
2/x^2)^(1/2))+I*c*e/(c^2*d^2-e^2)*dilog((-c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))-
I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2)))+I*c*e/(c^2*d^2-e^2)*d
ilog((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d
^2-e^2)^(1/2)))-c*e*arccsc(c*x)/(c^2*d^2-e^2)*ln((-c*d*(I/c/x+(1-1/c^2/x^2
)^(1/2))-I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2)))-c*e*arccsc(c
*x)/(c^2*d^2-e^2)*ln((c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))+I*e+(c^2*d^2-e^2)^(1
/2))/(I*e+(c^2*d^2-e^2)^(1/2)))+1/e*arccsc(c*x)/(c^2*d^2-e^2)*ln((-c*d*(I/
c/x+(1-1/c^2/x^2)^(1/2))-I*e+(c^2*d^2-e^2)^(1/2))/(-I*e+(c^2*d^2-e^2)^(1/2
)))*c^3*d^2+1/e*arccsc(c*x)/(c^2*d^2-e^2)*ln((c*d*(I/c/x+(1-1/c^2/x^2)^(1/
2))+I*e+(c^2*d^2-e^2)^(1/2))/(I*e+(c^2*d^2-e^2)^(1/2)))*c^3*d^2)
```

3.48.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e*x + d), x)
```

3.48.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx=\int \frac{a+b\operatorname{acsc}(cx)}{d+ex}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*acsc(c*x))/(d + e*x), x)
```

3.48.7 Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x + d), x) + a*log(
e*x + d)/e
```

3.48.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{d+e\,x}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x), x)
```

3.49 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx$

3.49.1 Optimal result

Integrand size = 16, antiderivative size = 102

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx=\frac{b\csc^{-1}(cx)}{de}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2-e^2}}$$

output
```
b*arccsc(c*x)/d/e+(-a-b*arccsc(c*x))/e/(e*x+d)+b*arctanh((c^2*d+e/x)/c/(c^
2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d/(c^2*d^2-e^2)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.38

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx=-\frac{a}{e(d+ex)}-\frac{b\csc^{-1}(cx)}{e(d+ex)}+\frac{b\arcsin\left(\frac{1}{cx}\right)}{de}+\frac{b\log(d+ex)}{d\sqrt{c^2d^2-e^2}}-\frac{b\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2-e^2}}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^2,x]
```

output
```
-(a/(e*(d + e*x))) - (b*ArcCsc[c*x])/(e*(d + e*x)) + (b*ArcSin[1/(c*x)])/(
d*e) + (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 - e^2]) - (b*Log[e + c*(c*d - Sqrt
[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 - e^2])
```

3.49.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5750, 1892, 1803, 605, 223, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx$$

$$\Big\downarrow \ 5750$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)}dx}{ce}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \ 1892$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)x^3}dx}{ce}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \ 1803$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)x}d\frac{1}{x}}{ce}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \ 605$$

$$\frac{b\left(\frac{\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{d}-\frac{e\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}\right)}{ce}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \ 223$$

$$\frac{b\left(\frac{c\arcsin\left(\frac{1}{cx}\right)}{d}-\frac{e\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}\right)}{ce}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \ 488$$

$$\frac{b\left(\frac{e\int \frac{1}{d^2-\frac{e^2}{c^2}-\frac{1}{x^2}}d\frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}}{d}+\frac{c\arcsin\left(\frac{1}{cx}\right)}{d}\right)}{ce}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}$$

$$\downarrow 219$$

$$\frac{b\left(\frac{c\arcsin\left(\frac{1}{cx}\right)}{d}+\frac{ce\operatorname{arctanh}\left(\frac{c\left(\frac{e}{c^2x}+d\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}}\right)}{ce}-\frac{a+b\csc^{-1}(cx)}{e(d+ex)}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x)^2,x]
```

output
```
-((a + b*ArcCsc[c*x])/(e*(d + e*x))) + (b*((c*ArcSin[1/(c*x)])/d + (c*e*Ar
cTanh[(c*(d + e/(c^2*x)))/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - 1/(c^2*x^2)])])/(d
*Sqrt[c^2*d^2 - e^2])))/(c*e)
```

3.49.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.49.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.88

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^2\,\mathrm{arccsc}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$
derivativedivides	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\mathrm{arccsc}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$
default	$\frac{-\frac{a\,c^2}{(cex+cd)e}+b\,c^2\left(-\frac{\mathrm{arccsc}(cx)}{(cex+cd)e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}-\ln\left(\frac{2\sqrt{\frac{c^2d^2-e^2}{e^2}}\,\sqrt{c^2x^2-1}\,e-2dx\,c^2-2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^2xd\sqrt{\frac{c^2d^2-e^2}{e^2}}}\right)}{c}$

input
```
int((a+b*arccsc(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arccsc(c*x)+1/e*(c^2*x^2-1)^(1/2)*(ar
ctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)-ln(2*(((c^2*d^2-e^2)/e
^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d)))/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x/d/((c^2*d^2-e^2)/e^2)^(1/2))
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(96) = 192.

Time = 0.33 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.66

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx$$

$$= \left[-\frac{ac^2d^3 - ade^2 - \sqrt{c^2d^2-e^2}(be^2x+bde)\log\left(\frac{c^3d^2x+cde+\sqrt{c^2d^2-e^2}(c^2dx+e)+\left(c^2d^2+\sqrt{c^2d^2-e^2}cd-e^2\right)\sqrt{c^2x^2-1}}{ex+d}\right) + \cdots}{c^2d^4e - d^2e^3 + (c^2d^3e\cdots}\right.$$

$$-\frac{ac^2d^3 - ade^2 + 2\sqrt{-c^2d^2+e^2}(be^2x+bde)\arctan\left(-\frac{\sqrt{-c^2d^2+e^2}\sqrt{c^2x^2-1}e-\sqrt{-c^2d^2+e^2}(cex+cd)}{c^2d^2-e^2}\right) + (bc^2d^3 - b\cdots}{c^2d^4e - d^2e^3 + (c^2d^3e^2 - d\cdots}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-(a*c^2*d^3 - a*d*e^2 - sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*log((c^3*d^
2*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 + sqrt(c^2*d^2
- e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + (b*c^2*d^3 - b*d*e^2)*ar
ccsc(c*x) + 2*(b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x
+ sqrt(c^2*x^2 - 1)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x), -(a
*c^2*d^3 - a*d*e^2 + 2*sqrt(-c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*arctan(-(sqr
t(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d)
)/(c^2*d^2 - e^2)) + (b*c^2*d^3 - b*d*e^2)*arccsc(c*x) + 2*(b*c^2*d^3 - b*
d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^
4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x)]
```

3.49.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\operatorname{acsc}(cx)}{(d+ex)^2}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x+d)**2,x)
```

output
```
Integral((a + b*acsc(c*x))/(d + e*x)**2, x)
```

3.49.7 Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-((c^2*e^2*x + c^2*d*e)*integrate(x*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)
)/(c^2*e^2*x^3 + c^2*d*e*x^2 - e^2*x - d*e + (c^2*e^2*x^3 + c^2*d*e*x^2 -
e^2*x - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1)))*b/(e^2*x + d*e) - a/(e^2*x + d*e)
```

3.49.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^2}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x)^2,x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x)^2, x)
```

3.50 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx$

3.50.1 Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx=-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{2d\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)}+\frac{b\csc^{-1}(cx)}{2d^2e}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}+\frac{b(2c^2d^2-e^2)\,\text{arctanh}\left(\frac{c^2d+\frac{e}{x}}{c\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2-e^2\right)^{3/2}}$$

output
```
1/2*b*arccsc(c*x)/d^2/e+1/2*(-a-b*arccsc(c*x))/e/(e*x+d)^2+1/2*b*(2*c^2*d^
2-e^2)*arctanh((c^2*d+e/x)/c/(c^2*d^2-e^2)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^2/
(c^2*d^2-e^2)^(3/2)-1/2*b*c*e*(1-1/c^2/x^2)^(1/2)/d/(c^2*d^2-e^2)/(e+d/x)
```

3.50.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.45

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx=\frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}\,x}{d\left(c^2d^2-e^2\right)(d+ex)}-\frac{b\csc^{-1}(cx)}{e(d+ex)^2}+\frac{b\arcsin\left(\frac{1}{cx}\right)}{d^2e}+\frac{b(2c^2d^2-e^2)\log(d+ex)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}-\frac{b(2c^2d^2-e^2)\log\left(e+c\left(cd-\sqrt{c^2d^2-e^2}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{d^2(cd-e)(cd+e)\sqrt{c^2d^2-e^2}}\right)$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^3,x]
```

output
```
(-(a/(e*(d + e*x)^2)) - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d^2 - e^2)
*(d + e*x)) - (b*ArcCsc[c*x])/(e*(d + e*x)^2) + (b*ArcSin[1/(c*x)])/(d^2*e
) + (b*(2*c^2*d^2 - e^2)*Log[d + e*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d
^2 - e^2]) - (b*(2*c^2*d^2 - e^2)*Log[e + c*(c*d - Sqrt[c^2*d^2 - e^2]*Sqr
t[1 - 1/(c^2*x^2)])*x])/(d^2*(c*d - e)*(c*d + e)*Sqrt[c^2*d^2 - e^2]))/2
```

3.50.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5750, 1892, 1803, 603, 719, 223, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{5750}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^2}dx}{2ce}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1892}$$

$$-\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)^2x^4}dx}{2ce}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1803}$$

$$\frac{b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)^2x^2}d\frac{1}{x}}{2ce}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{603}$$

$$\frac{b\left(-\frac{\int \frac{e-\frac{d-\frac{e^2}{c^2d}}{x}}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{719}$$

$$\frac{b\left(-\frac{e\left(2-\frac{e^2}{c^2d^2}\right)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}-\left(1-\frac{e^2}{c^2d^2}\right)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{b\left(-\frac{e\left(2-\frac{e^2}{c^2d^2}\right)\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{b\left(-\frac{-e\left(2-\frac{e^2}{c^2d^2}\right)\int \frac{1}{d^2-\frac{e^2}{c^2}-\frac{1}{x^2}}d\frac{d+\frac{e}{c^2x}}{\sqrt{1-\frac{1}{c^2x^2}}}-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(-\frac{-c\arcsin\left(\frac{1}{cx}\right)\left(1-\frac{e^2}{c^2d^2}\right)-\frac{ce\left(2-\frac{e^2}{c^2d^2}\right)\text{arctanh}\left(\frac{c\left(\frac{e}{c^2x}+d\right)}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{c^2d^2-e^2}}\right)}{\sqrt{c^2d^2-e^2}}}{d^2-\frac{e^2}{c^2}}-\frac{e^2\sqrt{1-\frac{1}{c^2x^2}}}{d\left(d^2-\frac{e^2}{c^2}\right)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\csc^{-1}(cx)}{2e(d+ex)^2}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x)^3,x]
```

output
```
-1/2*(a + b*ArcCsc[c*x])/(e*(d + e*x)^2) + (b*(-((e^2*Sqrt[1 - 1/(c^2*x^2)
])/(d*(d^2 - e^2/c^2)*(e + d/x))) - (-(c*(1 - e^2/(c^2*d^2))*ArcSin[1/(c*x
)]) - (c*e*(2 - e^2/(c^2*d^2))*ArcTanh[(c*(d + e/(c^2*x)))/(Sqrt[c^2*d^2 -
 e^2]*Sqrt[1 - 1/(c^2*x^2)])])/Sqrt[c^2*d^2 - e^2])/(d^2 - e^2/c^2)))/(2*c
*e)
```

3.50. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx$

3.50.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

3.50. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx$

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.50.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(159) = 318$.

Time = 3.24 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.33

method	result
parts	$-\frac{a}{2(ex+d)^2e}+\frac{b\left(-\frac{c^3\operatorname{arccsc}(cx)}{2(cex+cd)^2e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex \ldots\right.}{\ldots}\right.}{\ldots}$
derivativedivides	$\frac{-\frac{a\,c^3}{2(cex+cd)^2e}+b\,c^3\left(-\frac{\operatorname{arccsc}(cx)}{2(cex+cd)^2e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex \ldots\right.}{\ldots}\right.}{\ldots}$
default	$\frac{-\frac{a\,c^3}{2(cex+cd)^2e}+b\,c^3\left(-\frac{\operatorname{arccsc}(cx)}{2(cex+cd)^2e}+\frac{\sqrt{c^2x^2-1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^3+\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{\frac{c^2d^2-e^2}{e^2}}\,c^3d^2ex \ldots\right.}{\ldots}\right.}{\ldots}$

input
```
int((a+b*arccsc(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arccsc(c*x)+1/2/e*(c^2*x^
2-1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c^3*d^3+
arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*c^3*d^2*e*x-2*ln(2*(
((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*c^3*
d^3-2*ln(2*(((c^2*d^2-e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*
x+c*d))*c^3*d^2*e*x-arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*
c*d*e^2-arctan(1/(c^2*x^2-1)^(1/2))*((c^2*d^2-e^2)/e^2)^(1/2)*e^3*c*x-(c^2
*x^2-1)^(1/2)*((c^2*d^2-e^2)/e^2)^(1/2)*c*d*e^2+ln(2*(((c^2*d^2-e^2)/e^2)^
(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*c*d*e^2+ln(2*(((c^2*d^2-
e^2)/e^2)^(1/2)*(c^2*x^2-1)^(1/2)*e-d*x*c^2-e)/(c*e*x+c*d))*e^3*c*x)/((c^2
*x^2-1)/c^2/x^2)^(1/2)/x/d^2/((c^2*d^2-e^2)/e^2)^(1/2)/(c^2*d^2-e^2)/(c*e*
x+c*d))
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. 2(156) = 312.

Time = 0.56 (sec) , antiderivative size = 1111, normalized size of antiderivative = 6.46

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx$$

$$= \left[-\frac{ac^4d^6+bc^3d^5e-2\,ac^2d^4e^2-bcd^3e^3+ad^2e^4+(bc^3d^3e^3-bcde^5)x^2-(2\,bc^2d^4e-bd^2e^3+(2\,bc^2d^2e^3-}{}\right.$$

$$-\frac{ac^4d^6+bc^3d^5e-2\,ac^2d^4e^2-bcd^3e^3+ad^2e^4+(bc^3d^3e^3-bcde^5)x^2+2\,(2\,bc^2d^4e-bd^2e^3+(2\,bc^2d^2e^3-}{}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[-1/2*(a*c^4*d^6 + b*c^3*d^5*e - 2*a*c^2*d^4*e^2 - b*c*d^3*e^3 + a*d^2*e^4
 + (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2
*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*sqrt(c^2*d^2 - e^
2)*log((c^3*d^2*x + c*d*e + sqrt(c^2*d^2 - e^2)*(c^2*d*x + e) + (c^2*d^2 +
 sqrt(c^2*d^2 - e^2)*c*d - e^2)*sqrt(c^2*x^2 - 1))/(e*x + d)) + 2*(b*c^3*d
^4*e^2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arccsc
(c*x) + 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*
c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*
arctan(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3
*e^3 - b*d*e^5)*x)*sqrt(c^2*x^2 - 1))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5
 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^
5*e^4 + d^3*e^6)*x), -1/2*(a*c^4*d^6 + b*c^3*d^5*e - 2*a*c^2*d^4*e^2 - b*c
*d^3*e^3 + a*d^2*e^4 + (b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 + 2*(2*b*c^2*d^4*e
- b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4
)*x)*sqrt(-c^2*d^2 + e^2)*arctan(-(sqrt(-c^2*d^2 + e^2)*sqrt(c^2*x^2 - 1)*
e - sqrt(-c^2*d^2 + e^2)*(c*e*x + c*d))/(c^2*d^2 - e^2)) + 2*(b*c^3*d^4*e^
2 - b*c*d^2*e^4)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*arccsc(c*x)
 + 2*(b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d
^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*arcta
n(-c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*d^4*e^2 - b*d^2*e^4 + (b*c^2*d^3*e...
```

3.50.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{a+b\operatorname{acsc}(cx)}{(d+ex)^3}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x+d)**3,x)
```

output
```
Integral((a + b*acsc(c*x))/(d + e*x)**3, x)
```

3.50.7 Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(c^2*e^3*x^2 + 2*c^2*d*e^2*x + c^2*d^2*e)*integrate(1/2*x*e^(1/2*l
og(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^2*x
 - d^2*e + (c^2*d^2*e - e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 - 2*d*e^
2*x - d^2*e + (c^2*d^2*e - e^3)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x)
+ arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^2 + 2*d*e^2*x + d^2*e)
 - 1/2*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)
```

3.50.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^3}\,dx = \int \frac{a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^3}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x)^3,x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x)^3, x)
```

3.51 $\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 496

$$\begin{aligned}
&\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx\\
&=\frac{4bd\sqrt{d+ex}(1-c^2x^2)}{105c^3e\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{4b(d+ex)^{3/2}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}\\
&-\frac{4d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^3}+\frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^3}\\
&+\frac{4b(5c^2d^2-9e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}\\
&-\frac{4bd(9c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\
&-\frac{32bd^4\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

output
```
2/3*d^2*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3-4/5*d*(e*x+d)^(5/2)*(a+b*arccs
c(c*x))/e^3+2/7*(e*x+d)^(7/2)*(a+b*arccsc(c*x))/e^3-4/35*b*(e*x+d)^(3/2)*(
-c^2*x^2+1)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/105*b*d*(-c^2*x^2+1)*(e*x+d)^(1/
2)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/105*b*(5*c^2*d^2-9*e^2)*EllipticE(1/2*(-c
*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^
(1/2)/c^4/e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/105*b*d*(9
*c^2*d^2-e^2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/
2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2/x/(1-1/c^2/x^2)^(
1/2)/(e*x+d)^(1/2)-32/105*b*d^4*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^
(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^
3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.51.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 44.49 (sec) , antiderivative size = 870, normalized size of antiderivative = 1.75

$$\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx = -\frac{ad^3\sqrt{d+ex}B_{-\frac{ex}{d}}\left(3,\frac{3}{2}\right)}{e^3\sqrt{1+\frac{ex}{d}}} + \frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(-\frac{4\left(-5c^2d^2+9e^2\right)\sqrt{1-\frac{1}{c^2x^2}}}{105e^2}-\frac{16c^3d^3\csc^{-1}(cx)}{105e^3}-\frac{2}{7}c^3x^3\csc^{-1}(cx)-\frac{2c^2x^2\left(2e\sqrt{1-\frac{1}{c^2x^2}}+cd\csc^{-1}(cx)\right)}{35e}-\frac{8cx\left(cde\sqrt{1-\frac{1}{c^2x^2}}-c^2d\right.}{105e^2}\cdots\right)}{\sqrt{d+ex}}\cdots\right)}{\cdots}$$

input
```
Integrate[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

output

```
-((a*d^3*Sqrt[d + e*x]*Beta[-((e*x)/d), 3, 3/2])/(e^3*Sqrt[1 + (e*x)/d]))
+ (b*(-((c*(e + d/x)*x*((-4*(-5*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(1
05*e^2) - (16*c^3*d^3*ArcCsc[c*x])/(105*e^3) - (2*c^3*x^3*ArcCsc[c*x])/7 -
 (2*c^2*x^2*(2*e*Sqrt[1 - 1/(c^2*x^2)] + c*d*ArcCsc[c*x]))/(35*e) - (8*c*x
*(c*d*e*Sqrt[1 - 1/(c^2*x^2)] - c^2*d^2*ArcCsc[c*x]))/(105*e^2)))/Sqrt[d +
 e*x]) - (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(9*c^3*d^3*e - c*d*e^3)*Sqrt[(c*d
+ c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[
2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) +
 (2*(8*c^4*d^4 + 5*c^2*d^2*e^2 - 9*e^4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt
[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)
])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-5*c^3*d^3*e +
9*c*d*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt
[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]
/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*S
qrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x
)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x
)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c
*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[
Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c^2*x^2)]*Sqrt
[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(105*e^3*Sqrt[d + e*x])))/c^4
```

3.51.3 Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.10, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.095$, Rules used = {5770, 27, 7272, 2351, 634, 600, 508, 327, 511, 321, 632, 186, 413, 412, 687, 27, 687, 27, 600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$$

$$\Big\downarrow \text{5770}$$

$$\frac{b\int \frac{2(d+ex)^{3/2}(8d^2-12exd+15e^2x^2)}{105e^3\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{(d+ex)^{3/2}\left(8d^2-12exd+15e^2x^2\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{105ce^3}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 7272

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{(d+ex)^{3/2}\left(8d^2-12exd+15e^2x^2\right)}{x\sqrt{1-c^2x^2}}dx}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 2351

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx+\int \frac{(d+ex)^{3/2}\left(15e^2x-12de\right)}{\sqrt{1-c^2x^2}}dx\right)}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 634

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\int \frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)+\int \frac{(d+ex)^{3/2}\left(15e^2x-12de\right)}{\sqrt{1-c^2x^2}}dx\right)}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 600

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+e\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)+\int \frac{(d+ex)^{3/2}\left(15e^2x-12de\right)}{\sqrt{1-c^2x^2}}dx\right)}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 508

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2e\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)+\int \frac{(d+ex)^{3/2}(15\ldots}{\sqrt{1-c\ldots}}\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

3.51. $\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx$

$\Big\downarrow$ 327

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)+\int\frac{(d+ex)^{3/2}(1}{\sqrt{1-}}\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\Big\downarrow$ 511

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+}\right.}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\Big\downarrow$ 321

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\Big\downarrow$ 632

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(d^2\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\right.}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\Big\downarrow$ 186

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(-2d^2\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\text{ar}\right.}{c\sqrt{}}\right.\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

3.51. $\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

$\Big\downarrow$ 413

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E}{}\right.\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\Big\downarrow$ 412

$$\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{(d+ex)^{3/2}(15e^2x-12de)}{\sqrt{1-c^2x^2}}dx+8d^2\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\right.}{c\sqrt{d+}}\right.\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\Big\downarrow$ 687

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{2\int \frac{15e\sqrt{d+ex}\left(4d^2c^2+dexc^2-3e^2\right)}{2\sqrt{1-c^2x^2}}dx}{5c^2}+8d^2\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{Elli}}{}\right.\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\Big\downarrow$ 27

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3e\int \frac{\sqrt{d+ex}\left(4d^2c^2+dexc^2-3e^2\right)}{\sqrt{1-c^2x^2}}dx}{c^2}+8d^2\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{Ellipt}}{}\right.\right.}{105ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\Big\downarrow$ 687

3.51. $\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3e\left(-\frac{2\int -\frac{c^2\left(4d\left(3c^2d^2-2e^2\right)+e\left(13c^2d^2-9e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+8d^2\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\right.}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.\right.}{105ce^3x\sqrt{}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 27

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3e\left(\frac{1}{3}\int\frac{4d\left(3c^2d^2-2e^2\right)+e\left(13c^2d^2-9e^2\right)x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+8d^2\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-c}}{\sqrt{2}}\right.\right.}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.\right.}{105ce^3x\sqrt{1-}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 600

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3e\left(\frac{1}{3}\left(\left(13c^2d^2-9e^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx-d(cd-e)(cd+e)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)-\frac{2}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+8d^2\left(-\frac{2d^2\sqrt{1-\frac{e(}{}}}{}\right.\right.}{1}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 508

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3e\left(\frac{1}{3}\left(-\frac{2\left(13c^2d^2-9e^2\right)\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}-d(cd-e)(cd+e)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)-\frac{2}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+8\right.}{}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 327

3.51. $\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3e\left(\frac{1}{3}\left(-d(cd-e)(cd+e)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2(13c^2d^2-9e^2)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)-\frac{2}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+\right.}{}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{511}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3e\left(\frac{1}{3}\left(\frac{2d(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{2(13c^2d^2-9e^2)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)-\frac{2}{3}de\right.}{c^2}\right.}{}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{321}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{3e\left(\frac{1}{3}\left(\frac{2d(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2(13c^2d^2-9e^2)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)-\frac{2}{3}de\sqrt{1\right.}{c^2}\right.}{}$$

input
```
Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

3.51. $\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

output
```
(2*d^2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)
*(a + b*ArcCsc[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7
*e^3) + (2*b*Sqrt[1 - c^2*x^2]*((-6*e^2*(d + e*x)^(3/2)*Sqrt[1 - c^2*x^2])
/c^2 - (3*e*((-2*d*e*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2])/3 + ((-2*(13*c^2*d^2
 - 9*e^2)*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*
d + e)])/(c*Sqrt[(c*(d + e*x))/(c*d + e)]) + (2*d*(c*d - e)*(c*d + e)*Sqrt
[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(
c*d + e)])/(c*Sqrt[d + e*x]))/3))/c^2 + 8*d^2*((-2*e*Sqrt[d + e*x]*Ellipti
cE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/
(c*d + e)]) - (2*d*e*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]) - (2*d^2*Sqrt[1 - (e
*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)])/Sqrt[d + e/c - (e*(1 - c*x))/c])))/(105*c*e^3*Sqrt[1 - 1/(c^2*
x^2)]*x)
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

3.51. $\int x^2\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.51.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(445) = 890$.

Time = 12.33 (sec) , antiderivative size = 1204, normalized size of antiderivative = 2.43

method	result	size
derivativedivides	Expression too large to display	1204
default	Expression too large to display	1204
parts	Expression too large to display	1225

input
```
int(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*d*(e*x+d)^(5/2)+1/3*d^2*(e*x+d)^(3/2))+b*(
1/7*arccsc(c*x)*(e*x+d)^(7/2)-2/5*arccsc(c*x)*d*(e*x+d)^(5/2)+1/3*arccsc(c
*x)*d^2*(e*x+d)^(3/2)+2/105/c^4*(3*(c/(c*d-e))^(1/2)*c^3*(e*x+d)^(7/2)-7*(
c/(c*d-e))^(1/2)*c^3*d*(e*x+d)^(5/2)+4*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*
((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/
2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+5*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*(
(-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2
),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3-8*d^3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2
)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^
(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3+5*(c/(c*d-e))
^(1/2)*c^3*d^2*(e*x+d)^(3/2)-5*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*
x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*c^2*d^2*e+5*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e
*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*
d-e)/(c*d+e))^(1/2))*c^2*d^2*e-(c/(c*d-e))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+8*(
(-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellip
ticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2-9*((
-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellipt
icE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2-3*(c/
(c*d-e))^(1/2)*c*e^2*(e*x+d)^(3/2)+9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)...
```

3.51.5 Fricas [F]

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x + d), x)
```

3.51.6 Sympy [F]

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2(a+b\operatorname{acsc}(cx))\sqrt{d+ex}\,dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))*sqrt(d + e*x), x)
```

3.51.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.51.8 Giac [F]

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x^2, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int(x^2*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int(x^2*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.52 $\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

3.52.1 Optimal result

Integrand size = 19, antiderivative size = 404

$$\begin{aligned}
&\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} - \frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2} \\
&\quad + \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} - \frac{8bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&\quad + \frac{4b(3c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{8bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

output
```
-2/3*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^2+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c
*x))/e^2-4/15*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/x/(1-1/c^2/x^2)^(1/2)-8/15*
b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d
)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))
^(1/2)+4/15*b*(3*c^2*d^2-e^2)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)
*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e/x/(
1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+8/15*b*d^3*EllipticPi(1/2*(-c*x+1)^(1/2)*
2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1
)^(1/2)/c/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.42 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.91

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{15}\left(\frac{4b\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}+\frac{2a\sqrt{d+ex}(-2d^2+dex+3e^2x^2)}{e^2}+\frac{2b\sqrt{d+ex}(-2d^2+dex+3e^2x^2)\csc^{-1}(cx)}{e^2}-\frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-2cd(cd-e)E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(-c^2d^2-2cde+e^2)\,\text{Ellipti}\right.}{c^3e^2\sqrt{-\frac{c}{cd+}}}\right.$$

input
```
Integrate[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

output
```
((4*b*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^
2 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x
^2)*ArcCsc[c*x])/e^2 - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e -
 c*e*x)/(c*d + e)]*(-2*c*d*(c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d + e
))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (-(c^2*d^2) - 2*c*d*e + e^2)*El
lipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)
] + 2*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[
d + e*x]], (c*d + e)/(c*d - e)]))/(c^3*e^2*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1
/(c^2*x^2)]*x))/15
```

3.52.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.17, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.158, Rules used = {5770, 27, 7272, 2351, 27, 497, 27, 600, 508, 327, 511, 321, 634, 600, 508, 327, 511, 321, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5770}$$

$$\frac{b\int -\frac{2(2d-3ex)(d+ex)^{3/2}}{15e^2\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\int \frac{(2d-3ex)(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{15ce^2}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{7272}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{(2d-3ex)(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{2351}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\int -\frac{3e(d+ex)^{3/2}}{\sqrt{1-c^2x^2}}dx+2d\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx\right)}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx-3e\int \frac{(d+ex)^{3/2}}{\sqrt{1-c^2x^2}}dx\right)}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{497}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx-3e\left(-\frac{2\int -\frac{3d^2c^2+4dexc^2+e^2}{2\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}}{3c^2}\right)\right)}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx-3e\left(\frac{\int \frac{3d^2c^2+4dexc^2+e^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}}{3c^2}\right)\right)}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 600$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx-3e\left(\frac{4c^2d\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx-(cd-e)(cd+e)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}}{3c^2}\right)\right)}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 508$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx-3e\left(\frac{-\left((cd-e)(cd+e)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)-\frac{8cd\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{\frac{c(d+ex)}{cd+e}}}}{3c^2}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d \cdots}}{3c^2}\right.\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \; 327$$

3.52. $\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx-3e\left(\frac{-(cd-e)(cd+e)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{8cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}}{3c^2}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+\cdots}}{3c^2}\right.\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 511

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx-3e\left(\frac{\frac{2(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{8cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}}{3c^2}\cdots\right.\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 321

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx-3e\left(\frac{\frac{2(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{8cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}}{3c^2}\cdots\right.\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 634

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\int\frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)-3e\left(\frac{\frac{2(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+\cdots}\right.}{c\sqrt{d+ex}}\cdots}{3c^2}\right.\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 600

3.52. $\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+e\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)-3e\left(\frac{\frac{2(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\text{Ellip}}{c\sqrt{d}}}{}\right.\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 508

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2e\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)-3e\left(\frac{\frac{2(cd-e)(cd}{}}{}\right.\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 327

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+de\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)-3e\left(\frac{\frac{2(cd-e)(}{}}{}\right.\right.}{15ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 511

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(d^2\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{15ce^2x\sqrt{1}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 321

3.52. $\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right.}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{15ce^2x\sqrt{1-}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 632

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(d^2\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\right.}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{15ce^2x\sqrt{1}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 186

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(-2d^2\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}E\left(\right.}{c}\right.\right.}{15c}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 413

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-2e\sqrt{d+ex}E\right.\right.}{15}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 412

3.52. $\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\ dx$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-$$

$$\frac{2b\sqrt{1-c^2x^2}\left(2d\left(-\frac{2d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2e\sqrt{d+ex}}{}\right.\right.}{15}$$

input
```
Int[x*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

output
```
(-2*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a
 + b*ArcCsc[c*x]))/(5*e^2) - (2*b*Sqrt[1 - c^2*x^2]*(-3*e*((-2*e*Sqrt[d +
e*x]*Sqrt[1 - c^2*x^2])/(3*c^2) + ((-8*c*d*Sqrt[d + e*x]*EllipticE[ArcSin[
Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[(c*(d + e*x))/(c*d + e)] +
(2*(c*d - e)*(c*d + e)*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]))/(3*c^2)) + 2*d*((
-2*e*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e
)])/(c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (2*d*e*Sqrt[(c*(d + e*x))/(c*d + e
)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d +
e*x]) - (2*d^2*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt
[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[d + e/c - (e*(1 - c*x))/c])))/(
15*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

3.52. $\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.52. $\int x\sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(363) = 726.

Time = 9.99 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.04

method	result
derivativedivides	$\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)\right.}{}\right.}{}$
default	$\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}-2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)\right.}{}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{(ex+d)^{\frac{3}{2}}d}{3}\right)}{e^2}+\frac{2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}-\frac{4\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}}{15}\right.}{}$

input
```
int(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^2*(-a*(-1/5*(e*x+d)^(5/2)+1/3*(e*x+d)^(3/2)*d)-b*(-1/5*arccsc(c*x)*(e*
x+d)^(5/2)+1/3*arccsc(c*x)*(e*x+d)^(3/2)*d-2/15/c^3*((c/(c*d-e))^(1/2)*c^2
*(e*x+d)^(5/2)-2*(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^(3/2)-d^2*((-c*(e*x+d)+c*
d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(
1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2-2*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2+2*d^2*((-c*(e*x+d)+c*d-
e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1
/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c
^2+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^
(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e
))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e-2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/
2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^
(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2)
,((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c/(c*d
-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/
2)))
```

3.52.5 Fricas [F]

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x*arccsc(c*x) + a*x)*sqrt(e*x + d), x)
```

3.52.6 Sympy [F]

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x(a+b\,\mathrm{acsc}\,(cx))\sqrt{d+ex}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acsc(c*x))*sqrt(d + e*x), x)
```

3.52.7 Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.52.8 Giac [F]

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)*x, x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int(x*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int(x*(a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.53 $\int \sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

3.53.1 Optimal result

Integrand size = 18, antiderivative size = 315

$$\int \sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx = \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2
^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(
1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*d*EllipticF(1/2*(-c*x+1
)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2
*x^2+1)^(1/2)/c^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*d^2*EllipticPi
(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e
))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.53.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 657 vs. $2(315) = 630$.

Time = 32.09 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.09

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{2a(d+ex)^{3/2}}{3e} - \frac{b(cd+cex)\left(-\frac{2\left(2e\sqrt{1-\frac{1}{c^2x^2}}+cd\csc^{-1}(cx)+cex\csc^{-1}(cx)\right)}{e}+\frac{4d\sqrt{-c^2\left(1-\frac{1}{c^2x^2}\right)x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{(cd+e)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{cd+cex}{cd+e}}}-\frac{4(-cd+}{}\right.}{}$$

input
```
Integrate[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

output
```
(2*a*(d + e*x)^(3/2))/(3*e) - (b*(c*d + c*e*x)*((-2*(2*e*Sqrt[1 - 1/(c^2*x
^2)] + c*d*ArcCsc[c*x] + c*e*x*ArcCsc[c*x]))/e + (4*d*Sqrt[-(c^2*(1 - 1/(c
^2*x^2))*x^2)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/
((c*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d + c*e*x)/(c*d + e)]) - (4*(-(
c*d) + e)*Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*EllipticPi[2, ArcSin[Sqrt[1 -
 c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*d +
 c*e*x)/(c*d + e)]) + ((c^2*(1 - 1/(c^2*x^2))*x^2*(c*d + c*e*x) + c^2*d*x*
Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*Sqrt[(c*d + c*e*x)/(c*d + e)]*EllipticF
[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e
- c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[Arc
Sin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[Arc
Sin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*
x))/(-(c*d) + e)] + c*e*x*Sqrt[-(c^2*(1 - 1/(c^2*x^2))*x^2)]*Sqrt[(c*d + c
*e*x)/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
 e)])*Sec[2*ArcCsc[c*x]]*Sin[4*ArcCsc[c*x]])/(c^2*(1 - 1/(c^2*x^2))*(e + d
/x)*x^2)))/(3*c^2*Sqrt[d + e*x])
```

3.53.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5750, 1898, 634, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \; 5750$$

$$\frac{2b\int \frac{(d+ex)^{3/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{3ce}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 1898$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{(d+ex)^{3/2}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 634$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\int \frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 600$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+e\int \frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 509$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{e\sqrt{1-c^2x^2}\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 508$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$\downarrow$ 327

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+de\int \frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$\downarrow$ 512

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{de\sqrt{1-c^2x^2}\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$\downarrow$ 511

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$\downarrow$ 321

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

3.53. $\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\Big\downarrow \ 633$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^2\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\text{arcsin}\ldots\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\ldots}}\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \ 632$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^2\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\ldots\right.}{c\sqrt{x^2-\frac{1}{c^2}}\ldots}\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \ 186$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\int\frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2e\sqrt{1-c^2x\ldots}}{\ldots}\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \ 413$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\ldots\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \ 412$$

$$\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2de\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\ldots\right.}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

3.53. $\int\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx$

input
```
Int[Sqrt[d + e*x]*(a + b*ArcCsc[c*x]),x]
```

output
```
(2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e) + (2*b*Sqrt[-c^(-2) + x^2]*(
(-2*e*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[
2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2) + x^2
]) - (2*d*e*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcS
in[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2)
 + x^2]) - (2*d^2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*Elli
pticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[-c^(-2) +
 x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.53.3.1 Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

3.53. $\int \sqrt{d+ex}(a+b\csc^{-1}(cx))\,dx$

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.53.4 Maple [A] (verified)

Time = 8.52 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\frac{2\left(2d\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\operatorname{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\operatorname{Ellip}\ldots}{\ldots}\right.}{\ldots}$
default	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\frac{2\left(2d\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\operatorname{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\operatorname{Ellip}\ldots}{\ldots}\right.}{\ldots}$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\frac{2\left(2d\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c-\operatorname{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)cd-d\operatorname{E}\ldots}{\ldots}\right.}{\ldots}$

input
```
int((a+b*arccsc(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arccsc(c*x)+2/3/c^2*(2*d*Ell
ipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c-Elliptic
E((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-d*EllipticP
i((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-
e))^(1/2))*c+EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(
1/2))*e-EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))
*e)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/
(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x
^2)^(1/2)))
```

3.53.5 Fricas [F]

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a), x)
```

3.53.6 Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx=\int (a+b\,\mathrm{acsc}\,(cx))\sqrt{d+ex}\,dx$$

input
```
integrate((a+b*acsc(c*x))*(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))*sqrt(d + e*x), x)
```

3.53.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.53.8 Giac [F]

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \sqrt{ex+d}(b\operatorname{arccsc}(cx)+a)\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int((a + b*asin(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int((a + b*asin(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.54 $\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \text{Int}\left(\frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

3.54.2 Mathematica [N/A]

Not integrable

Time = 38.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx$$

input
```
Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x, x]
```

3.54.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}dx$$

input
```
Int[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
$Aborted
```

3.54.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.54.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{ex+d}}{x}dx$$

input
```
int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

output
```
int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x)
```

3.54.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x, x)
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))*(e*x+d)**(1/2)/x,x)
```

output
```
Timed out
```

3.54.7 Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.48

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

output
```
a*sqrt(d)*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)) + b*integrate(sqr
t(e*x + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + 2*sqrt(e*x + d)
*a
```

3.54.8 Giac [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x, x)
```

3.54.9 Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x}\,dx$$

input
```
int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x,x)
```

output
```
int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x, x)
```

3.55 $\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2} dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2} dx = \text{Int}\left(\frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

3.55.2 Mathematica [N/A]

Not integrable

Time = 5.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2} dx = \int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2} dx$$

input
```
Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2, x]
```

3.55.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2} dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2} dx$$

input
```
Int[(Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.55.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.55.4 Maple [N/A] (verified)

Not integrable

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\sqrt{ex+d}}{x^2} dx$$

input
```
int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

output
```
int((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x)
```

3.55.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x^2, x)
```

3.55.6 Sympy [N/A]

Not integrable

Time = 22.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\sqrt{d+ex}}{x^2}\,dx$$

input
```
integrate((a+b*acsc(c*x))*(e*x+d)**(1/2)/x**2,x)
```

output
```
Integral((a + b*acsc(c*x))*sqrt(d + e*x)/x**2, x)
```

3.55.7 Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 88, normalized size of antiderivative = 4.19

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*(a*e*x*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)) + 2*b*sqrt(d)*x*
integrate(sqrt(e*x + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x) -
2*sqrt(e*x + d)*a*sqrt(d))/(sqrt(d)*x)
```

3.55.8 Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsc(c*x) + a)/x^2, x)
```

3.55.9 Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x^2}\,dx$$

input
```
int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x^2,x)
```

output
```
int(((a + b*asin(1/(c*x)))*(d + e*x)^(1/2))/x^2, x)
```

3.56 $\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) dx$

3.56.1 Optimal result

Integrand size = 18, antiderivative size = 372

$$\begin{aligned}\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) dx &= -\frac{4be\sqrt{d+ex}(1-c^2x^2)}{15c^3\sqrt{1-\frac{1}{c^2x^2}}x} \\ &+ \frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e} - \frac{28bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{15c^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &- \frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ &- \frac{4bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

output
```
2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*x))/e-4/15*b*e*(-c^2*x^2+1)*(e*x+d)^(1/2)/
c^3/x/(1-1/c^2/x^2)^(1/2)-28/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2
^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/x/(1-1/c^2/
x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*
(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2
)*(-c^2*x^2+1)^(1/2)/c^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*d^3*Ell
ipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)
/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.56.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.33 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.90

$$\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{15}\left(\frac{4be\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}\right.$$
$$+\frac{6a(d+ex)^{5/2}}{e}+\frac{6b(d+ex)^{5/2}\csc^{-1}(cx)}{e}$$
$$-\frac{4ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(-7cd(cd-e)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)+(9c^2d^2-7cde+e^2)\,\text{EllipticF}\right.}{c^3e\sqrt{-\frac{c}{cd+e}}\sqrt{}}$$

input
```
Integrate[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
((4*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (6*a*(d + e*x)^(5/2))/e
 + (6*b*(d + e*x)^(5/2)*ArcCsc[c*x])/e - ((4*I)*b*Sqrt[(e*(1 + c*x))/(-(c*
d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(-7*c*d*(c*d - e)*EllipticE[I*ArcSinh
[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + (9*c^2*d^2 -
7*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c
*d + e)/(c*d - e)] - 3*c^2*d^2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/
(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)]))/(c^3*e*Sqrt[-(c/(c*d +
e))]*Sqrt[1 - 1/(c^2*x^2)]*x))/15
```

3.56.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.06, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.944, Rules used = {5750, 1898, 634, 633, 632, 186, 413, 412, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \; 5750$$

$$\frac{2b\int \frac{(d+ex)^{5/2}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{5ce}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{1898}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{(d+ex)^{5/2}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{5cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{634}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(d^3\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{633}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^3\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{632}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{d^3\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{186}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}}-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{413}$$

3.56. $\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 412$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\int\frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 2185$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\int-\frac{e^3\left(9d^2c^2+7dexc^2+e^2\right)}{2c^2\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{3e^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2}{3}e^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\int\frac{9d^2c^2+7dexc^2+e^2}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2}{3}e^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}+$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 600$$

3.56. $\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+7c^2d\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}\cdots$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$\big\downarrow$ 509

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{7c^2d\sqrt{1-c^2x^2}\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\cdots\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}\cdots$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$\big\downarrow$ 508

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\cdots\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\cdots}}\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}\cdots$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$\big\downarrow$ 327

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\left(2c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(\cdots\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}\cdots}}\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}\cdots$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

3.56. $\int(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\Big\downarrow \text{512}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(\frac{\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{Ell}}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-}}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{511}$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{14cd\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{2d}{}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{321}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2d^3\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\left(2c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-c}}{\sqrt{2}}\right.\right.}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\right.}{3}\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

3.56. $\int (d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

output
```
(2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) + (2*b*Sqrt[-c^(-2) + x^2]*(
(2*e^2*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2])/3 + (e*((-14*c*d*Sqrt[d + e*x]*S
qrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]
)/(Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2) + x^2]) - (2*(2*c^2*d^2 + e^
2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1
 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]))
)/(3*c^2) - (2*d^3*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*Ell
ipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[-c^(-2)
+ x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(5*c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.56.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

3.56. $\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.56. $\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right) dx$

3.56.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(335) = 670$.

Time = 9.27 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}+\frac{6d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-}}\right.}{5}\right.}{}$
default	$\frac{\frac{2a(ex+d)^{\frac{5}{2}}}{5}+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}+\frac{6d^2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-}}\right.}{5}\right.}{}$
parts	$\frac{2a(ex+d)^{\frac{5}{2}}}{5e}+\frac{2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\sqrt{\frac{c}{cd-e}}\,c^2(ex+d)^{\frac{5}{2}}}{15}-\frac{4\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^{\frac{3}{2}}}{15}+\frac{6d^2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-c}{cd+e}}}{}\right.}{}$

input
```
int((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*arccsc(c*x)*(e*x+d)^(5/2)+2/15/c^3*((c/(c*
d-e))^(1/2)*c^2*(e*x+d)^(5/2)+9*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-
c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),
((c*d-e)/(c*d+e))^(1/2))*c^2-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*
x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d
-e)/(c*d+e))^(1/2))*c^2*d^2-3*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*
(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1
/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2-2*(c/(c*d-e))^(1/2)*
c^2*d*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+
e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c*d*e-7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/
(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))
^(1/2))*c*d*e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*(e*x+
d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)
/c^2/e^2/x^2)^(1/2)))
```

3.56.5 Fricas [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
Timed out
```

3.56.6 Sympy [F]

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx = \int \left(a+b\operatorname{acsc}(cx)\right)(d+ex)^{\frac{3}{2}}\, dx$$

input
```
integrate((e*x+d)**(3/2)*(a+b*acsc(c*x)),x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x)**(3/2), x)
```

3.56.7 Maxima [F(-2)]

Exception generated.

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.56.8 Giac [F]

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx=\int (ex+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)^{3/2}\,dx$$

input
```
int((a + b*asin(1/(c*x)))*(d + e*x)^(3/2),x)
```

output
```
int((a + b*asin(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.57 $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

3.57.1 Optimal result

Integrand size = 21, antiderivative size = 714

$$\begin{aligned}
&\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx \\
&= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{35c^3e\sqrt{1-\frac{1}{c^2x^2}}} + \frac{4bd\sqrt{d+ex}(1-c^2x^2)}{21c^3e^2\sqrt{1-\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} \\
&+ \frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4} - \frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4} \\
&+ \frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4} - \frac{24bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&+ \frac{4b(2c^2d^2-9e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{105c^4e^3\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} \\
&+ \frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35c^2e^3\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&- \frac{32bd(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{105c^4e^3\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&+ \frac{64bd^4\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{35ce^4\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}
\end{aligned}$$

output
```
2*d^2*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^4-6/5*d*(e*x+d)^(5/2)*(a+b*arccsc(
c*x))/e^4+2/7*(e*x+d)^(7/2)*(a+b*arccsc(c*x))/e^4-2*d^3*(a+b*arccsc(c*x))*
(e*x+d)^(1/2)/e^4-4/35*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e/(1-1/c^2/x^2)^(1
/2)+4/21*b*d*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1-1/c^2/x^2)^(1/2)-24/3
5*b*d^2*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e
*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c
*d+e))^(1/2)+4/105*b*(2*c^2*d^2-9*e^2)*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2
),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1
-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+64/35*b*d^3*EllipticF(1/2*(-c*
x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-
c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/105*b*d*(c
*d-e)*(c*d+e)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/
2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x^2)^(
1/2)/(e*x+d)^(1/2)+64/35*b*d^4*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(
1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4
/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.57.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 33.90 (sec) , antiderivative size = 873, normalized size of antiderivative = 1.22

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \frac{ad^4\sqrt{1+\frac{ex}{d}}B_{-\frac{ex}{d}}\left(4,\frac{1}{2}\right)}{e^4\sqrt{d+ex}}$$

$$+\frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(-\frac{4\left(16c^2d^2+9e^2\right)\sqrt{1-\frac{1}{c^2x^2}}}{105e^3}+\frac{32c^3d^3\csc^{-1}(cx)}{35e^4}-\frac{2c^3x^3\csc^{-1}(cx)}{7e}-\frac{4c^2x^2\left(e\sqrt{1-\frac{1}{c^2x^2}}-3cd\csc^{-1}(cx)\right)}{35e^2}+\frac{4cx\left(5cde\sqrt{1-\frac{1}{c^2x^2}}-12c^2\cdots\right)}{105e^3}\cdots\right)}{\sqrt{d+ex}}\cdots\right)}{\cdots}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

output
```
(a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (
b*(-((c*(e + d/x)*x*((-4*(16*c^2*d^2 + 9*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(105*
e^3) + (32*c^3*d^3*ArcCsc[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsc[c*x])/(7*e) -
 (4*c^2*x^2*(e*Sqrt[1 - 1/(c^2*x^2)] - 3*c*d*ArcCsc[c*x]))/(35*e^2) + (4*c
*x*(5*c*d*e*Sqrt[1 - 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsc[c*x]))/(105*e^3)))/S
qrt[d + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*((2*(40*c^3*d^3*e + 8*c*d*e^3)*
Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x
)^(3/2)) + (2*(48*c^4*d^4 + 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[(c*d + c*e*x)/(c*
d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*
e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(-16
*c^3*d^3*e - 9*c*d*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) +
 c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[
Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)
/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt
[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt
[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c
*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Elliptic
Pi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(c*d*Sqrt[1 - 1/(c
^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(105*e^4*Sqrt[d + e*...
```

3.57.3 Rubi [A] (verified)

Time = 2.35 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.68, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5770, 27, 7272, 2351, 637, 2009, 2185, 27, 687, 27, 600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b \csc^{-1}(cx)\right)}{\sqrt{d+ex}} \, dx$$

$$\Big\downarrow \; 5770$$

$$\frac{b \int -\frac{2\sqrt{d+ex}\left(16d^3-8exd^2+6e^2x^2d-5e^3x^3\right)}{35e^4\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c} - \frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4} - \frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\int \frac{\sqrt{d+ex}\left(16d^3-8exd^2+6e^2x^2d-5e^3x^3\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{35ce^4}-\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 7272

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{\sqrt{d+ex}\left(16d^3-8exd^2+6e^2x^2d-5e^3x^3\right)}{x\sqrt{1-c^2x^2}}dx}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 2351

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx+\int \frac{\sqrt{d+ex}\left(-5x^2e^3+6dxe^2-8d^2e\right)}{\sqrt{1-c^2x^2}}dx\right)}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 637

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx+\int \frac{\sqrt{d+ex}\left(-5x^2e^3+6dxe^2-8d^2e\right)}{\sqrt{1-c^2x^2}}dx\right)}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 2009

$$-\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{\sqrt{d+ex}\left(-5x^2e^3+6dxe^2-8d^2e\right)}{\sqrt{1-c^2x^2}}dx+16d^3\left(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2d\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\ldots}{\ldots}\right.\right.}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 2185

3.57. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(-\frac{2\int\frac{5e^3\sqrt{d+ex}\left(8d^2c^2-8dexc^2+3e^2\right)}{2\sqrt{1-c^2x^2}}dx}{5c^2e^2}+16d^3\left(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2d\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{Ellipti}\ldots}{}\right.\right.}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+$$
$$\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\int\frac{\sqrt{d+ex}\left(8d^2c^2-8dexc^2+3e^2\right)}{\sqrt{1-c^2x^2}}dx}{c^2}+16d^3\left(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2d\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\ldots}{\sqrt{\ldots}}\right.\right.}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+$$
$$\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{687}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(\frac{16}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}-\frac{2\int-\frac{c^2\left(d\left(24c^2d^2+e^2\right)+e\left(16c^2d^2+9e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2}\right)}{c^2}+16d^3\left(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\ldots}}{\sqrt{\ldots}}\right.\right.}{c\sqrt{d+ex}}\right.\right.}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+$$
$$\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{27}$$

3.57. $\int\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\Bigg(-\frac{e\left(\frac{1}{3}\int \frac{d(24c^2d^2+e^2)+e(16c^2d^2+9e^2)x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+\frac{16}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+16d^3\Bigg(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\ \text{EllipticF}\Big(\arcsin\Big(\frac{\sqrt{1-cx}}{\sqrt{2}}\Big),\ldots}{c\sqrt{d+ex}}\ldots}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}\ldots}+\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4}+\frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4}-\frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4}$$

$$\Big\downarrow \text{600}$$

$$-\frac{2b\sqrt{1-c^2x^2}\Bigg(-\frac{e\left(\frac{1}{3}\left(8d(c^2d^2-e^2)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+(16c^2d^2+9e^2)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)+\frac{16}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+16d^3\Bigg(-\frac{2e\sqrt{\frac{c(d+\ldots}{cd\ldots}}\ldots}{\ldots}}{35ce^4x\sqrt{1-\frac{1}{c^2x^2}}\ldots}+\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4}+\frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4}-\frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4}$$

$$\Big\downarrow \text{508}$$

$$-\frac{2b\sqrt{1-c^2x^2}\Bigg(-\frac{e\left(\frac{1}{3}\left(8d(c^2d^2-e^2)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2(16c^2d^2+9e^2)\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)+\frac{16}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+16\ldots}{35ce^4x\sqrt{\ldots}}+\frac{2d^3\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}+\frac{2d^2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{e^4}+\frac{2(d+ex)^{7/2}(a+b\csc^{-1}(cx))}{7e^4}-\frac{6d(d+ex)^{5/2}(a+b\csc^{-1}(cx))}{5e^4}$$

$$\Big\downarrow \text{327}$$

3.57. $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(\frac{1}{3}\left(8d(c^2d^2-e^2)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2\left(16c^2d^2+9e^2\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)+\frac{16}{3}de\sqrt{1-c^2x^2}\sqrt{d+ex}\right)}{c^2}+1\right.}{35ce^4x\sqrt{}}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 511

$$-\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(\frac{1}{3}\left(-\frac{16d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{2\left(16c^2d^2+9e^2\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)+\frac{1}{3}\right.}{c^2}\right.}{}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 321

$$-\frac{2d^3\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}-\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(\frac{1}{3}\left(-\frac{16d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2\left(16c^2d^2+9e^2\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)+\frac{16}{3}de\sqrt{1}\right.}{c^2}\right.}{3}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

3.57. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

output
```
(-2*d^3*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a
 + b*ArcCsc[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^4)
 + (2*(d + e*x)^(7/2)*(a + b*ArcCsc[c*x]))/(7*e^4) - (2*b*Sqrt[1 - c^2*x^2
]*((2*e^2*(d + e*x)^(3/2)*Sqrt[1 - c^2*x^2])/c^2 - (e*((16*d*e*Sqrt[d + e*
x]*Sqrt[1 - c^2*x^2])/3 + ((-2*(16*c^2*d^2 + 9*e^2)*Sqrt[d + e*x]*Elliptic
E[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(
c*d + e)]) - (16*d*(c^2*d^2 - e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF
[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]))/3))/c
^2 + 16*d^3*((-2*e*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 -
 c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]) - (2*d*Sqrt[(c*(d + e*
x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e
)])/Sqrt[d + e*x])))/(35*c*e^4*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

3.57. $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.57.4 Maple [A] (verified)

Time = 10.84 (sec) , antiderivative size = 1233, normalized size of antiderivative = 1.73

method	result	size
derivativedivides	Expression too large to display	1233
default	Expression too large to display	1233
parts	Expression too large to display	1251

input
```
int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^4*(-a*(-1/7*(e*x+d)^(7/2)+3/5*d*(e*x+d)^(5/2)-d^2*(e*x+d)^(3/2)+d^3*(e
*x+d)^(1/2))-b*(-1/7*arccsc(c*x)*(e*x+d)^(7/2)+3/5*arccsc(c*x)*d*(e*x+d)^(
5/2)-arccsc(c*x)*d^2*(e*x+d)^(3/2)+arccsc(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^4
*(-3*(c/(c*d-e))^(1/2)*c^3*(e*x+d)^(7/2)+14*(c/(c*d-e))^(1/2)*c^3*d*(e*x+d
)^(5/2)-19*(c/(c*d-e))^(1/2)*c^3*d^2*(e*x+d)^(3/2)+24*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3+16*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*
(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3-48*d^3*((-c*(e*x+d)+c*d
-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(
1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*
c^3+8*(c/(c*d-e))^(1/2)*c^3*d^3*(e*x+d)^(1/2)-16*((-c*(e*x+d)+c*d-e)/(c*d-
e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c
*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e+16*((-c*(e*x+d)+c*d-e)/(c*
d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/
(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e+3*(c/(c*d-e))^(1/2)*c*e^
2*(e*x+d)^(3/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*
d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1
/2))*c*d*e^2+9*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d
+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^...
```

3.57.5 Fricas [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^3*arccsc(c*x) + a*x^3)/sqrt(e*x + d), x)
```

3.57.6 Sympy [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x^3(a+b\operatorname{acsc}(cx))}{\sqrt{d+ex}}\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))/sqrt(d + e*x), x)
```

3.57.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.57.8 Giac [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^3}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x + d), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.58 $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

3.58.1 Optimal result

Integrand size = 21, antiderivative size = 530

$$\begin{aligned}
\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx &= -\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{2d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}\\
&\quad+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+\frac{4bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{5c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}}\\
&\quad-\frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\
&\quad+\frac{4b(cd-e)(cd+e)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\
&\quad-\frac{32bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

output
```
-4/3*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c
*x))/e^3+2*d^2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/15*b*(-c^2*x^2+1)*(e*
x+d)^(1/2)/c^3/e/x/(1-1/c^2/x^2)^(1/2)+4/5*b*d*EllipticE(1/2*(-c*x+1)^(1/2
)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/
e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-32/15*b*d^2*EllipticF(
1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^
(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*
b*(c*d-e)*(c*d+e)*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))
^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^4/e^2/x/(1-1/c^2/x^
2)^(1/2)/(e*x+d)^(1/2)-32/15*b*d^3*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2
,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c
/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.58.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.01 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.48

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = -\frac{ad^3\sqrt{1+\frac{ex}{d}}B_{-\frac{ex}{d}}\left(3,\frac{1}{2}\right)}{e^3\sqrt{d+ex}}$$

$$+ \frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4cd\sqrt{1-\frac{1}{c^2x^2}}}{5e^2}-\frac{16c^2d^2\csc^{-1}(cx)}{15e^3}-\frac{2c^2x^2\csc^{-1}(cx)}{5e}-\frac{4cx\left(e\sqrt{1-\frac{1}{c^2x^2}}-2cd\csc^{-1}(cx)\right)}{15e^2}\right)}{\sqrt{d+ex}} - \frac{2\sqrt{e+\frac{d}{x}}\sqrt{cx}\left(\frac{2\left(7c^2d^2e+e^3\right)\sqrt{\frac{cd+\cdots}{cd\cdots}}\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

output
```
-((a*d^3*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 3, 1/2])/(e^3*Sqrt[d + e*x]))
+ (b*(-((c*(e + d/x)*x*((4*c*d*Sqrt[1 - 1/(c^2*x^2)])/(5*e^2) - (16*c^2*d^
2*ArcCsc[c*x])/(15*e^3) - (2*c^2*x^2*ArcCsc[c*x])/(5*e) - (4*c*x*(e*Sqrt[1
 - 1/(c^2*x^2)] - 2*c*d*ArcCsc[c*x]))/(15*e^2)))/Sqrt[d + e*x]) - (2*Sqrt[
e + d/x]*Sqrt[c*x]*((2*(7*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*S
qrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]
)/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*c^3*d^3 + 3*c*
d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSi
n[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e
+ d/x]*(c*x)^(3/2)) - (6*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2
*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[
ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e -
 c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcS
in[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcS
in[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x
))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*E
llipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1
/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(15*e^3*Sqrt[d + e*x
])))/c^3
```

3.58.3 Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.78, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5770, 27, 7272, 2351, 637, 687, 27, 600, 508, 327, 511, 321, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b \csc^{-1}(cx)\right)}{\sqrt{d+ex}} \, dx$$

$$\Big\downarrow \text{5770}$$

$$\frac{b \int \frac{2\sqrt{d+ex}\left(8d^2-4exd+3e^2x^2\right)}{15e^3\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{\sqrt{d+ex}\left(8d^2-4exd+3e^2x^2\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{15ce^3}+\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 7272$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{\sqrt{d+ex}\left(8d^2-4exd+3e^2x^2\right)}{x\sqrt{1-c^2x^2}}dx}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 2351$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx+\int \frac{\sqrt{d+ex}\left(3e^2x-4de\right)}{\sqrt{1-c^2x^2}}dx\right)}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 637$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx+\int \frac{\sqrt{d+ex}\left(3e^2x-4de\right)}{\sqrt{1-c^2x^2}}dx\right)}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 687$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{2\int \frac{3e\left(4d^2c^2+3dexc^2-e^2\right)}{2\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2}+8d^2\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx-\frac{2e^2\sqrt{1-c^2x^2}\sqrt{d+ex}}{c^2}\right)}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \ 27$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\int \frac{4d^2c^2+3dexc^2-e^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{c^2}+8d^2\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx-\frac{2e^2\sqrt{1-c^2x^2}\sqrt{d+ex}}{c^2}\right)}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

3.58. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}dx$

$$\Big\downarrow \text{600}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(\left(c^2d^2-e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+3c^2d\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{c^2}+8d^2\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx-\frac{2e^2\sqrt{}}{}\right.}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{508}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(\left(c^2d^2-e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{6cd\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2}+8d^2\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right.\right.}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{327}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(\left(c^2d^2-e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{6cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2}+8d^2\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2}}\right.\right.}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{511}$$

3.58. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(-\frac{2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{6cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2}+8d^2\int\left(\frac{d}{x\sqrt{d+ex}\ldots}\right.\right.}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 321

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int\left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx-\frac{e\left(-\frac{2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{6cd\sqrt{d+\ldots}}{\ldots}\right.}{c^2}\right.}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 2009

$$\frac{2d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{e\left(-\frac{2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{6cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2}+8d^2\left(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\ldots}{\ldots}\right.\right.}{15ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

3.58. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

output
```
(2*d^2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a +
b*ArcCsc[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3)
+ (2*b*Sqrt[1 - c^2*x^2]*((-2*e^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2])/c^2 - (
e*((-6*c*d*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c
*d + e)])/Sqrt[(c*(d + e*x))/(c*d + e)] - (2*(c^2*d^2 - e^2)*Sqrt[(c*(d +
e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]
)/(c*Sqrt[d + e*x])))/c^2 + 8*d^2*((-2*e*Sqrt[(c*(d + e*x))/(c*d + e)]*Ell
ipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x])
- (2*d*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sq
rt[2]], (2*e)/(c*d + e)])/Sqrt[d + e*x])))/(15*c*e^3*Sqrt[1 - 1/(c^2*x^2)]
*x)
```

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.58. $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

3.58.4 Maple [A] (verified)

Time = 9.16 (sec) , antiderivative size = 850, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)d^2\sqrt{ex+d}+\frac{2\sqrt{\frac{c}{cd-e}}}{}\right.}{}$
default	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)+2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)d^2\sqrt{ex+d}+\frac{2\sqrt{\frac{c}{cd-e}}}{}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-\frac{2(ex+d)^{\frac{3}{2}}d}{3}+d^2\sqrt{ex+d}\right)}{e^3}+\frac{2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\frac{2\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d}{3}+\operatorname{arccsc}(cx)d^2\sqrt{ex+d}+\frac{2\sqrt{\frac{c}{cd-}}}{}\right.}{}$

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^3*(a*(1/5*(e*x+d)^(5/2)-2/3*(e*x+d)^(3/2)*d+d^2*(e*x+d)^(1/2))+b*(1/5*
arccsc(c*x)*(e*x+d)^(5/2)-2/3*arccsc(c*x)*(e*x+d)^(3/2)*d+arccsc(c*x)*d^2*
(e*x+d)^(1/2)+2/15/c^3*((c/(c*d-e))^(1/2)*c^2*(e*x+d)^(5/2)+4*d^2*((-c*(e*
x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e
*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2+3*((-c*(e*x+d)+
c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)
^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2-8*d^2*((-c*(e*x+
d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(
1/2))*c^2-2*(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c
*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c
/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+3*((-c*(e*x+d)+c*d-e)/(c*d-
e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c
*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x
+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^
(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e
^2-(c/(c*d-e))^(1/2)*e^2*(e*x+d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^
2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

3.58.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex}}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.58.6 Sympy [F]

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex}}\,dx = \int \frac{x^2(a + b\,\mathrm{acsc}\,(cx))}{\sqrt{d + ex}}\,dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))/sqrt(d + e*x), x)
```

3.58.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.58.8 Giac [F]

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex}}\, dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^2}{\sqrt{ex + d}}\, dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x + d), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{\sqrt{d + ex}}\, dx = \int \frac{x^2\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d + e\,x}}\, dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.59 $\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

3.59.1 Optimal result

Integrand size = 19, antiderivative size = 344

$$\begin{aligned}\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = & -\frac{2d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2}+\frac{2(d+ex)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^2} \\ & -\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\ & +\frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ & +\frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

output
```
2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^2-2*d*(a+b*arccsc(c*x))*(e*x+d)^(1/2
)/e^2-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2)
)*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/
(c*d+e))^(1/2)+8/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*
d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2
/x^2)^(1/2)/(e*x+d)^(1/2)+8/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),
2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/
c/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.11

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(a(-2d+ex)(d+ex)+b(-2d+ex)(d+ex)\csc^{-1}(cx)-\frac{2bde\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{1-c^2x^2}}\right.}{}$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

output
```
(2*(a*(-2*d + e*x)*(d + e*x) + b*(-2*d + e*x)*(d + e*x)*ArcCsc[c*x] - (2*b
*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSi
n[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[1 - c^2*x^2] + (2*b*e*Sqr
t[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d - e)]*Sqrt[(e - c*e*x)/(c*d +
 e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)
/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)
/(c*d + e)]))/(c*(-1 + c*x)*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]) - (4*b*c*d^2
*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSi
n[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[1 - c^2*x^2]))/(3*e^2*Sqr
t[d + e*x])
```

3.59.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.80, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {5770, 27, 7272, 2351, 25, 27, 508, 327, 637, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \text{ 5770}$$
$$\frac{b\int -\frac{2(2d-ex)\sqrt{d+ex}}{3e^2\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$
$$\Big\downarrow \text{ 27}$$

$$-\frac{2b\int \frac{(2d-ex)\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{3ce^2}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 7272}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{(2d-ex)\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 2351}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\int -\frac{e\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx+2d\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx-\int \frac{e\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx-e\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 508}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx+\frac{2e\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 327}$$

3.59. $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex}}dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}}dx+\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow 637$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}+\frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx+\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow 2009$$

$$\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}+2d\left(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2d\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticPi}\left(2,\text{ar}\right.}{\sqrt{d+ex}}\right.\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x],x]
```

output
```
(-2*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*A
rcCsc[c*x]))/(3*e^2) - (2*b*Sqrt[1 - c^2*x^2]*((2*e*Sqrt[d + e*x]*Elliptic
E[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(
c*d + e)]) + 2*d*((-2*e*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]) - (2*d*Sqrt[(c*(d
 + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*
d + e)])/Sqrt[d + e*x])))/(3*c*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.59. $\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx$

3.59.4 Maple [A] (verified)

Time = 8.05 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\operatorname{arccsc}(cx)d\sqrt{ex+d}+\frac{2\left(d\,\text{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c+\text{El}\right.}{}\right.}{}$
default	$\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}+\operatorname{arccsc}(cx)d\sqrt{ex+d}+\frac{2\left(d\,\text{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c+\text{El}\right.}{}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-d\sqrt{ex+d}\right)}{e^2}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{2\left(d\,\text{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c+\text{Ellip}\right.}{}\right.}{}$

input
```
int(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^2*(-a*(-1/3*(e*x+d)^(3/2)+d*(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccs
c(c*x)+arccsc(c*x)*d*(e*x+d)^(1/2)+2/3/c^2*(d*EllipticF((e*x+d)^(1/2)*(c/(
c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c+EllipticE((e*x+d)^(1/2)*(c/(c*d-e
))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d-2*d*EllipticPi((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c-EllipticF(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e+EllipticE((e*x+
d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e)*((-c*(e*x+d)+c*d+e)
/(c*d+e))^(1/2)*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c
^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

3.59.5 Fricas [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x*arccsc(c*x) + a*x)/sqrt(e*x + d), x)
```

3.59.6 Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x(a+b\operatorname{acsc}(cx))}{\sqrt{d+ex}}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acsc(c*x))/sqrt(d + e*x), x)
```

3.59.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.59.8 Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x/sqrt(e*x + d), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2),x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.60 $\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx$

3.60.1 Optimal result

Integrand size = 18, antiderivative size = 212

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

output
```
2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/
2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)
/c^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(-c*x+1)^(1/
2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^
2+1)^(1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.60.2 Mathematica [A] (warning: unable to verify)

Time = 5.23 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\left(\frac{a(d+ex)}{e}+\frac{b\left((d+ex)\csc^{-1}(cx)+\frac{2cd\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{1-c^2x^2}}\right)}{e}+\frac{2bcx^2\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\text{arcsi}\right.}{}\right.}{\sqrt{d+ex}}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*((a*(d + e*x))/e + (b*((d + e*x)*ArcCsc[c*x] + (2*c*d*Sqrt[1 - 1/(c^2*x
^2)]*x*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sq
rt[2]], (2*e)/(c*d + e)])/Sqrt[1 - c^2*x^2]))/e + (2*b*c*x^2*Sqrt[1 + c*x]
*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)]*(Cos[ArcCsc[c*x]/2] - Sin[ArcCsc[c*x]/2])^3*(Cos[ArcCsc[c*x
]/2] + Sin[ArcCsc[c*x]/2]))/(Sqrt[1 - c*x]*(-1 + c^2*x^2))))/Sqrt[d + e*x]
```

3.60.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5750, 1898, 637, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \; 5750$$
$$\frac{2b\int \frac{\sqrt{d+ex}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e}$$
$$\Big\downarrow \; 1898$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{\sqrt{d+ex}}{x\sqrt{x^2-\frac{1}{c^2}}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e}$$

$$\Big\downarrow \ 637$$

$$\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \left(\frac{d}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}+\frac{e}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}\right)dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e}+\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2e\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2d\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e + (2*b*Sqrt[-c^(-2) + x^2]*((-2*e*
Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]) - (
2*d*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[S
qrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2
])))/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.60.3.1 Defintions of rubi rules used

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.60.4 Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{2a\sqrt{ex+d}+2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\left(\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}}{e}$
default	$\frac{2a\sqrt{ex+d}+2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{EllipticPi}\left(\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}}{e}$
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)-\operatorname{Elliptic}\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,x\sqrt{\frac{c}{cd-e}}}}{e}$

input
```
int((a+b*arccsc(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(a*(e*x+d)^(1/2)+b*((e*x+d)^(1/2)*arccsc(c*x)+2/c*((-c*(e*x+d)+c*d-e)/
(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)
*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c
*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*
x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2)
))
```

3.60.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+d}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/sqrt(e*x + d), x)
```

3.60.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{\sqrt{d+ex}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/sqrt(d + e*x), x)
```

3.60.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.60.8 Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{\sqrt{ex+d}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/sqrt(e*x + d), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex}}\,dx=\int \frac{a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)}{\sqrt{d+e\,x}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x)^(1/2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.61 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

3.61.2 Mathematica [N/A]

Not integrable

Time = 3.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]), x]
```

3.61.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x]),x]
```

output
```
$Aborted
```

3.61.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.61.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\sqrt{ex+d}}dx$$

input
```
int((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x)
```

3.61.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{\sqrt{ex+d}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e*x^2 + d*x), x)
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.61.7 Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.19

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{\sqrt{ex+d}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
(b*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(e*x + d
)*x), x) + a*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))/sqrt(d)
```

3.61.8 Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+d}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(sqrt(e*x + d)*x), x)
```

3.61.9 Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{d+e\,x}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(1/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(1/2)), x)
```

3.62 $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

3.62.2 Mathematica [N/A]

Not integrable

Time = 6.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]), x]
```

3.62.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \text{5772}$$
$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x]),x]
```

output
```
$Aborted
```

3.62.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.62.4 Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x^2\sqrt{ex+d}}dx$$

input
```
int((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x)
```

3.62.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+dx^2}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e*x^3 + d*x^2), x)
```

3.62.6 Sympy [N/A]

Not integrable

Time = 19.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^2\sqrt{d+ex}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x)), x)
```

3.62.7 Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.24

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+dx^2}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*b*d^(3/2)*x*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt
(e*x + d)*x^2), x) - a*e*x*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d))
- 2*sqrt(e*x + d)*a*sqrt(d))/(d^(3/2)*x)
```

3.62.8 Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex+d}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(sqrt(e*x + d)*x^2), x)
```

3.62.9 Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{d+e\,x}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(1/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(1/2)), x)
```

3.63 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 551

$$\begin{aligned}\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = &-\frac{4b\sqrt{d+ex}(1-c^2x^2)}{15c^3e^2\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{2d^3(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} \\ &+\frac{6d^2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4} \\ &+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+\frac{32bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &-\frac{8bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ &-\frac{4b(2c^2d^2+e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15c^4e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ &-\frac{64bd^3\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

output
```
-2*d*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^4+2/5*(e*x+d)^(5/2)*(a+b*arccsc(c*x
))/e^4+2*d^3*(a+b*arccsc(c*x))/e^4/(e*x+d)^(1/2)+6*d^2*(a+b*arccsc(c*x))*(
e*x+d)^(1/2)/e^4-4/15*b*(-c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1-1/c^2/x^2)
^(1/2)+32/15*b*d*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^
(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/c^2/x^2)^(1/2)/(c*(
e*x+d)/(c*d+e))^(1/2)-8*b*d^2*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)
*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x
/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*(2*c^2*d^2+e^2)*EllipticF(1/2*(-
c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*
(-c^2*x^2+1)^(1/2)/c^4/e^3/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-64/5*b*d^3*
EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x
+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^
(1/2)
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.38 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.48

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{ad^4\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(4,-\frac{1}{2}\right)}{e^4(d+ex)^{3/2}}$$

$$+\frac{b\left(-\frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(\frac{32cd\sqrt{1-\frac{1}{c^2x^2}}}{15e^3}-\frac{32c^2d^2\csc^{-1}(cx)}{5e^4}+\frac{2c^2d^2\csc^{-1}(cx)}{e^3\left(e+\frac{d}{x}\right)}-\frac{2c^2x^2\csc^{-1}(cx)}{5e^2}-\frac{2cx\left(2e\sqrt{1-\frac{1}{c^2x^2}}-9cd\csc^{-1}(cx)\right)}{15e^3}\right)}{(d+ex)^{3/2}}-\frac{2\left(e+\frac{d}{x}\right)^{3/2}(cx\cdots}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2)
) + (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 - 1/(c^2*x^2)])/(15*e^3) -
(32*c^2*d^2*ArcCsc[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsc[c*x])/(e^3*(e + d/x))
 - (2*c^2*x^2*ArcCsc[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 - 1/(c^2*x^2)] - 9
*c*d*ArcCsc[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c*x)^
(3/2)*((2*(32*c^2*d^2*e + e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*
x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 -
1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(48*c^3*d^3 + 8*c*d*e^2)*Sqrt
[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 -
c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x
)^(3/2)) - (16*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^
2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqr
t[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c
*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c
*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c
*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d)
 + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[
2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)
]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(15*e^4*(d + e*x)^(3/2))))/c^4
```

3.63.3 Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.71, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5770, 27, 7272, 2351, 632, 186, 413, 412, 2185, 27, 600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b \csc^{-1}(cx)\right)}{(d+ex)^{3/2}} \, dx$$

$$\Big\downarrow \ 5770$$

$$\frac{b \int \frac{2\left(16 d^3+8 e x d^2-2 e^2 x^2 d+e^3 x^3\right)}{5 e^4 \sqrt{1-\frac{1}{c^2 x^2}} x^2 \sqrt{d+e x}}dx}{c}+\frac{2 d^3 \left(a+b \csc^{-1}(cx)\right)}{e^4 \sqrt{d+e x}}+\frac{6 d^2 \sqrt{d+e x} \left(a+b \csc^{-1}(cx)\right)}{e^4}-\frac{2 d (d+e x)^{3/2} \left(a+b \csc^{-1}(cx)\right)}{e^4}+\frac{2 (d+e x)^{5/2} \left(a+b \csc^{-1}(cx)\right)}{5 e^4}$$

$$\Big\downarrow \ 27$$

$$\frac{2b\int \frac{16d^3+8exd^2-2e^2x^2d+e^3x^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{5ce^4}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 7272

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{16d^3+8exd^2-2e^2x^2d+e^3x^3}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 2351

$$\frac{2b\sqrt{1-c^2x^2}\left(16d^3\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 632

$$\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+16d^3\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 186

$$\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-32d^3\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\Big\downarrow$ 413

3.63. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 412

$$\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 2185

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{2\int -\frac{e^3\left(24d^2c^2-8dexc^2+e^2\right)}{2\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2e^2}-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2e^2\sqrt{1-c^2x^2}\sqrt{d+ex}}{3c^2}\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 27

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\int \frac{24d^2c^2-8dexc^2+e^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{3c^2}-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2e^2\sqrt{1-c^2x^2}\sqrt{d+ex}}{3c^2}\right)}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 600

3.63. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(\left(32c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-8c^2d\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{3c^2}-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-2e^2\sqrt{\cdots}\right.}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\big\downarrow$ 508

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(\left(32c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+\frac{16cd\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\cdots\right.}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\big\downarrow$ 327

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(\left(32c^2d^2+e^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+\frac{16cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3c^2}-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\cdots\right.\right.}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right.}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$\big\downarrow$ 511

3.63. $\int\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(\frac{16cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2\left(32c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}\right)}{3c^2}-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{E}}{\sqrt{}}\right.}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \; 321$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^4}+$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(\frac{16cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2\left(32c^2d^2+e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}\right)}{3c^2}-\frac{32d^3\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{Ellip}}{\sqrt{-\frac{e}{}}}\right.}{5ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(2*d^3*(a + b*ArcCsc[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a
+ b*ArcCsc[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/e^4 + (2
*(d + e*x)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^4) + (2*b*Sqrt[1 - c^2*x^2]*((-
2*e^2*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2])/(3*c^2) + (e*((16*c*d*Sqrt[d + e*x]
*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[(c*(d + e
*x))/(c*d + e)] - (2*(32*c^2*d^2 + e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Elli
pticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x])))
/(3*c^2) - (32*d^3*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[
Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[d + e/c - (e*(1 - c*x))/c])
)/(5*c*e^4*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.63. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

3.63.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

3.63. $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

3.63. $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.63.4 Maple [A] (verified)

Time = 10.08 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+(ex+d)^{\frac{3}{2}}d-3d^{2}\sqrt{ex+d}-\frac{d^{3}}{\sqrt{ex+d}}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d-3\operatorname{arccsc}(cx)d^{2}\sqrt{\right.}{}$
default	$\frac{-2a\left(-\frac{(ex+d)^{\frac{5}{2}}}{5}+(ex+d)^{\frac{3}{2}}d-3d^{2}\sqrt{ex+d}-\frac{d^{3}}{\sqrt{ex+d}}\right)-2b\left(-\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}+\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d-3\operatorname{arccsc}(cx)d^{2}\sqrt{\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{5}{2}}}{5}-(ex+d)^{\frac{3}{2}}d+3d^{2}\sqrt{ex+d}+\frac{d^{3}}{\sqrt{ex+d}}\right)}{e^{4}}+\frac{2b\left(\frac{\operatorname{arccsc}(cx)(ex+d)^{\frac{5}{2}}}{5}-\operatorname{arccsc}(cx)(ex+d)^{\frac{3}{2}}d+3\operatorname{arccsc}(cx)d^{2}\sqrt{ex\right.}{}$

input
```
int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^4*(-a*(-1/5*(e*x+d)^(5/2)+(e*x+d)^(3/2)*d-3*d^2*(e*x+d)^(1/2)-d^3/(e*x
+d)^(1/2))-b*(-1/5*arccsc(c*x)*(e*x+d)^(5/2)+arccsc(c*x)*(e*x+d)^(3/2)*d-3
*arccsc(c*x)*d^2*(e*x+d)^(1/2)-arccsc(c*x)*d^3/(e*x+d)^(1/2)-2/15/c^3*((c/
(c*d-e))^(1/2)*c^2*(e*x+d)^(5/2)+24*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)
*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*c^2+8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c
*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),(
(c*d-e)/(c*d+e))^(1/2))*c^2*d^2-48*d^2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*
((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2-2*(c/(c*d-e))^(
1/2)*c^2*d*(e*x+d)^(3/2)-8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)
+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/
(c*d+e))^(1/2))*c*d*e+8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*
d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*
d+e))^(1/2))*c*d*e+(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)^(1/2)+((-c*(e*x+d)+c*
d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(
1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^2-(c/(c*d-e))^(1/2)*e^2*
(e*x+d)^(1/2))/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2
-e^2)/c^2/e^2/x^2)^(1/2)))
```

3.63.5 Fricas [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^3*arccsc(c*x) + a*x^3)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^
2), x)
```

3.63.6 Sympy [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \int \frac{x^3(a + b\,\mathrm{acsc}\,(cx))}{(d + ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

3.63.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.63.8 Giac [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{(ex + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \int \frac{x^3\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(d + e\,x\right)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.64 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 369

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = -\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{20bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{32bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
2/3*(e*x+d)^(3/2)*(a+b*arccsc(c*x))/e^3-2*d^2*(a+b*arccsc(c*x))/e^3/(e*x+d
)^(1/2)-4*d*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/3*b*EllipticE(1/2*(-c*x+
1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/
2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+20/3*b*d*Ellipt
icF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+
e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+3
2/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2
))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c^2/x^2)^(1/2
)/(e*x+d)^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.08 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.03

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = -\frac{ad^3\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(3,-\frac{1}{2}\right)}{e^3(d+ex)^{3/2}}$$

$$+\frac{b\left(-\frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(-\frac{4\sqrt{1-\frac{1}{c^2x^2}}}{3e^2}+\frac{16cd\csc^{-1}(cx)}{3e^3}-\frac{2cd\csc^{-1}(cx)}{e^2\left(e+\frac{d}{x}\right)}-\frac{2cx\csc^{-1}(cx)}{3e^2}\right)}{(d+ex)^{3/2}}+\frac{2\left(e+\frac{d}{x}\right)^{3/2}(cx)^{3/2}\left(\frac{10cde\sqrt{\frac{cd+cex}{cd+e}}\sqrt{1-c^2x^2}\,\mathrm{EllipticF}\left(\mathrm{arcs}\cdots\right.}{\sqrt{1-\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}(cx)\cdots}\right.}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

output
```
-((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/
2))) + (b*(-((c^2*(e + d/x)^2*x^2*((-4*Sqrt[1 - 1/(c^2*x^2)])/(3*e^2) + (1
6*c*d*ArcCsc[c*x])/(3*e^3) - (2*c*d*ArcCsc[c*x])/(e^2*(e + d/x)) - (2*c*x*
ArcCsc[c*x])/(3*e^2)))/(d + e*x)^(3/2)) + (2*(e + d/x)^(3/2)*(c*x)^(3/2)*(
(10*c*d*e*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e +
 d/x]*(c*x)^(3/2)) + (2*(8*c^2*d^2 + e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sq
rt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cos[2*ArcCsc
[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e
)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d +
 e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d
- e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e
)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e
)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)
/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
 (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c
^2*x^2))))/(3*e^3*(d + e*x)^(3/2))))/c^3
```

3.64.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5770, 27, 7272, 2351, 600, 508, 327, 511, 321, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b \csc^{-1}(c x)\right)}{(d+e x)^{3/2}} \, dx$$

$$\Big\downarrow \text{5770}$$

$$\frac{b \int -\frac{2 \left(8 d^2+4 e x d-e^2 x^2\right)}{3 e^3 \sqrt{1-\frac{1}{c^2 x^2}} x^2 \sqrt{d+e x}}dx}{c}-\frac{2 d^2 \left(a+b \csc^{-1}(c x)\right)}{e^3 \sqrt{d+e x}}-\frac{4 d \sqrt{d+e x} \left(a+b \csc^{-1}(c x)\right)}{e^3}+\frac{2 (d+e x)^{3/2} \left(a+b \csc^{-1}(c x)\right)}{3 e^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2 b \int \frac{8 d^2+4 e x d-e^2 x^2}{\sqrt{1-\frac{1}{c^2 x^2}} x^2 \sqrt{d+e x}}dx}{3 c e^3}-\frac{2 d^2 \left(a+b \csc^{-1}(c x)\right)}{e^3 \sqrt{d+e x}}-\frac{4 d \sqrt{d+e x} \left(a+b \csc^{-1}(c x)\right)}{e^3}+\frac{2 (d+e x)^{3/2} \left(a+b \csc^{-1}(c x)\right)}{3 e^3}$$

$$\Big\downarrow \text{7272}$$

$$-\frac{2 b \sqrt{1-c^2 x^2} \int \frac{8 d^2+4 e x d-e^2 x^2}{x \sqrt{d+e x} \sqrt{1-c^2 x^2}}dx}{3 c e^3 x \sqrt{1-\frac{1}{c^2 x^2}}}-\frac{2 d^2 \left(a+b \csc^{-1}(c x)\right)}{e^3 \sqrt{d+e x}}-\frac{4 d \sqrt{d+e x} \left(a+b \csc^{-1}(c x)\right)}{e^3}+\frac{2 (d+e x)^{3/2} \left(a+b \csc^{-1}(c x)\right)}{3 e^3}$$

$$\Big\downarrow \text{2351}$$

$$-\frac{2 b \sqrt{1-c^2 x^2} \left(8 d^2 \int \frac{1}{x \sqrt{d+e x} \sqrt{1-c^2 x^2}}dx+\int \frac{4 d e-e^2 x}{\sqrt{d+e x} \sqrt{1-c^2 x^2}}dx\right)}{3 c e^3 x \sqrt{1-\frac{1}{c^2 x^2}}}-\frac{2 d^2 \left(a+b \csc^{-1}(c x)\right)}{e^3 \sqrt{d+e x}}-\frac{4 d \sqrt{d+e x} \left(a+b \csc^{-1}(c x)\right)}{e^3}+\frac{2 (d+e x)^{3/2} \left(a+b \csc^{-1}(c x)\right)}{3 e^3}$$

$$\Big\downarrow \text{600}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+5de\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-e\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 508

$$-\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+5de\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+\frac{2e\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 327

$$-\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+5de\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 511

$$-\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{10de\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}+\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 321

$$-\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{10de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}+\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 632

3.64. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx-\frac{10de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}+\frac{2e\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+}\right.}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}+\frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$

$\downarrow$ 186

$$-\frac{2b\sqrt{1-c^2x^2}\left(-16d^2\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}-\frac{10de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}+\frac{2e\sqrt{d+ex}E\left(\text{arcs}\right.}{c\sqrt{\frac{c}{}}}\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}+\frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$

$\downarrow$ 413

$$-\frac{2b\sqrt{1-c^2x^2}\left(-\frac{16d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{10de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}+\frac{2e\sqrt{d+ex}E\left(a\right.}{c}\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}+\frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$

$\downarrow$ 412

$$-\frac{2d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}+\frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}-$$
$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{16d^2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{10de\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}+\frac{2e\sqrt{d+ex}\left(\right.}{}\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

3.64. $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

output
```
(-2*d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
 b*ArcCsc[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (
2*b*Sqrt[1 - c^2*x^2]*((2*e*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/S
qrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (10*d*e*Sqr
t[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)])/(c*Sqrt[d + e*x]) - (16*d^2*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*
EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[d + e/
c - (e*(1 - c*x))/c]))/(3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

3.64. $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.64.4 Maple [A] (verified)

Time = 9.30 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\frac{2\left(4d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\right.\right.}{}\right.}{}$
default	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\frac{2\left(4d\,\mathrm{EllipticF}\left(\sqrt{ex+d}\right.\right.}{}\right.}{}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)}{e^3}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsc}(cx)}{3}-2\operatorname{arccsc}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsc}(cx)d^2}{\sqrt{ex+d}}-\frac{2\left(4d\,\mathrm{EllipticF}\left(\sqrt{ex}\right.\right.}{}\right.}{}$

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^3*(a*(1/3*(e*x+d)^(3/2)-2*d*(e*x+d)^(1/2)-d^2/(e*x+d)^(1/2))+b*(1/3*(e
*x+d)^(3/2)*arccsc(c*x)-2*arccsc(c*x)*d*(e*x+d)^(1/2)-arccsc(c*x)*d^2/(e*x
+d)^(1/2)-2/3/c^2*(4*d*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/
(c*d+e))^(1/2))*c+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c*d-8*d*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/
d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c-EllipticF((e*x+d)^(1/2)*(c/(c*d-e
))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e+EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*e)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*((-c*(e
*x+d)+c*d-e)/(c*d-e))^(1/2)/(c/(c*d-e))^(1/2)/x/((c^2*(e*x+d)^2-2*c^2*d*(e
*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

3.64.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.64.6 Sympy [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\int \frac{x^2(a+b\,\mathrm{acsc}\,(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.64.8 Giac [F]

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^2}{(ex + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(3/2), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex)^{3/2}}\,dx = \int \frac{x^2\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(d + e\,x)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.65 $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

3.65.1 Optimal result

Integrand size = 19, antiderivative size = 238

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{2d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^2} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{8bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
2*d*(a+b*arccsc(c*x))/e^2/(e*x+d)^(1/2)+2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/
e^2-4*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c
*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e/x/(1-1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)-8*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e
))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x/(1-1/c^2/x^
2)^(1/2)/(e*x+d)^(1/2)
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.57 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.95

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{a(2d+ex)}{\sqrt{d+ex}}+\frac{b(2d+ex)\csc^{-1}(cx)}{\sqrt{d+ex}}-\frac{2ib\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right),\frac{cd+e}{cd-e}\right)\cdots\right)}{c\sqrt{-\frac{c}{cd+e}}\sqrt{\cdots}}\right)}{e^2}$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(2*((a*(2*d + e*x))/Sqrt[d + e*x] + (b*(2*d + e*x)*ArcCsc[c*x])/Sqrt[d + e
*x] - ((2*I)*b*Sqrt[(e*(1 + c*x))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)
]*(EllipticF[I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d
 - e)] - 2*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d +
 e*x]], (c*d + e)/(c*d - e)]))/(c*Sqrt[-(c/(c*d + e))]*Sqrt[1 - 1/(c^2*x^2
)]*x)))/e^2
```

3.65.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {5770, 27, 7272, 2351, 27, 511, 321, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \; 5770$$

$$\frac{b\int \frac{2(2d+ex)}{e^2\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{c}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\int \frac{2d+ex}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{ce^2}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \; 7272$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{2d+ex}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 2351}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{e}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+2d\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(e\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+2d\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 511}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 632}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 186}$$

3.65. $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(-4d\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(-\frac{4d\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2b\sqrt{1-c^2x^2}\left(-\frac{2e\sqrt{\frac{c(d+ex)}{cd+e}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{4d\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(2*d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*Ar
cCsc[c*x]))/e^2 + (2*b*Sqrt[1 - c^2*x^2]*((-2*e*Sqrt[(c*(d + e*x))/(c*d +
e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d +
 e*x]) - (4*d*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[
1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/Sqrt[d + e/c - (e*(1 - c*x))/c]))/(c*
e^2*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.65. $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

3.65. $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx$

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.65.4 Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.18

method	result
parts	$\frac{2a\left(\sqrt{ex+d}+\frac{d}{\sqrt{ex+d}}\right)}{e^2}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arccsc}(cx)+\frac{\operatorname{arccsc}(cx)d}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+d}\,,\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(}{c^2e^2}}}\right.}{e^2}$
derivativedivides	$\frac{-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsc}(cx)-\frac{\operatorname{arccsc}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c}{c}}}\right.}{e^2}$
default	$\frac{-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsc}(cx)-\frac{\operatorname{arccsc}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\left(\operatorname{EllipticF}\left(\sqrt{ex+\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c}{c}}}\right.}{e^2}$

input
```
int(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/e^2*((e*x+d)^(1/2)+d/(e*x+d)^(1/2))+2*b/e^2*((e*x+d)^(1/2)*arccsc(c*x)
+arccsc(c*x)*d/(e*x+d)^(1/2)+2/c*(-(c*(e*x+d)-c*d+e)/(c*d-e))^(1/2)*(-(c*(
e*x+d)-c*d-e)/(c*d+e))^(1/2)*(EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((
c*d-e)/(c*d+e))^(1/2))-2*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c
*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)))/((c^2*(e*x+d)^2-2*c^2*d*(e*x
+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/(c/(c*d-e))^(1/2))
```

3.65.5 Fricas [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x*arccsc(c*x) + a*x)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d^2),
x)
```

3.65.6 Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\int \frac{x(a+b\,\mathrm{acsc}\,(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

3.65.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.65.8 Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(3/2), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{3/2}}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2),x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.66 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

3.66.1 Optimal result

Integrand size = 18, antiderivative size = 119

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{ce\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

output

```
-2*(a+b*arccsc(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(
1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(
1/2)/c/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx = \frac{-2(-1+c^2x^2)\left(a+b\csc^{-1}(cx)\right)+4bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\Big(2,}{e\sqrt{d+ex}\,(-1+c^2x^2)}$$

input

```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*(-1 + c^2*x^2)*(a + b*ArcCsc[c*x]) + 4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqr
t[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 -
 c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x]*(-1 + c^2*x^2))
```

3.66.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {5750, 1898, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{5750}$$

$$-\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{ce}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{1898}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{633}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{632}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{186}$$

$$\frac{4b\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{413}$$

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int\frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{412}$$

$$\frac{4b\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{cex\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{2(a+b\csc^{-1}(cx))}{e\sqrt{d+ex}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcCsc[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[1 - c^2*x^2]*Sqrt[1
 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (
2*e)/(c*d + e)])/(c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[d + e/c - (e*(1 - c*x))
/c])
```

3.66.3.1 Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.66.4 Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.81

method	result	s
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+\frac{2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{c}{cd-e}},\frac{cd-e}{cd},\frac{\sqrt{\frac{c}{cd+e}}}{\sqrt{\frac{c}{cd-e}}}\right)}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2-e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{c}{cd-e}}}\right)}{e}$	2

input
```
int((a+b*arccsc(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arccsc(c*x)+2/c/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)/x/d/(c/(c*d-e))^(1/2)*((-c
*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elliptic
Pi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d
-e))^(1/2))))
```

3.66.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.66.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(d + e*x)**(3/2), x)
```

3.66.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.66.8 Giac [F]

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{(ex + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(e*x + d)^(3/2), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex)^{3/2}}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{3/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x)^(3/2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.67 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

3.67.2 Mathematica [N/A]

Not integrable

Time = 11.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)), x]
```

3.67.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(3/2)),x]
```

output
```
$Aborted
```

3.67.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.67.4 Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\left(ex+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

output
```
int((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x)
```

3.67.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{a + b\csc^{-1}(cx)}{x(d + ex)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{(ex + d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^3 + 2*d*e*x^2 + d^2*x),
x)
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x(d + ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x+d)**(3/2),x)
```

output
```
Timed out
```

3.67.7 Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.62

$$\int \frac{a + b\csc^{-1}(cx)}{x(d + ex)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{(ex + d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
((b*d^(3/2)*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e*x^2 + d*
x)*sqrt(e*x + d)), x) + a*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))*
sqrt(e*x + d) + 2*a*sqrt(d))/(sqrt(e*x + d)*d^(3/2))
```

3.67.8 Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x + d)^(3/2)*x), x)
```

3.67.9 Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,(d+e\,x)^{3/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(3/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(3/2)), x)
```

3.68 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

3.68.2 Mathematica [N/A]

Not integrable

Time = 15.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

3.68.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{5772}$$

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(3/2)),x]
```

output
```
$Aborted
```

3.68.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.68.4 Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x^{2}\left(ex+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x)
```

3.68.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2)
, x)
```

3.68.6 Sympy [N/A]

Not integrable

Time = 84.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^2\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x**2*(d + e*x)**(3/2)), x)
```

3.68.7 Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 145, normalized size of antiderivative = 6.90

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*(b*d^2*e*x^2 + b*d^3*x)*sqrt(d)*integrate(arctan2(1, sqrt(c*x + 1)*
sqrt(c*x - 1))/((e*x^3 + d*x^2)*sqrt(e*x + d)), x) - 2*(3*a*e*x + a*d)*sqr
t(e*x + d)*sqrt(d) - 3*(a*e^2*x^2 + a*d*e*x)*log(e*x/(e*x + 2*sqrt(e*x + d
)*sqrt(d) + 2*d)))/((d^2*e*x^2 + d^3*x)*sqrt(d))
```

3.68.8 Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\operatorname{arccsc}(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x + d)^(3/2)*x^2), x)
```

3.68.9 Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(3/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(3/2)), x)
```

3.69 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 602

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{4bd^2(1-c^2x^2)}{3ce^2(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d^3(a+b\csc^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\csc^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}(a+b\csc^{-1}(cx))}{3e^4} + \frac{8bd^2\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^3(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b(2c^2d^2-e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3c^2e^3(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c^2e^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{64bd^2\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^4\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
2/3*d^3*(a+b*arccsc(c*x))/e^4/(e*x+d)^(3/2)+2/3*(e*x+d)^(3/2)*(a+b*arccsc(
c*x))/e^4-6*d^2*(a+b*arccsc(c*x))/e^4/(e*x+d)^(1/2)-4/3*b*d^2*(-c^2*x^2+1)
/c/e^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-6*d*(a+b*arccsc(c
*x))*(e*x+d)^(1/2)/e^4+8/3*b*d^2*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1
/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/e^3/(c^2*d^2-e^2)/
x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4/3*b*(2*c^2*d^2-e^2)*Elli
pticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*
(-c^2*x^2+1)^(1/2)/c^2/e^3/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/
(c*d+e))^(1/2)+32/3*b*d*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c
*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^3/x/(1-1/
c^2/x^2)^(1/2)/(e*x+d)^(1/2)+64/3*b*d^2*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1
/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1
/2)/c/e^4/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.21 (sec) , antiderivative size = 887, normalized size of antiderivative = 1.47

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \frac{ad^4\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(4,-\frac{3}{2}\right)}{e^4(d+ex)^{5/2}}$$

$$+\frac{b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(-\frac{4\sqrt{1-\frac{1}{c^2x^2}}}{3e\left(-c^2d^2+e^2\right)}+\frac{32cd\csc^{-1}(cx)}{3e^4}-\frac{2cd\csc^{-1}(cx)}{3e^2\left(e+\frac{d}{x}\right)^2}-\frac{2cx\csc^{-1}(cx)}{3e^3}-\frac{2\left(-2c^2d^2e\sqrt{1-\frac{1}{c^2x^2}}-7c^3d^3\csc^{-1}(cx)+7cde^2\csc^{-1}(cx)\right)}{3e^3\left(-c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right)}{(d+ex)^{5/2}}+ \cdots\right)}{\cdots}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

output
```
(a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2)
) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 - 1/(c^2*x^2)])/(3*e*(-(c^2*d^2
) + e^2)) + (32*c*d*ArcCsc[c*x])/(3*e^4) - (2*c*d*ArcCsc[c*x])/(3*e^2*(e +
 d/x)^2) - (2*c*x*ArcCsc[c*x])/(3*e^3) - (2*(-2*c^2*d^2*e*Sqrt[1 - 1/(c^2*
x^2)] - 7*c^3*d^3*ArcCsc[c*x] + 7*c*d*e^2*ArcCsc[c*x]))/(3*e^3*(-(c^2*d^2)
 + e^2)*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((2
*(8*c^3*d^3*e - 8*c*d*e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]
*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c
^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(16*c^4*d^4 - 16*c^2*d^2*e^2 - e^
4)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/
x]*(c*x)^(3/2)) + (2*e^3*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2)
+ c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin
[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x
)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqr
t[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqr
t[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(
c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipti
cPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(Sqrt[1 - 1/(c^2*
x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*(c*d - e)*e^4*(c*d +...
```

3.69.3 Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.93, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.048$, Rules used = {5770, 27, 7272, 2351, 635, 25, 27, 498, 27, 508, 327, 632, 186, 413, 412, 2182, 27, 600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b \csc^{-1}(cx)\right)}{(d+ex)^{5/2}} \, dx$$

$$\Big\downarrow \text{5770}$$

$$\frac{b \int -\frac{2\left(16 d^3+24 e x d^2+6 e^2 x^2 d-e^3 x^3\right)}{3 e^4 \sqrt{1-\frac{1}{c^2 x^2}} x^2 (d+e x)^{3/2}}dx}{c}+\frac{2 d^3 \left(a+b \csc^{-1}(cx)\right)}{3 e^4 (d+e x)^{3/2}}-\frac{6 d^2 \left(a+b \csc^{-1}(cx)\right)}{e^4 \sqrt{d+e x}}-\frac{6 d \sqrt{d+e x} \left(a+b \csc^{-1}(cx)\right)}{e^4}+\frac{2 (d+e x)^{3/2} \left(a+b \csc^{-1}(cx)\right)}{3 e^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\int \frac{16d^3+24exd^2+6e^2x^2d-e^3x^3}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce^4}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \text{7272}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{16d^3+24exd^2+6e^2x^2d-e^3x^3}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \text{2351}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \text{635}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}\right)+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \text{27}$$

3.69. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{d}\right)+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\Big\downarrow$ 498

$$-\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}\right)}{d}\right)\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\Big\downarrow$ 27

$$-\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{d}\right)\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\Big\downarrow$ 508

3.69. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \ 327$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \ 632$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\left(\frac{\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \ 186$$

3.69. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\left(-\frac{2\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\int \frac{-x^2}{(d+\cdots}\right.}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 413

$$-\frac{2b\sqrt{1-c^2x^2}\left(16d^3\left(-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)\right.}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 412

$$-\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+16d^3\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{Elli}\cdots}{d\sqrt{\cdots}}\right.\right.}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 2182

3.69. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\frac{2\int \frac{e\left(d\left(24c^2d^2-7e^2\right)+e\left(16c^2d^2+e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+16d^3\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}-\frac{2\sqrt{1-\cdots}}{\cdots}\cdots\right.\cdots}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\int \frac{d\left(24c^2d^2-7e^2\right)+e\left(16c^2d^2+e^2\right)x}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+16d^3\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}-\frac{2\sqrt{1-\frac{e(\cdots}{c\cdots}}}{\cdots}\cdots\right.\cdots}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 600

$$-\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(8d(c^2d^2-e^2)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+(16c^2d^2+e^2)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{c^2d^2-e^2}+16d^3\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\cdots\right.\right.}{\left(c^2d^2-e^2\right)\sqrt{\cdots}}\cdots\right.}{d}\cdots\right.\cdots}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 508

3.69. $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(8d(c^2d^2-e^2)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2\left(16c^2d^2+e^2\right)\sqrt{d+ex}\int\frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2d^2-e^2}+16d^3\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\cdots}{\cdots}\right.}{\cdots}\right.\right.\cdots}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}\cdots}\cdots$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 327$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(8d(c^2d^2-e^2)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{2\left(16c^2d^2+e^2\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2d^2-e^2}+16d^3\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\cdots\right.}{\cdots}\right.\right.\cdots}{3ce^4x\sqrt{1-\frac{1}{c^2x^2}}\cdots}\cdots$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 511$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(-\frac{16d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{2\left(16c^2d^2+e^2\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2d^2-e^2}+16d^3\cdots\right.}{3ce^4x\sqrt{\cdots}}\cdots$$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 321$$

3.69. $\int\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2d^3\left(a+b\csc^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\csc^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^4}-\frac{2b\sqrt{1-c^2x^2}\left(\frac{e\left(-\frac{16d\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}}-\frac{2\left(16c^2d^2+e^2\right)\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2d^2-e^2}+16d^3\left(-\frac{e}{\cdots}\right.\cdots\right)}{3ce^4x\sqrt{1-\cdots}}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

output
```
(2*d^3*(a + b*ArcCsc[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^2*(a + b*ArcCsc
[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^4
+ (2*(d + e*x)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^4) - (2*b*Sqrt[1 - c^2*x^2]
*((34*d^2*e^2*Sqrt[1 - c^2*x^2])/((c^2*d^2 - e^2)*Sqrt[d + e*x]) + (e*((-2
*(16*c^2*d^2 + e^2)*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
 (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (16*d*(c^2*d^2 - e^
2)*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(c*Sqrt[d + e*x])))/(c^2*d^2 - e^2) + 16*d^3*(-((e*((2*e
*Sqrt[1 - c^2*x^2])/((c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*c*Sqrt[d + e*x]*E
llipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c^2*d^2 - e^2)
*Sqrt[(c*(d + e*x))/(c*d + e)])))/d) - (2*Sqrt[1 - (e*(1 - c*x))/(c*d + e)
]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d*Sqrt[d
 + e/c - (e*(1 - c*x))/c]))))/(3*c*e^4*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

3.69. $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.69. $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.69.4 Maple [A] (verified)

Time = 9.56 (sec) , antiderivative size = 1067, normalized size of antiderivative = 1.77

method	result	size
derivativedivides	Expression too large to display	1067
default	Expression too large to display	1067
parts	Expression too large to display	1080

input
```
int(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^4*(-a*(-1/3*(e*x+d)^(3/2)+3*d*(e*x+d)^(1/2)+3*d^2/(e*x+d)^(1/2)-1/3*d^
3/(e*x+d)^(3/2))-b*(-1/3*(e*x+d)^(3/2)*arccsc(c*x)+3*arccsc(c*x)*d*(e*x+d)
^(1/2)+3*arccsc(c*x)*d^2/(e*x+d)^(1/2)-1/3*arccsc(c*x)*d^3/(e*x+d)^(3/2)+2
/3/c^2*(8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^
(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c
^3*d^3*(e*x+d)^(1/2)-16*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*
d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)
/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-(c/(c*d-e))^
(1/2)*c^3*d^2*(e*x+d)^2+2*(c/(c*d-e))^(1/2)*c^3*d^3*(e*x+d)-7*((-c*(e*x+d)
+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d
)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-(
(-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Ellip
ticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e^2*(e*x
+d)^(1/2)+16*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+
e))^(1/2)/(c/(c*d-e))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^3*d
^4+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*E
llipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*e^3*(e*x
+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^
(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2)...
```

3.69.5 Fricas [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^3*arccsc(c*x) + a*x^3)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2
+ 3*d^2*e*x + d^3), x)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

output
```
Timed out
```

3.69.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.69.8 Giac [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.70 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 440

$$\begin{aligned}\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx &= \frac{4bd(1-c^2x^2)}{3ce\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ &- \frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}} + \frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3} \\ &- \frac{4bd\sqrt{d+ex}\sqrt{1-c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e^2\left(c^2d^2-e^2\right)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} \\ &- \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c^2e^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ &- \frac{32bd\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^3\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

output
```
-2/3*d^2*(a+b*arccsc(c*x))/e^3/(e*x+d)^(3/2)+4*d*(a+b*arccsc(c*x))/e^3/(e*
x+d)^(1/2)+4/3*b*d*(-c^2*x^2+1)/c/e/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e
*x+d)^(1/2)+2*(a+b*arccsc(c*x))*(e*x+d)^(1/2)/e^3-4/3*b*d*EllipticE(1/2*(-
c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)
^(1/2)/e^2/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)-4
*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x
+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c^2/e^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d
)^(1/2)-32/3*b*d*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e
))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^3/x/(1-1/c^2/x^
2)^(1/2)/(e*x+d)^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.07 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.95

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{ad^3\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(3,-\frac{3}{2}\right)}{e^3(d+ex)^{5/2}}$$

$$+ \frac{b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(\frac{4cd\sqrt{1-\frac{1}{c^2x^2}}}{3e^2\left(-c^2d^2+e^2\right)}-\frac{16\csc^{-1}(cx)}{3e^3}+\frac{2\csc^{-1}(cx)}{3e\left(e+\frac{d}{x}\right)^2}+\frac{4\left(-cde\sqrt{1-\frac{1}{c^2x^2}}-2c^2d^2\csc^{-1}(cx)+2e^2\csc^{-1}(cx)\right)}{3e^2\left(-c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right)}{(d+ex)^{5/2}} - \frac{2\left(e+\frac{d}{x}\right)^{5/2}(cx)^{5/2}\left(\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

output
```
-((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/
2))) + (b*(-((c^3*(e + d/x)^3*x^3*((4*c*d*Sqrt[1 - 1/(c^2*x^2)])/(3*e^2*(-
(c^2*d^2) + e^2)) - (16*ArcCsc[c*x])/(3*e^3) + (2*ArcCsc[c*x])/(3*e*(e + d
/x)^2) + (4*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) - 2*c^2*d^2*ArcCsc[c*x] + 2*e^
2*ArcCsc[c*x]))/(3*e^2*(-(c^2*d^2) + e^2)*(e + d/x))))/(d + e*x)^(5/2)) -
(2*(e + d/x)^(5/2)*(c*x)^(5/2)*((2*(3*c^2*d^2*e - 3*e^3)*Sqrt[(c*d + c*e*x
)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2
*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(8*
c^3*d^3 - 9*c*d*e^2)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellip
ticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2
*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*c*d*e*Cos[2*ArcCsc[c*x]]*((c*d + c*
e*x)*(-1 + c^2*x^2) + c^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x
^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 +
c*x)*Sqrt[(e - c*e*x)/(c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*
EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*
EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sq
rt[(e*(1 + c*x))/(-(c*d) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[
1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]
))/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(-2 + c^2*x^2))))/(3*(c*
d - e)*e^3*(c*d + e)*(d + e*x)^(5/2))))/c^3
```

3.70.3 Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.18, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.048$, Rules used = {5770, 27, 7272, 2351, 635, 25, 27, 498, 27, 508, 327, 632, 186, 413, 412, 688, 27, 600, 508, 327, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b \csc ^{-1}(c x)\right)}{(d+e x)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5770}$$

$$\frac{b \int \frac{2 \left(8 d^2+12 e x d+3 e^2 x^2\right)}{3 e^3 \sqrt{1-\frac{1}{c^2 x^2}} x^2 (d+e x)^{3/2}}dx}{c}-\frac{2 d^2 \left(a+b \csc ^{-1}(c x)\right)}{3 e^3 (d+e x)^{3/2}}+\frac{4 d \left(a+b \csc ^{-1}(c x)\right)}{e^3 \sqrt{d+e x}}+\frac{2 \sqrt{d+e x} \left(a+b \csc ^{-1}(c x)\right)}{e^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b\int \frac{8d^2+12exd+3e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce^3}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{7272}$$

$$\frac{2b\sqrt{1-c^2x^2}\int \frac{8d^2+12exd+3e^2x^2}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{2351}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{635}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{498}$$

3.70. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}\right)}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 508$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 327$$

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

3.70. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$

$\Big\downarrow$ 632

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}d\ldots}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$

$\Big\downarrow$ 186

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(-\frac{2\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\int \frac{3xe\ldots}{(d+ex)^{3}\ldots}\ldots}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$

$\Big\downarrow$ 413

$$\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\ldots}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$

$\Big\downarrow$ 412

$$\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+8d^2\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticP}\ldots}{d\sqrt{-\frac{e(1\ldots}{}}}\right.\ldots}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\csc^{-1}(cx))}{e^3}$$

3.70. $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

$$\Big\downarrow \; 688$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{2\int \frac{3e\left(4d^2c^2+3dexc^2-e^2\right)}{2\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+8d^2\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{Elliptic}\ldots}{d\sqrt{-\ldots}}\right.\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{3e\int \frac{4d^2c^2+3dexc^2-e^2}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+8d^2\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\ldots}{d\sqrt{-\frac{e(1-\ldots}{c\ldots}}}\right.\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 600$$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{3e\left(\left(c^2d^2-e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx+3c^2d\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx\right)}{c^2d^2-e^2}+8d^2\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\ldots\right.}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right.}{d}\right.\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 508$$

3.70. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{3e\left(\left(c^2d^2-e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{6cd\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2d^2-e^2}+8d^2\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\right.\right.}{\left(c^2d^2-e^2\right)}\right.}{d}\right.\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 327

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{3e\left(\left(c^2d^2-e^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx-\frac{6cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2d^2-e^2}+8d^2\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\right.}{\left(c^2d^2-e^2\right.}\right.}{d}\right.\right.}{3ce^3x\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 511

$$\frac{2b\sqrt{1-c^2x^2}\left(\frac{3e\left(-\frac{2\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int \frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{d+ex}}-\frac{6cd\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{c^2d^2-e^2}+8d^2\left(-\frac{e\left(\frac{2e\sqrt{\ }}{\left(c^2d^2-\right.}\right.}{}\right.\right.}{3ce^3x\sqrt{1-\frac{}{c^2}}}$$

$$\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 321

3.70. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2d^2\left(a+b\csc^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{2b\sqrt{1-c^2x^2}\left(8d^2\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)\right)}{3ce^3x\sqrt{1-\frac{1}{c^2x}}}$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

output
```
(-2*d^2*(a + b*ArcCsc[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a + b*ArcCsc[
c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsc[c*x]))/e^3 + (
2*b*Sqrt[1 - c^2*x^2]*((18*d*e^2*Sqrt[1 - c^2*x^2])/((c^2*d^2 - e^2)*Sqrt[
d + e*x]) + (3*e*((-6*c*d*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqr
t[2]], (2*e)/(c*d + e)])/Sqrt[(c*(d + e*x))/(c*d + e)] - (2*(c^2*d^2 - e^2
)*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (
2*e)/(c*d + e)])/(c*Sqrt[d + e*x])))/(c^2*d^2 - e^2) + 8*d^2*(-((e*((2*e*S
qrt[1 - c^2*x^2])/((c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*c*Sqrt[d + e*x]*Ell
ipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c^2*d^2 - e^2)*S
qrt[(c*(d + e*x))/(c*d + e)])))/d) - (2*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*
EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d*Sqrt[d +
 e/c - (e*(1 - c*x))/c]))))/(3*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

3.70. $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

3.70. $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(401) = 802$.

Time = 9.03 (sec) , antiderivative size = 1026, normalized size of antiderivative = 2.33

method	result	size
derivativedivides	Expression too large to display	1026
default	Expression too large to display	1026
parts	Expression too large to display	1041

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^3*(a*((e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2)+2*d/(e*x+d)^(1/2))+b*((e*x+d
)^(1/2)*arccsc(c*x)-1/3*arccsc(c*x)*d^2/(e*x+d)^(3/2)+2*arccsc(c*x)*d/(e*x
+d)^(1/2)+2/3/c*(4*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/
(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))
^(1/2))*c^2*d^2*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x
+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-
e)/(c*d+e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1
/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e)
)^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)
^(1/2)-(c/(c*d-e))^(1/2)*c^2*d*(e*x+d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2
)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(
1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d
-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(
c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(
1/2)*c^2*d^2*(e*x+d)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d
+e))^(1/2))*e^2*(e*x+d)^(1/2)+8*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e
*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c
*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d-
e))^(1/2)*c^2*d^3+(c/(c*d-e))^(1/2)*d*e^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e...
```

3.70.5 Fricas [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2
+ 3*d^2*e*x + d^3), x)
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

output
```
Timed out
```

3.70.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.70.8 Giac [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.71 $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

3.71.1 Optimal result

Integrand size = 19, antiderivative size = 314

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{4b(1-c^2x^2)}{3c(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3e(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{8b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3ce^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

output
```
2/3*d*(a+b*arccsc(c*x))/e^2/(e*x+d)^(3/2)-2*(a+b*arccsc(c*x))/e^2/(e*x+d)^
(1/2)-4/3*b*(-c^2*x^2+1)/c/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/
2)+4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(
e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/e/(c^2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(
e*x+d)/(c*d+e))^(1/2)+8/3*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2
)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/e^2/x/
(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.71.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.46 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \frac{4bc\sqrt{1-\frac{1}{c^2x^2}}x}{3(c^2d^2-e^2)\sqrt{d+ex}} - \frac{2a(2d+3ex)}{3e^2(d+ex)^{3/2}} - \frac{2b(2d+3ex)\csc^{-1}(cx)}{3e^2(d+ex)^{3/2}} + \frac{4ib\sqrt{-\frac{c}{cd+e}}\sqrt{\frac{e(1+cx)}{-cd+e}}\sqrt{\frac{e-cex}{cd+e}}\left(cdE\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{d+ex}\right)|\frac{cd+e}{cd-e}\right)-cd\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd+e}}\sqrt{\right.\right.\right.}{3c^2de^2\sqrt{1-\frac{1}{c^2x^2}}x}$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

output
```
(4*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(3*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*a*(
2*d + 3*e*x))/(3*e^2*(d + e*x)^(3/2)) - (2*b*(2*d + 3*e*x)*ArcCsc[c*x])/(3
*e^2*(d + e*x)^(3/2)) + (((4*I)/3)*b*Sqrt[-(c/(c*d + e))]*Sqrt[(e*(1 + c*x
))/(-(c*d) + e)]*Sqrt[(e - c*e*x)/(c*d + e)]*(c*d*EllipticE[I*ArcSinh[Sqrt
[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] - c*d*EllipticF[I*Ar
cSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]], (c*d + e)/(c*d - e)] + 2*(c*d +
 e)*EllipticPi[1 + e/(c*d), I*ArcSinh[Sqrt[-(c/(c*d + e))]*Sqrt[d + e*x]],
 (c*d + e)/(c*d - e)]))/(c^2*d*e^2*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.71.3 Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.32, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.053$, Rules used = {5770, 27, 7272, 2351, 27, 498, 27, 508, 327, 635, 25, 27, 498, 27, 508, 327, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{5770}$$

$$\frac{b\int -\frac{2(2d+3ex)}{3e^2\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{c} - \frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\int \frac{2d+3ex}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce^2}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{7272}$$

$$-\frac{2b\sqrt{1-c^2x^2}\int \frac{2d+3ex}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{2351}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(\int \frac{3e}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(3e\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{498}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}\right)+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(3e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}\right)+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{508}$$

3.71. $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \ 327$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)\right)}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \ 635$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}\right)+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\right.}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx\right)+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2}{cd}\right.}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}}dx}{d}\right)+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right.}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right.\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$-\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

3.71. $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\Big\downarrow \text{498}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}\right)}{d}\right)+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(a\cdots\right.}{(c^2d^2-\cdots}\right.\cdots}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{c^2d^2-e^2}+\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{d}\right)+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\text{arcsi}\cdots\right.}{(c^2d^2-e^2\cdots}\right.\cdots}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{508}$$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\cdots}{\cdots}\right.\cdots}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\csc^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \text{327}$$

3.71. $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\right.\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 632

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(\frac{\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}\right.\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 186

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(-\frac{2\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+3e\left(\frac{2}{(c^2d}\right.\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 413

$$-\frac{2b\sqrt{1-c^2x^2}\left(2d\left(-\frac{2\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d}\right)+\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$
$$\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 412

3.71. $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

$$-\frac{2(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\csc^{-1}(cx))}{3e^2(d+ex)^{3/2}}-\frac{2b\sqrt{1-c^2x^2}\left(3e\left(\frac{2e\sqrt{1-c^2x^2}}{(c^2d^2-e^2)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}\right)+2d\left(-\frac{e\left(\frac{2e\sqrt{1-c^2x^2}}{\left(c^2d^2-e^2\right)\sqrt{d+ex}}-\frac{2c\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right.}{\left(c^2d^2-e^2\right)\sqrt{\frac{c(d+ex}{cd+e}}}\right.}{d}\right.\right.}{3ce^2x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x)^(5/2),x]
```

output
```
(2*d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCsc[c*x])
)/(e^2*Sqrt[d + e*x]) - (2*b*Sqrt[1 - c^2*x^2]*(3*e*((2*e*Sqrt[1 - c^2*x^2
])/((c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*c*Sqrt[d + e*x]*EllipticE[ArcSin[S
qrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c^2*d^2 - e^2)*Sqrt[(c*(d + e*x
))/(c*d + e)])) + 2*d*(-((e*((2*e*Sqrt[1 - c^2*x^2])/((c^2*d^2 - e^2)*Sqrt
[d + e*x]) - (2*c*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (
2*e)/(c*d + e)])/((c^2*d^2 - e^2)*Sqrt[(c*(d + e*x))/(c*d + e)])))/d) - (2
*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt
[2]], (2*e)/(c*d + e)])/(d*Sqrt[d + e/c - (e*(1 - c*x))/c]))))/(3*c*e^2*Sq
rt[1 - 1/(c^2*x^2)]*x)
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

3.71. $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx$

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 899 vs. $2(284) = 568$.

Time = 9.51 (sec) , antiderivative size = 900, normalized size of antiderivative = 2.87

method	result
derivativedivides	$-2a\left(\frac{1}{\sqrt{ex+d}}-\frac{d}{3(ex+d)^{\frac{3}{2}}}\right)-2b\left(\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}-\frac{\operatorname{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^2-\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\mathrm{Elli}\right.}{}\right.$
default	$-2a\left(\frac{1}{\sqrt{ex+d}}-\frac{d}{3(ex+d)^{\frac{3}{2}}}\right)-2b\left(\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}-\frac{\operatorname{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^2-\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\mathrm{Elli}\right.}{}\right.$
parts	$\frac{2a\left(-\frac{1}{\sqrt{ex+d}}+\frac{d}{3(ex+d)^{\frac{3}{2}}}\right)}{e^2}+\frac{2b\left(-\frac{\operatorname{arccsc}(cx)}{\sqrt{ex+d}}+\frac{\operatorname{arccsc}(cx)d}{3(ex+d)^{\frac{3}{2}}}+\frac{\frac{2\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^2}{3}-\frac{2\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\sqrt{-\frac{c(ex+d)-cd-}{cd+e}}}{}}{}\right.}{}$

input
```
int(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^2*(-a*(1/(e*x+d)^(1/2)-1/3*d/(e*x+d)^(3/2))-b*(1/(e*x+d)^(1/2)*arccsc(
c*x)-1/3*arccsc(c*x)*d/(e*x+d)^(3/2)-2/3/c*((c/(c*d-e))^(1/2)*c^2*d*(e*x+d
)^2-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*
EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2
*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d
+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/
2))*c^2*d^2*(e*x+d)^(1/2)+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d
)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*
d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-2*(c/(c*
d-e))^(1/2)*c^2*d^2*(e*x+d)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d
)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)
/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((
-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2)
,((c*d-e)/(c*d+e))^(1/2))*c*d*e*(e*x+d)^(1/2)+(c/(c*d-e))^(1/2)*c^2*d^3-2*
((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*Elli
pticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/
(c*d-e))^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*d*e^2)/d/(c*d-e)/(c/(c
*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/x/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d
^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

3.71.5 Fricas [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x*arccsc(c*x) + a*x)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*
d^2*e*x + d^3), x)
```

3.71.6 Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{x(a+b\operatorname{acsc}(cx))}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral(x*(a + b*acsc(c*x))/(d + e*x)**(5/2), x)
```

3.71.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.71.8 Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x/(e*x + d)^(5/2), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2),x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.72 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

3.72.1 Optimal result

Integrand size = 18, antiderivative size = 298

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{4be(1-c^2x^2)}{3cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4b\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{3d\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3cde\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

output
```
-2/3*(a+b*arccsc(c*x))/e/(e*x+d)^(3/2)+4/3*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-e
^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*EllipticE(1/2*(-c*x+1)^(1/2)
*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^
2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c*d+e))^(1/2)+4/3*b*EllipticP
i(1/2*(-c*x+1)^(1/2)*2^(1/2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+
e))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.72.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 608 vs. $2(298) = 596$.

Time = 21.29 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.04

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \frac{2\left(-\frac{a}{e}+\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}(d+ex)^2}{c^2d^3-de^2}-\frac{bex^2\csc^{-1}(cx)}{d^2}-\frac{b(d+ex)^2\csc^{-1}(cx)}{d^2e}+\frac{2bx(d+ex)\left(-cde\sqrt{1-\frac{1}{c^2x^2}}+(\cdots\right.}{c^2d^4-d^2e\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2),x]
```

output
```
(2*(-(a/e) + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x)^2)/(c^2*d^3 - d*e^2) -
 (b*e*x^2*ArcCsc[c*x])/d^2 - (b*(d + e*x)^2*ArcCsc[c*x])/(d^2*e) + (2*b*x*
(d + e*x)*(-(c*d*e*Sqrt[1 - 1/(c^2*x^2)]) + (c^2*d^2 - e^2)*ArcCsc[c*x]))/
(c^2*d^4 - d^2*e^2) + (2*b*d*((c*(d + e*x))/(c*d + e))^(3/2)*Sqrt[1 - c^2*
x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/((c*d
- e)*e*Sqrt[1 - 1/(c^2*x^2)]*x) - (2*b*c*(d + e*x)*Cos[2*ArcCsc[c*x]]*((d
+ e*x)*(-1 + c^2*x^2) + c*d*x*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 - c^2*x
^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (x*(1 + c*
x)*Sqrt[(c*(d + e*x))/(c*d - e)]*Sqrt[(e - c*e*x)/(c*d + e)]*((c*d + e)*El
lipticE[ArcSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)/(c*d + e)] - e*El
lipticF[ArcSin[Sqrt[(c*(d + e*x))/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt
[(e*(1 + c*x))/(-(c*d) + e)] + e*x*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[1 -
c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)]))/(
d*(c*d - e)*(c*d + e)*Sqrt[1 - 1/(c^2*x^2)]*(-2 + c^2*x^2))))/(3*(d + e*x)
^(3/2))
```

3.72.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5750, 1898, 635, 25, 27, 498, 27, 509, 508, 327, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx$$

$\big\downarrow$ 5750

$$-\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 1898

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 635

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 25

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 27

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 498

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2\int -\frac{\sqrt{d+ex}}{2\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 27

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{\int \frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 509

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 508

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 327

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 633

3.72. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 632

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 186

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d\sqrt{x^2-\frac{1}{c^2}}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\big\downarrow$ 413

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

3.72. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$\Big\downarrow 412$$

$$-\frac{2(a+b\csc^{-1}(cx))}{3e(d+ex)^{3/2}}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(-\frac{2\sqrt{1-c^2x^2}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}}{\left(d^2-\frac{e^2}{c^2}\right)\sqrt{d+ex}}\right)}{d}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right.}{d\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}}{3cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcCsc[c*x]))/(3*e*(d + e*x)^(3/2)) - (2*b*Sqrt[-c^(-2) + x^2]*
(-((e*((-2*e*Sqrt[-c^(-2) + x^2])/((d^2 - e^2/c^2)*Sqrt[d + e*x]) - (2*Sqr
t[d + e*x]*Sqrt[1 - c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e
)/(c*d + e)])/(c*(d^2 - e^2/c^2)*Sqrt[(c*(d + e*x))/(c*d + e)]*Sqrt[-c^(-2
) + x^2])))/d) - (2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)]*El
lipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d*Sqrt[-c^(-
2) + x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(3*c*e*Sqrt[1 - 1/(c^2*x^2)]*
x)
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
 - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

3.72. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(270) = 540$.

Time = 7.16 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.94

method	result
derivativedivides	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}}{3}-\frac{2\sqrt{-}}{}\right.$
default	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}+\frac{2\sqrt{\frac{-c(ex+d)+cd-e}{cd-e}}\,\sqrt{\frac{-c(ex+d)+cd+e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\sqrt{\frac{cd-e}{cd+e}}\right)c^2d^2\sqrt{ex+d}}{3}-\frac{2\sqrt{-}}{}\right.$
parts	$-\frac{2a}{3(ex+d)^{\frac{3}{2}}e}+\frac{2b\left(-\frac{\operatorname{arccsc}(cx)}{3(ex+d)^{\frac{3}{2}}}-\frac{2\left(\sqrt{\frac{c}{cd-e}}\,c^2d(ex+d)^2-\sqrt{-\frac{c(ex+d)-cd+e}{cd-e}}\,\sqrt{-\frac{c(ex+d)-cd-e}{cd+e}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\,\sqrt{\frac{c}{cd-e}},\right.\right.}{}\right.}{}$

input

```
int((a+b*arccsc(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsc(c*x)+2/3/c*(((-c*(e
*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((
e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*(e*x+d)^(1
/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*
EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2
*(e*x+d)^(1/2)+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d
+e))^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*
d+e))^(1/2)/(c/(c*d-e))^(1/2))*c^2*d^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2
*d*(e*x+d)^2+((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e
))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2)
)*c*d*e*(e*x+d)^(1/2)-((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+
e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+
e))^(1/2))*c*d*e*(e*x+d)^(1/2)+2*(c/(c*d-e))^(1/2)*c^2*d^2*(e*x+d)-((-c*(e
*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi(
(e*x+d)^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e)
)^(1/2))*e^2*(e*x+d)^(1/2)-(c/(c*d-e))^(1/2)*c^2*d^3+(c/(c*d-e))^(1/2)*d*e
^2)/(c*d-e)/(c/(c*d-e))^(1/2)/(e*x+d)^(1/2)/(c*d+e)/d^2/x/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2-e^2)/c^2/e^2/x^2)^(1/2)))
```

3.72. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

3.72.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*
e*x + d^3), x)
```

3.72.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(d + e*x)**(5/2), x)
```

3.72.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.72.8 Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(e*x + d)^(5/2), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{5/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x)^(5/2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.73 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

3.73.2 Mathematica [N/A]

Not integrable

Time = 30.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)), x]
```

3.73.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \ 5772$$

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*(d + e*x)^(5/2)),x]
```

output
```
$Aborted
```

3.73.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.73.4 Maple [N/A] (verified)

Not integrable

Time = 0.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\left(ex+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

output
```
int((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x)
```

3.73.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*
e*x^2 + d^3*x), x)
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x+d)**(5/2),x)
```

output
```
Timed out
```

3.73.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 177, normalized size of antiderivative = 8.43

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(3*(b*d^2*e^2*x^2 + 2*b*d^3*e*x + b*d^4)*sqrt(d)*integrate(arctan2(1,
sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^3 + 2*d*e*x^2 + d^2*x)*sqrt(e*x + d))
, x) + 2*(3*a*e*x + 4*a*d)*sqrt(e*x + d)*sqrt(d) + 3*(a*e^2*x^2 + 2*a*d*e*
x + a*d^2)*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)))/((d^2*e^2*x^2 +
 2*d^3*e*x + d^4)*sqrt(d))
```

3.73.8 Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x + d)^(5/2)*x), x)
```

3.73.9 Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,(d+e\,x)^{5/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(5/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x)^(5/2)), x)
```

3.74 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

3.74.2 Mathematica [N/A]

Not integrable

Time = 27.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

3.74.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5772}$$

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

output
```
$Aborted
```

3.74.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.74.4 Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x^{2}\left(ex+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x)
```

3.74.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsc(c*x) + a)/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*
e*x^3 + d^3*x^2), x)
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x**2/(e*x+d)**(5/2),x)
```

output
```
Timed out
```

3.74.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 179, normalized size of antiderivative = 8.52

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(3*(2*(b*d^3*e*x^2 + b*d^4*x)*sqrt(d)*integrate(arctan2(1, sqrt(c*x +
1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^3 + d^2*x^2)*sqrt(e*x + d)), x) - 5*
(a*e^2*x^2 + a*d*e*x)*log(e*x/(e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d))*sqrt
(e*x + d) - 2*(15*a*e^2*x^2 + 20*a*d*e*x + 3*a*d^2)*sqrt(d))/((d^3*e*x^2 +
 d^4*x)*sqrt(e*x + d)*sqrt(d))
```

3.74.8 Giac [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x + d)^(5/2)*x^2), x)
```

3.74.9 Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,(d+e\,x)^{5/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(5/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x)^(5/2)), x)
```

3.75 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

3.75.1 Optimal result

Integrand size = 18, antiderivative size = 540

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx &= \frac{4be(1-c^2x^2)}{15cd\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}}\\
&+\frac{16bce(1-c^2x^2)}{15\,(c^2d^2-e^2)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{4be(1-c^2x^2)}{5cd^2\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\\
&-\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}-\frac{4b(7c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{15\,(c^2d^3-de^2)^2\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{\frac{c(d+ex)}{cd+e}}}\\
&+\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d\,(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\\
&+\frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\end{aligned}$$

output
```
-2/5*(a+b*arccsc(c*x))/e/(e*x+d)^(5/2)+4/15*b*e*(-c^2*x^2+1)/c/d/(c^2*d^2-
e^2)/x/(e*x+d)^(3/2)/(1-1/c^2/x^2)^(1/2)+16/15*b*c*e*(-c^2*x^2+1)/(c^2*d^2
-e^2)^2/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*e*(-c^2*x^2+1)/c/d^2/(c^
2*d^2-e^2)/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*(7*c^2*d^2-3*e^2)*El
lipticE(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e))^(1/2))*(e*x+d)^(1/2
)*(-c^2*x^2+1)^(1/2)/(c^2*d^3-d*e^2)^2/x/(1-1/c^2/x^2)^(1/2)/(c*(e*x+d)/(c
*d+e))^(1/2)+4/15*b*EllipticF(1/2*(-c*x+1)^(1/2)*2^(1/2),2^(1/2)*(e/(c*d+e
))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/2)/d/(c^2*d^2-e^2)/x/(
1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*EllipticPi(1/2*(-c*x+1)^(1/2)*2^(1/
2),2,2^(1/2)*(e/(c*d+e))^(1/2))*(c*(e*x+d)/(c*d+e))^(1/2)*(-c^2*x^2+1)^(1/
2)/c/d^2/e/x/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.75.2 Mathematica [A] (warning: unable to verify)

Time = 34.05 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.86

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2a}{5e(d+ex)^{5/2}}$$

$$+\frac{b\left(-\frac{c^4\left(e+\frac{d}{x}\right)^4x^4\left(\frac{4\left(-7c^2d^2+3e^2\right)\sqrt{1-\frac{1}{c^2x^2}}}{15c^2d^2\left(-c^2d^2+e^2\right)^2}+\frac{2\csc^{-1}(cx)}{5c^3d^3e}-\frac{2e^2\csc^{-1}(cx)}{5c^3d^3\left(e+\frac{d}{x}\right)^3}-\frac{2\left(2cde^2\sqrt{1-\frac{1}{c^2x^2}}-9c^2d^2e\csc^{-1}(cx)+9e^3\csc^{-1}(cx)\right)}{15c^3d^3\left(c^2d^2-e^2\right)\left(e+\frac{d}{x}\right)^2}-\frac{2\left(-16c^3d^3e\sqrt{1-\cdots}\right.}{\cdots}\right)}{(d+ex)^{7/2}}\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2),x]
```

output
```
(-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((4*(-7*c^2*d^2
+ 3*e^2)*Sqrt[1 - 1/(c^2*x^2)])/(15*c^2*d^2*(-(c^2*d^2) + e^2)^2) + (2*Arc
Csc[c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsc[c*x])/(5*c^3*d^3*(e + d/x)^3) - (2
*(2*c*d*e^2*Sqrt[1 - 1/(c^2*x^2)] - 9*c^2*d^2*e*ArcCsc[c*x] + 9*e^3*ArcCsc
[c*x]))/(15*c^3*d^3*(c^2*d^2 - e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[
1 - 1/(c^2*x^2)] + 8*c*d*e^3*Sqrt[1 - 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsc[c*x]
 - 18*c^2*d^2*e^2*ArcCsc[c*x] + 9*e^4*ArcCsc[c*x]))/(15*c^3*d^3*(c^2*d^2 -
 e^2)^2*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*((2
*(c^2*d^2*e - e^3)*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipti
cF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]
*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(3*c^3*d^3 + c*d*e^2)*Sqrt[(c*d + c*e*x)/
(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]],
(2*e)/(c*d + e)])/(Sqrt[1 - 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*(
-7*c^2*d^2*e + 3*e^3)*Cos[2*ArcCsc[c*x]]*((c*d + c*e*x)*(-1 + c^2*x^2) + c
^2*d*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)] - (c*x*(1 + c*x)*Sqrt[(e - c*e*x)/(
c*d + e)]*Sqrt[(c*d + c*e*x)/(c*d - e)]*((c*d + e)*EllipticE[ArcSin[Sqrt[(
c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)] - e*EllipticF[ArcSin[Sqrt[(
c*d + c*e*x)/(c*d - e)]], (c*d - e)/(c*d + e)]))/Sqrt[(e*(1 + c*x))/(-(c*d
) + e)] + c*e*x*Sqrt[(c*d + c*e*x)/(c*d + e)]*Sqrt[1 - c^2*x^2]*Ellipti...
```

3.75.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 516, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {5750, 1898, 635, 633, 632, 186, 413, 412, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{5750}$$

$$-\frac{2b\int \frac{1}{\sqrt{1-\frac{1}{c^2x^2}}x^2(d+ex)^{5/2}}dx}{5ce}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1898}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\int \frac{1}{x(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 635}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 633}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 632}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx+\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{1-cx}\sqrt{cx+1}\sqrt{d+ex}}dx}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 186}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\int \frac{1}{cx\sqrt{cx+1}\sqrt{d+\frac{e}{c}-\frac{e(1-cx)}{c}}}d\sqrt{1-cx}}{d^2\sqrt{x^2-\frac{1}{c^2}}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 413}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\int \frac{1}{cx\sqrt{cx+1}\sqrt{1-\frac{e(1-cx)}{cd+e}}}d\sqrt{1-cx}}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 412}$$

3.75. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 688

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{2\int \frac{e\left(3d\left(2-\frac{e^2}{c^2d^2}\right)-ex\right)}{2d(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2e^2\sqrt{x^2-\frac{1}{c^2}}}{3d\left(d^2-\frac{e^2}{c^2}\right)(d+ex)^{3/2}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 27

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\int \frac{3\left(2d-\frac{e^2}{c^2d}\right)-ex}{(d+ex)^{3/2}\sqrt{x^2-\frac{1}{c^2}}}dx}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}+\frac{2e^2\sqrt{x^2-\frac{1}{c^2}}}{3d\left(d^2-\frac{e^2}{c^2}\right)(d+ex)^{3/2}}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 688

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(-\frac{2\int -\frac{6d^2-\frac{2e^2}{c^2}+e\left(7d-\frac{3e^2}{c^2d}\right)x}{2\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\ldots}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\ldots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 27

3.75. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\int \frac{2\left(3d^2-\frac{e^2}{c^2}\right)+e\left(7d-\frac{3e^2}{c^2d}\right)x}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+\cdots}\cdots\right.}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(1-cx)}{c}+\frac{e}{c}+d}}\cdots\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}\cdots}$$

$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 600}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\left(7d-\frac{3e^2}{c^2d}\right)\int\frac{\sqrt{d+ex}}{\sqrt{x^2-\frac{1}{c^2}}}dx-\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx)}{cd+e}}\,\text{EllipticP}\cdots}{d^2\sqrt{x^2-\frac{1}{c^2}}\sqrt{-\frac{e(\cdots}{}}}\cdots\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}\cdots}$$

$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 509}$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\frac{\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\int\frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-\frac{2\sqrt{1-c^2x^2}\sqrt{1-\frac{e(1-cx\cdots}{cd+e}}\cdots}{d^2\sqrt{x^2\cdots}}\cdots\right.}{5cex\sqrt{1-\frac{1}{c^2x^2}}\cdots}$$

$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 508}$$

3.75. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{-\left(\left(d^2-\frac{e^2}{c^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx\right)-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}\int \frac{\sqrt{1-\frac{e(1-cx)}{cd+e}}}{\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-2\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 327$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{-\left(d^2-\frac{e^2}{c^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{x^2-\frac{1}{c^2}}}dx-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}-2\sqrt{\cdots}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 512$$

3.75. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(-\frac{\sqrt{1-c^2x^2}\left(d^2-\frac{e^2}{c^2}\right)\int\frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx}{\sqrt{x^2-\frac{1}{c^2}}}-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\right)}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{x^2-\frac{1}{c^2}}\left(7c^2d^2-3e^2\right)\cdots}{d\left(c^2d^2-e^2\right)\sqrt{d+ex}}\cdots}{3d\left(d^2-\frac{e^2}{c^2}\right)}\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}\cdots}$$

$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 511$$

$$-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{2\sqrt{1-c^2x^2}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}\int\frac{1}{\sqrt{1-\frac{e(1-cx)}{cd+e}}\sqrt{\frac{1}{2}(cx-1)+1}}d\frac{\sqrt{1-cx}}{\sqrt{2}}}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}\cdots\right)}{d^2-\frac{e^2}{c^2}}\cdots}{3d\left(d^2-\frac{e^2}{c^2}\right)}\cdots\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}\cdots}$$

$$\frac{2\left(a+b\csc^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 321$$

3.75. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2(a+b\csc^{-1}(cx))}{5e(d+ex)^{5/2}}-\frac{2b\sqrt{x^2-\frac{1}{c^2}}\left(-\frac{e\left(\frac{\frac{2\sqrt{1-c^2x^2}\left(d^2-\frac{e^2}{c^2}\right)\sqrt{\frac{c(d+ex)}{cd+e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{d+ex}}-\frac{2\sqrt{1-c^2x^2}\left(7d-\frac{3e^2}{c^2d}\right)\sqrt{d+ex}\,E\left(\arcsin\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)|\frac{2e}{cd+e}\right)}{c\sqrt{x^2-\frac{1}{c^2}}\sqrt{\frac{c(d+ex)}{cd+e}}}}{d^2-\frac{e^2}{c^2}}-\frac{2e\sqrt{}}{d}\right)}{3d\left(d^2-\frac{e^2}{c^2}\right)}\right)}{5cex\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x)^(7/2),x]
```

output
```
(-2*(a + b*ArcCsc[c*x]))/(5*e*(d + e*x)^(5/2)) - (2*b*Sqrt[-c^(-2) + x^2]*
((2*e^2*Sqrt[-c^(-2) + x^2])/(3*d*(d^2 - e^2/c^2)*(d + e*x)^(3/2)) - (e*((
-2*e*(7*c^2*d^2 - 3*e^2)*Sqrt[-c^(-2) + x^2])/(d*(c^2*d^2 - e^2)*Sqrt[d +
e*x]) + ((-2*(7*d - (3*e^2)/(c^2*d))*Sqrt[d + e*x]*Sqrt[1 - c^2*x^2]*Ellip
ticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[(c*(d + e*x)
)/(c*d + e)]*Sqrt[-c^(-2) + x^2]) + (2*(d^2 - e^2/c^2)*Sqrt[(c*(d + e*x))/
(c*d + e)]*Sqrt[1 - c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e
)/(c*d + e)])/(c*Sqrt[d + e*x]*Sqrt[-c^(-2) + x^2]))/(d^2 - e^2/c^2)))/(3*
d*(d^2 - e^2/c^2)) - (2*Sqrt[1 - c^2*x^2]*Sqrt[1 - (e*(1 - c*x))/(c*d + e)
]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(d^2*Sqrt
[-c^(-2) + x^2]*Sqrt[d + e/c - (e*(1 - c*x))/c])))/(5*c*e*Sqrt[1 - 1/(c^2*
x^2)]*x)
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
 - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

3.75. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

3.75. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

rule 5750
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsc[c*x])/(e*(m + 1))), x] + Simp[b/
(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(491) = 982$.

Time = 9.34 (sec) , antiderivative size = 1618, normalized size of antiderivative = 3.00

method	result	size
derivativedivides	Expression too large to display	1618
default	Expression too large to display	1618
parts	Expression too large to display	1642

input
```
int((a+b*arccsc(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsc(c*x)-2/15/c*(7*(c/(
c*d-e))^(1/2)*c^4*d^3*(e*x+d)^3-6*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*
(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((
c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e)
)^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d
-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^4*d^4*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c
*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)
^(1/2)*(c/(c*d-e))^(1/2),1/c*(c*d-e)/d,(c/(c*d+e))^(1/2)/(c/(c*d-e))^(1/2)
)*c^4*d^4*(e*x+d)^(3/2)-13*(c/(c*d-e))^(1/2)*c^4*d^4*(e*x+d)^2-7*((-c*(e*x
+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticF((e*
x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d^3*e*(e*x+d)^(3
/2)+7*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2
)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d+e))^(1/2))*c^3*d
^3*e*(e*x+d)^(3/2)+5*(c/(c*d-e))^(1/2)*c^4*d^5*(e*x+d)-3*(c/(c*d-e))^(1/2)
*c^2*d*e^2*(e*x+d)^3+2*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d
+e)/(c*d+e))^(1/2)*EllipticF((e*x+d)^(1/2)*(c/(c*d-e))^(1/2),((c*d-e)/(c*d
+e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)-3*((-c*(e*x+d)+c*d-e)/(c*d-e))^(1/2)
*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticE((e*x+d)^(1/2)*(c/(c*d-e))^(1
/2),((c*d-e)/(c*d+e))^(1/2))*c^2*d^2*e^2*(e*x+d)^(3/2)+6*((-c*(e*x+d)+c*d-
e)/(c*d-e))^(1/2)*((-c*(e*x+d)+c*d+e)/(c*d+e))^(1/2)*EllipticPi((e*x+d)...
```

3.75.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/(e*x+d)**(7/2),x)
```

output
```
Timed out
```

3.75.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e+c*d>0)', see `assume?` for mor
e details)
```

3.75.8 Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(e*x + d)^(7/2), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(d+e\,x)^{7/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x)^(7/2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.76 $\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

3.76.1 Optimal result

Integrand size = 19, antiderivative size = 206

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(42c^2d+25e)\,x^2\sqrt{-1+c^2x^2}}{560c^5\sqrt{c^2x^2}}+\frac{b(42c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}}+\frac{bex^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{b(42c^2d+25e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{560c^6\sqrt{c^2x^2}}$$

output
```
1/5*d*x^5*(a+b*arccsc(c*x))+1/7*e*x^7*(a+b*arccsc(c*x))+1/560*b*(42*c^2*d+
25*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)+1/560*b*(42*c^2
*d+25*e)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)+1/840*b*(42*c^2*d+25*e)
*x^4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/42*b*e*x^6*(c^2*x^2-1)^(1/2)/
c/(c^2*x^2)^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.68

$$\int x^4\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{48ac^7x^5(7d+5ex^2)+bc^2\sqrt{1-\frac{1}{c^2x^2}}x^2(75e+2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4))+48bc^7x^5(7d+5ex}{1680c^7}$$

input
```
Integrate[x^4*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*Sqrt[1 - 1/(c^2*x^2)]*x^2*(75*e + 2*
c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5
*e*x^2)*ArcCsc[c*x] + 3*b*(42*c^2*d + 25*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)]
)*x])/(1680*c^7)
```

3.76.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5762, 27, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \; 5762$$

$$\frac{bcx\int \frac{x^4(5ex^2+7d)}{35\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{bcx\int \frac{x^4(5ex^2+7d)}{\sqrt{c^2x^2-1}}dx}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 363$$

$$\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\int \frac{x^4}{\sqrt{c^2x^2-1}}dx+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 262$$

$$\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\int \frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 262$$

$$\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 224$$

$$\frac{bcx\left(\frac{1}{6}\left(\frac{25e}{c^2}+42d\right)\left(\frac{3\left(\frac{\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}+\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{5}dx^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\csc^{-1}(cx)\right)+\frac{bcx\left(\frac{1}{6}\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{x^3\sqrt{c^2x^2-1}}{4c^2}\right)\left(\frac{25e}{c^2}+42d\right)+\frac{5ex^5\sqrt{c^2x^2-1}}{6c^2}\right)}{35\sqrt{c^2x^2}}$$

input
```
Int[x^4*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(d*x^5*(a + b*ArcCsc[c*x]))/5 + (e*x^7*(a + b*ArcCsc[c*x]))/7 + (b*c*x*((5
*e*x^5*Sqrt[-1 + c^2*x^2])/(6*c^2) + ((42*d + (25*e)/c^2)*((x^3*Sqrt[-1 +
c^2*x^2])/(4*c^2) + (3*((x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]]/(2*c^3)))/(4*c^2)))/6))/(35*Sqrt[c^2*x^2])
```

3.76. $\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.76.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.59

method	result
parts	$a\left(\frac{1}{7}e\,x^{7}+\frac{1}{5}d\,x^{5}\right)+\frac{b\,\mathrm{arccsc}(cx)e\,x^{7}}{7}+\frac{b\,\mathrm{arccsc}(cx)x^{5}d}{5}+\frac{b(c^{2}x^{2}-1)x^{4}e}{42c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{b(c^{2}x^{2}-1)x^{2}d}{20c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b(c^{2}x^{2}-1)}{168c^{5}\sqrt{\frac{c^{2}x^{2}}{c^{2}}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\,\mathrm{arccsc}(cx)d\,c^{5}x^{5}}{5}+\frac{b\,c^{5}\,\mathrm{arccsc}(cx)e\,x^{7}}{7}+\frac{b\left(c^{2}x^{2}-1\right)c^{2}x^{2}d}{20\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{b\,c^{2}\left(c^{2}x^{2}-1\right)x^{4}e}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{3b\left(c^{2}x^{2}-1\right)d}{40\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\left(c^{2}x^{2}\right.}{168\sqrt{}}}{c^{5}}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\,\mathrm{arccsc}(cx)d\,c^{5}x^{5}}{5}+\frac{b\,c^{5}\,\mathrm{arccsc}(cx)e\,x^{7}}{7}+\frac{b\left(c^{2}x^{2}-1\right)c^{2}x^{2}d}{20\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{b\,c^{2}\left(c^{2}x^{2}-1\right)x^{4}e}{42\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{3b\left(c^{2}x^{2}-1\right)d}{40\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{5b\left(c^{2}x^{2}\right.}{168\sqrt{}}}{c^{5}}$

input
```
int(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e*x^7+1/5*d*x^5)+1/7*b*arccsc(c*x)*e*x^7+1/5*b*arccsc(c*x)*x^5*d+1/
42*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^4*e+1/20*b/c^3*(c^2*x^2
-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*d+5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)
/c^2/x^2)^(1/2)*x^2*e+3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d
+5/112*b/c^7*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e+3/40*b/c^6*(c^2*x^2
-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln(c*x+(c^2*x^2-1)^(1/2))+5/112*
b/c^8*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e*ln(c*x+(c^2*x^2-1)
^(1/2))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.93

$$\int x^4\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7ex^7+336\,ac^7dx^5+48\left(5\,bc^7ex^7+7\,bc^7dx^5-7\,bc^7d-5\,bc^7e\right)\mathrm{arccsc}\left(cx\right)-96\left(7\,bc^7d+5\,bc^7e\right)\mathrm{arct}}{}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(5*b*c^7*e*x^7 + 7*b*c^7*d*
x^5 - 7*b*c^7*d - 5*b*c^7*e)*arccsc(c*x) - 96*(7*b*c^7*d + 5*b*c^7*e)*arct
an(-c*x + sqrt(c^2*x^2 - 1)) - 3*(42*b*c^2*d + 25*b*e)*log(-c*x + sqrt(c^2
*x^2 - 1)) + (40*b*c^5*e*x^5 + 2*(42*b*c^5*d + 25*b*c^3*e)*x^3 + 3*(42*b*c
^3*d + 25*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^7
```

3.76.6 Sympy [A] (verification not implemented)

Time = 10.79 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.98

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{adx^5}{5}+\frac{aex^7}{7}+\frac{bdx^5\operatorname{acsc}(cx)}{5}+\frac{bex^7\operatorname{acsc}(cx)}{7}+\frac{bd\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}+\frac{be\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**4*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

output
```
a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*acsc(c*x)/5 + b*e*x**7*acsc(c*x)/7 + b*
d*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1
)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x*
*2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**
2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), Tr
ue))/(5*c) + b*e*Piecewise((c*x**7/(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sq
rt(c**2*x**2 - 1)) + 5*x**3/(48*c**3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*s
qrt(c**2*x**2 - 1)) + 5*acosh(c*x)/(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x
**7/(6*sqrt(-c**2*x**2 + 1)) - I*x**5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x*
*3/(48*c**3*sqrt(-c**2*x**2 + 1)) + 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) -
 5*I*asin(c*x)/(16*c**6), True))/(7*c)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.44

$$\int x^4 (d + ex^2) \left(a + b \csc^{-1}(cx)\right) dx = \frac{1}{7} aex^7 + \frac{1}{5} adx^5 + \frac{1}{80} \left(16\, x^5 \operatorname{arccsc}(cx) - \frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2 + 2\,c^4\left(\frac{1}{c^2x^2}-1\right) + c^4} - \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c} \right) bd + \frac{1}{672} \left(96\, x^7 \operatorname{arccsc}(cx) + \frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3 + 3\,c^6\left(\frac{1}{c^2x^2}-1\right)^2 + 3\,c^6\left(\frac{1}{c^2x^2}-1\right) + c^6} + \frac{15\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6} - \frac{15\,\log\left(\sqrt{-\frac{1}{c^2 \cdots}\right.}{c^6}}{c} \right.$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2)
 + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(
1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sq
rt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arccsc(c*x) + (2*(15
*(-1/(c^2*x^2) + 1)^(5/2) - 40*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*
x^2) + 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1
/(c^2*x^2) - 1) + c^6) + 15*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(s
qrt(-1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e
```

3.76.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1166 vs. 2(178) = 356.

Time = 1.66 (sec) , antiderivative size = 1166, normalized size of antiderivative = 5.66

$$\int x^4 (d + ex^2) \left(a + b \csc^{-1}(cx)\right) dx = \text{Too large to display}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/13440*(15*b*e*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 15*
a*e*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + 5*b*e*x^6*(sqrt(-1/(c^2*x^2) +
1) + 1)^6/c^2 + 84*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/
c + 84*a*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 105*b*e*x^5*(sqrt(-1/(c^
2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 105*a*e*x^5*(sqrt(-1/(c^2*x^2) +
1) + 1)^5/c^3 + 42*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^2 + 45*b*e*x^4
*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^4 + 420*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^3*arcsin(1/(c*x))/c^3 + 420*a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^
3 + 315*b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^5 + 315*a
*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^5 + 336*b*d*x^2*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^2/c^4 + 225*b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^6 + 840*b*
d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 840*a*d*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)/c^5 + 525*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/
(c*x))/c^7 + 525*a*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^7 + 1008*b*d*log(sqr
t(-1/(c^2*x^2) + 1) + 1)/c^6 - 1008*b*d*log(1/(abs(c)*abs(x)))/c^6 + 600*b
*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^8 - 600*b*e*log(1/(abs(c)*abs(x)))/c^
8 + 840*b*d*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 840*a*d
/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 525*b*e*arcsin(1/(c*x))/(c^9*x*(sq
rt(-1/(c^2*x^2) + 1) + 1)) + 525*a*e/(c^9*x*(sqrt(-1/(c^2*x^2) + 1) + 1))
- 336*b*d/(c^8*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 225*b*e/(c^10*x^2*...
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^4\left(e\,x^2+d\right)\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^4*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^4*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.77 $\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

3.77.1 Optimal result

Integrand size = 19, antiderivative size = 161

$$\begin{aligned}\int x^2\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx&=\frac{b(20c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}}+\frac{bex^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}}\\&\quad+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)\\&\quad+\frac{b(20c^2d+9e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}\end{aligned}$$

output
```
1/3*d*x^3*(a+b*arccsc(c*x))+1/5*e*x^5*(a+b*arccsc(c*x))+1/120*b*(20*c^2*d+
9*e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^4/(c^2*x^2)^(1/2)+1/120*b*(20*c^2*
d+9*e)*x^2*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/20*b*e*x^4*(c^2*x^2-1)^
(1/2)/c/(c^2*x^2)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int x^2\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$=\frac{c^2x^2\left(8ac^3x(5d+3ex^2)+b\sqrt{1-\frac{1}{c^2x^2}}(9e+c^2(20d+6ex^2))\right)+8bc^5x^3(5d+3ex^2)\csc^{-1}(cx)+b(20c^2d+}{120c^5}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) + b*Sqrt[1 - 1/(c^2*x^2)]*(9*e + c^2*(
20*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcCsc[c*x] + b*(20*c^2*d
+ 9*e)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(120*c^5)
```

3.77.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5762, 27, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d+ex^2) (a+b\csc^{-1}(cx)) \, dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int \frac{x^2(3ex^2+5d)}{15\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{3}dx^3(a+b\csc^{-1}(cx))+\frac{1}{5}ex^5(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{x^2(3ex^2+5d)}{\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3(a+b\csc^{-1}(cx))+\frac{1}{5}ex^5(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{363}$$

$$\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\int \frac{x^2}{\sqrt{c^2x^2-1}}dx+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3(a+b\csc^{-1}(cx))+\frac{1}{5}ex^5(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{262}$$

$$\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\left(\frac{\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3(a+b\csc^{-1}(cx))+\frac{1}{5}ex^5(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{224}$$

$$\frac{bcx\left(\frac{1}{4}\left(\frac{9e}{c^2}+20d\right)\left(\frac{\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}dx^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\csc^{-1}(cx)\right)+\frac{bcx\left(\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)\left(\frac{9e}{c^2}+20d\right)+\frac{3ex^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcCsc[c*x]))/3 + (e*x^5*(a + b*ArcCsc[c*x]))/5 + (b*c*x*((3
*e*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + ((20*d + (9*e)/c^2)*((x*Sqrt[-1 + c^2
*x^2])/(2*c^2) + ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]]/(2*c^3)))/4))/(15*Sqrt[
c^2*x^2])
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.77.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.58

method	result
parts	$a\left(\frac{1}{5}e\,x^{5}+\frac{1}{3}d\,x^{3}\right)+\frac{b\,\operatorname{arccsc}(cx)e\,x^{5}}{5}+\frac{b\,\operatorname{arccsc}(cx)x^{3}d}{3}+\frac{b(c^{2}x^{2}-1)x^{2}e}{20c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{b(c^{2}x^{2}-1)d}{6c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{3b(c^{2}x^{2}-1)e}{40c^{5}\sqrt{\frac{c^{2}x^{2}-}{c^{2}x^{2}}}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^{5}x^{3}+\frac{1}{5}e\,c^{5}x^{5}\right)}{c^{2}}+\frac{b\,\operatorname{arccsc}(cx)d\,c^{3}x^{3}}{3}+\frac{b\,c^{3}\operatorname{arccsc}(cx)e\,x^{5}}{5}+\frac{b\left(c^{2}x^{2}-1\right)d}{6\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{b\left(c^{2}x^{2}-1\right)x^{2}e}{20\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{b\sqrt{c^{2}x^{2}-1}\,d\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{6\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{3}}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^{5}x^{3}+\frac{1}{5}e\,c^{5}x^{5}\right)}{c^{2}}+\frac{b\,\operatorname{arccsc}(cx)d\,c^{3}x^{3}}{3}+\frac{b\,c^{3}\operatorname{arccsc}(cx)e\,x^{5}}{5}+\frac{b\left(c^{2}x^{2}-1\right)d}{6\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{b\left(c^{2}x^{2}-1\right)x^{2}e}{20\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}}+\frac{b\sqrt{c^{2}x^{2}-1}\,d\ln\left(cx+\sqrt{c^{2}x^{2}-1}\right)}{6\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}}{c^{3}}$

input
```
int(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*d*x^3)+1/5*b*arccsc(c*x)*e*x^5+1/3*b*arccsc(c*x)*x^3*d+1/
20*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e+1/6*b/c^3*(c^2*x^2-
1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d+3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x
^2)^(1/2)*e+1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d*ln
(c*x+(c^2*x^2-1)^(1/2))+3/40*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/x*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{24\,ac^5ex^5+40\,ac^5dx^3+8\,(3\,bc^5ex^5+5\,bc^5dx^3-5\,bc^5d-3\,bc^5e)\,\mathrm{arccsc}\,(cx)-16\,(5\,bc^5d+3\,bc^5e)\arctan}{120}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3
- 5*b*c^5*d - 3*b*c^5*e)*arccsc(c*x) - 16*(5*b*c^5*d + 3*b*c^5*e)*arctan(-
c*x + sqrt(c^2*x^2 - 1)) - (20*b*c^2*d + 9*b*e)*log(-c*x + sqrt(c^2*x^2 -
1)) + (6*b*c^3*e*x^3 + (20*b*c^3*d + 9*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^5
```

3.77.6 Sympy [A] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.83

$$\begin{aligned}\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx &= \frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\,\mathrm{acsc}\,(cx)}{3}+\frac{bex^5\,\mathrm{acsc}\,(cx)}{5}\\ &+\frac{bd\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}\\ &+\frac{be\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\mathrm{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\mathrm{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}\end{aligned}$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

output
```
a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*acsc(c*x)/3 + b*e*x**5*acsc(c*x)/5 + b*
d*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x
**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2
+ 1)) - I*asin(c*x)/(2*c**2), True))/(3*c) + b*e*Piecewise((c*x**5/(4*sqrt
(c**2*x**2 - 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*
x**2 - 1)) + 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqr
t(-c**2*x**2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqr
t(-c**2*x**2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.44

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{5}\,aex^5+\frac{1}{3}\,adx^3 + \frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\left(cx\right)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd + \frac{1}{80}\left(16\,x^5\,\mathrm{arccsc}\left(cx\right)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2)
 + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2
- log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsc(c*x) -
 (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x
^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2) + 1)
 + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

3.77.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(139) = 278$.

Time = 1.16 (sec) , antiderivative size = 822, normalized size of antiderivative = 5.11

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{960}\left(\frac{6\,bex^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{6\,aex^5\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^5}{c}+\frac{3\,bex^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c^2}+\right.$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/960*(6*b*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 6*a*e*
x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 3*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) +
 1)^4/c^2 + 40*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c +
40*a*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 30*b*e*x^3*(sqrt(-1/(c^2*x^2
) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 30*a*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3/c^3 + 40*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 24*b*e*x^2*(sqrt
(-1/(c^2*x^2) + 1) + 1)^2/c^4 + 120*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arc
sin(1/(c*x))/c^3 + 120*a*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 60*b*e*x*(
sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 60*a*e*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^5 + 160*b*d*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 160*b*d*
log(1/(abs(c)*abs(x)))/c^4 + 72*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 -
72*b*e*log(1/(abs(c)*abs(x)))/c^6 + 120*b*d*arcsin(1/(c*x))/(c^5*x*(sqrt(-
1/(c^2*x^2) + 1) + 1)) + 120*a*d/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60
*b*e*arcsin(1/(c*x))/(c^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*a*e/(c^7*x*
(sqrt(-1/(c^2*x^2) + 1) + 1)) - 40*b*d/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) +
1)^2) - 24*b*e/(c^8*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 40*b*d*arcsin(1/
(c*x))/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 40*a*d/(c^7*x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3) + 30*b*e*arcsin(1/(c*x))/(c^9*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3) + 30*a*e/(c^9*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) - 3*b*e/
(c^10*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 6*b*e*arcsin(1/(c*x))/(c^11...
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.78 $\int (d+ex^2)(a+b\csc^{-1}(cx))\,dx$

3.78.1 Optimal result

Integrand size = 16, antiderivative size = 109

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{bex^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)+\frac{b(6c^2d+e)\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}$$

output
```
d*x*(a+b*arccsc(c*x))+1/3*e*x^3*(a+b*arccsc(c*x))+1/6*b*(6*c^2*d+e)*x*arct
anh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)^(1/2)+1/6*b*e*x^2*(c^2*x^2-1)^(1/
2)/c/(c^2*x^2)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.37

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=adx+\frac{1}{3}aex^3+\frac{bex^2\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{6c}+bdx\csc^{-1}(cx)+\frac{1}{3}bex^3\csc^{-1}(cx)+\frac{bd\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}+\frac{be\log\left(x\left(1+\sqrt{\frac{-1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
a*d*x + (a*e*x^3)/3 + (b*e*x^2*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d
*x*ArcCsc[c*x] + (b*e*x^3*ArcCsc[c*x])/3 + (b*d*Sqrt[1 - 1/(c^2*x^2)]*x*Ar
cTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2] + (b*e*Log[x*(1 + Sqrt
[(-1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

3.78.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5752, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx$$

$$\Big\downarrow \text{5752}$$

$$\frac{bcx\int \frac{ex^2+3d}{3\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{ex^2+3d}{\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{299}$$

$$\frac{bcx\left(\frac{\left(6c^2d+e\right)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{bcx\left(\frac{\left(6c^2d+e\right)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}+dx\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{219}$$

$$dx(a + b\csc^{-1}(cx)) + \frac{1}{3}ex^3(a + b\csc^{-1}(cx)) + \frac{bcx\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(6c^2d+e)}{2c^3} + \frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
d*x*(a + b*ArcCsc[c*x]) + (e*x^3*(a + b*ArcCsc[c*x]))/3 + (b*c*x*((e*x*Sqr
t[-1 + c^2*x^2])/(2*c^2) + ((6*c^2*d + e)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]
])/(2*c^3)))/(3*Sqrt[c^2*x^2])
```

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 5752
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCsc[c*x])   u,
x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.78.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.21

method	result
parts	$a\left(\frac{1}{3}x^3e+dx\right)+\frac{b\left(\frac{c\,\mathrm{arccsc}(cx)x^3e}{3}+\mathrm{arccsc}(cx)dxc+\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2\right.\right.}}{6c^3x\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c}$
derivativedivides	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arccsc}(cx)d\,c^3x+\frac{\mathrm{arccsc}(cx)e\,c^3x^3}{3}+\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)\right.}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^2}}{c}$
default	$\frac{\frac{a\left(c^3dx+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\mathrm{arccsc}(cx)d\,c^3x+\frac{\mathrm{arccsc}(cx)e\,c^3x^3}{3}+\frac{\sqrt{c^2x^2-1}\left(6d\,c^2\ln\left(cx+\sqrt{c^2x^2-1}\right)+ecx\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)\right.}{6cx\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right.}{c^2}}{c}$

input
```
int((e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*x^3*e+d*x)+b/c*(1/3*c*arccsc(c*x)*x^3*e+arccsc(c*x)*d*x*c+1/6/c^3*(
c^2*x^2-1)^(1/2)*(6*d*c^2*ln(c*x+(c^2*x^2-1)^(1/2))+e*c*x*(c^2*x^2-1)^(1/2
)+e*ln(c*x+(c^2*x^2-1)^(1/2)))/x/((c^2*x^2-1)/c^2/x^2)^(1/2))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.29

$$\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3ex^3+6\,ac^3dx+\sqrt{c^2x^2-1}bcex+2\left(bc^3ex^3+3\,bc^3dx-3\,bc^3d-bc^3e\right)\mathrm{arccsc}\left(cx\right)-4\left(3\,bc^3d+bc^3e\right)}{6\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x + sqrt(c^2*x^2 - 1)*b*c*e*x + 2*(b*c^3*e*
x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*arccsc(c*x) - 4*(3*b*c^3*d + b*c^
3*e)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^2*d + b*e)*log(-c*x + sqrt(
c^2*x^2 - 1)))/c^3
```

3.78.6 Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= adx + \frac{aex^3}{3} + bdx\,\mathrm{acsc}\,(cx) + \frac{bex^3\,\mathrm{acsc}\,(cx)}{3} + \frac{bd\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{be\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*acsc(c*x)),x)
```

output
```
a*d*x + a*e*x**3/3 + b*d*x*acsc(c*x) + b*e*x**3*acsc(c*x)/3 + b*d*Piecewis
e((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + b*e*Piecewis
e((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1),
 (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*
asin(c*x)/(2*c**2), True))/(3*c)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{1}{3}aex^3 + \frac{1}{12}\left(4\,x^3\,\mathrm{arccsc}\,(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be + adx + \frac{\left(2\,cx\,\mathrm{arccsc}\,(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd}{2\,c}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/12*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/
(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/
(c^2*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arccsc(c*x) + log(sqr
t(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d/c
```

3.78.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(95) = 190$.

Time = 0.94 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.34

$$\int (d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{24}\left(\frac{bex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{aex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c}+\frac{bex^2\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^2}{c^2}+\frac{12\,bc\right.$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/24*(b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + a*e*x^3*(
sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c
^2 + 12*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 12*a*d*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)/c + 3*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin
(1/(c*x))/c^3 + 3*a*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 24*b*d*log(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^2 - 24*b*d*log(1/(abs(c)*abs(x)))/c^2 + 4*b*e*lo
g(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 4*b*e*log(1/(abs(c)*abs(x)))/c^4 + 12*
b*d*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 12*a*d/(c^3*x*(
sqrt(-1/(c^2*x^2) + 1) + 1)) + 3*b*e*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*
x^2) + 1) + 1)) + 3*a*e/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - b*e/(c^6*x^
2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + b*e*arcsin(1/(c*x))/(c^7*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3) + a*e/(c^7*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))*c
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

output
```
int((d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.79 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2} dx$

3.79.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2} dx = -\frac{bcd\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{x} + ex(a+b\csc^{-1}(cx)) + \frac{bex\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

output
```
-d*(a+b*arccsc(c*x))/x+e*x*(a+b*arccsc(c*x))+b*e*x*arctanh(c*x/(c^2*x^2-1)
^(1/2))/(c^2*x^2)^(1/2)-b*c*d*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2} dx = -\frac{ad}{x} + aex - bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}} - \frac{bd\csc^{-1}(cx)}{x} + bex\csc^{-1}(cx) + \frac{be\sqrt{1-\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{-1+c^2x^2}}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x - b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcCsc[c*
x])/x + b*e*x*ArcCsc[c*x] + (b*e*Sqrt[1 - 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqr
t[-1 + c^2*x^2]])/Sqrt[-1 + c^2*x^2]
```

3.79.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5762, 25, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2}dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int -\frac{d-ex^2}{x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{x}+ex(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{25}$$

$$-\frac{bcx\int \frac{d-ex^2}{x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{x}+ex(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{358}$$

$$-\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x}-e\int \frac{1}{\sqrt{c^2x^2-1}}dx\right)}{\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{x}+ex(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{224}$$

$$-\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x}-e\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{x}+ex(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{219}$$

$$-\frac{d(a+b\csc^{-1}(cx))}{x}+ex(a+b\csc^{-1}(cx))-\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}}{x}-\frac{e\,\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)}{\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcCsc[c*x]))/x) + e*x*(a + b*ArcCsc[c*x]) - (b*c*x*((d*Sqrt[-
1 + c^2*x^2])/x - (e*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/c))/Sqrt[c^2*x^2]
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.79.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

method	result	si
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(-\frac{\operatorname{arccsc}(cx)d}{xc}+\frac{\operatorname{arccsc}(cx)ex}{c}-\frac{\sqrt{c^2x^2-1}\left(d\,c^2\sqrt{c^2x^2-1}-e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^4x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)$	11
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arccsc}(cx)xe-\frac{\operatorname{arccsc}(cx)dc}{x}+\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	12
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^2}+\frac{b\left(c\operatorname{arccsc}(cx)xe-\frac{\operatorname{arccsc}(cx)dc}{x}+\frac{\sqrt{c^2x^2-1}\left(-d\,c^2\sqrt{c^2x^2-1}+e\ln\left(cx+\sqrt{c^2x^2-1}\right)cx\right)}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}\right)}{c^2}\right)$	12

input
```
int((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(e*x-d/x)+b*c*(-arccsc(c*x)*d/x/c+1/c*arccsc(c*x)*e*x-1/c^4*(c^2*x^2-1)^
(1/2)*(d*c^2*(c^2*x^2-1)^(1/2)-e*ln(c*x+(c^2*x^2-1)^(1/2))*c*x)/x^2/((c^2*
x^2-1)/c^2/x^2)^(1/2))
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.44

$$\int\frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx = -\frac{bc^2dx-acex^2+bex\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}bcd+acd-2\left(bcd-bce\right)x\arctan\left(-cx+\sqrt{c\right.}}{cx}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

output
```
-(b*c^2*d*x - a*c*e*x^2 + b*e*x*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x
^2 - 1)*b*c*d + a*c*d - 2*(b*c*d - b*c*e)*x*arctan(-c*x + sqrt(c^2*x^2 - 1
)) - (b*c*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*arccsc(c*x))/(c*x)
```

3.79.6 Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx = -\frac{ad}{x} + aex - bcd\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd\,\operatorname{acsc}(cx)}{x} + bex\,\operatorname{acsc}(cx) + \frac{be\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

input
```
integrate((e*x**2+d)*(a+b*acsc(c*x))/x**2,x)
```

output
```
-a*d/x + a*e*x - b*c*d*sqrt(1 - 1/(c**2*x**2)) - b*d*acsc(c*x)/x + b*e*x*a
csc(c*x) + b*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x),
True))/c
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^2}\,dx = -\left(c\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\operatorname{arccsc}(cx)}{x}\right)bd + aex + \frac{\left(2\,cx\operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be}{2\,c} - \frac{ad}{x}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

output
```
-(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arccs
c(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1
))*b*e/c - a*d/x
```

3.79.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(79) = 158$.

Time = 0.57 (sec) , antiderivative size = 1052, normalized size of antiderivative = 12.09

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

output
```
1/2*(b*e*arcsin(1/(c*x))/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3)) + a*e/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) +
 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3)) - 2*b*c*d/(x*(sqrt(-1/(c^2*x^2)
 + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^
2) + 1) + 1)^3))) + 2*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x*(sqrt(-1/(c
^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/
(c^2*x^2) + 1) + 1)^3))) - 2*b*e*log(1/(abs(c)*abs(x)))/(c*x*(sqrt(-1/(c^2
*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c
^2*x^2) + 1) + 1)^3))) - 4*b*d*arcsin(1/(c*x))/(x^2*(sqrt(-1/(c^2*x^2) + 1
) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^3))) - 4*a*d/(x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1
/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*b*e
*arcsin(1/(c*x))/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c
^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*a*e/(c
^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1))
+ 1/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) + 2*b*d/(c*x^3*(sqrt(-1/(c^2*
x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x^3*(sqrt(-1/(
c^2*x^2) + 1) + 1)^3))) + 2*b*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c^3*x^3*(
sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 1/(c*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3))) - 2*b*e*log(1/(abs(c)*abs(x)))/(c^...
```

3.79.9 Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^2}\,dx = a\,e\,x - \frac{a\,d}{x} + \frac{b\,e\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-\frac{1}{c^2\,x^2}}}\right)}{c} - b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}} - \frac{b\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x} + b\,e\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)$$

input
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^2,x)
```

output
```
a*e*x - (a*d)/x + (b*e*atanh(1/(1 - 1/(c^2*x^2))^(1/2)))/c - b*c*d*(1 - 1/
(c^2*x^2))^(1/2) - (b*d*asin(1/(c*x)))/x + b*e*x*asin(1/(c*x))
```

3.79. $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^2}\,dx$

3.80 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx$

3.80.1 Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{bc(2c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{3x^3} - \frac{e(a+b\csc^{-1}(cx))}{x}$$

output
```
-1/3*d*(a+b*arccsc(c*x))/x^3-e*(a+b*arccsc(c*x))/x-1/9*b*c*(2*c^2*d+9*e)*(
c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/9*b*c*d*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)
^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{3a(d+3ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+9ex^2)+3b(d+3ex^2)\csc^{-1}(cx)}{9x^3}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

output
```
-1/9*(3*a*(d + 3*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 9
*e*x^2) + 3*b*(d + 3*e*x^2)*ArcCsc[c*x])/x^3
```

3.80.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5762, 27, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^4}dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int -\frac{3ex^2+d}{3x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{3ex^2+d}{x^4\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}-\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{359}$$

$$-\frac{bcx\left(\frac{1}{3}(2c^2d+9e)\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{d\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{242}$$

$$-\frac{d\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\csc^{-1}(cx)\right)}{x}-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}(2c^2d+9e)}{3x}+\frac{d\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

output
```
-1/3*(b*c*x*((d*Sqrt[-1 + c^2*x^2])/(3*x^3) + ((2*c^2*d + 9*e)*Sqrt[-1 + c
^2*x^2])/(3*x)))/Sqrt[c^2*x^2] - (d*(a + b*ArcCsc[c*x]))/(3*x^3) - (e*(a +
 b*ArcCsc[c*x]))/x
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.80.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arccsc}(cx)e}{c^3x}-\frac{\operatorname{arccsc}(cx)d}{3x^3c^3}-\frac{(c^2x^2-1)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^4}\right)$	108
derivativedivides	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{3c\,x^3}-\frac{\operatorname{arccsc}(cx)e}{cx}-\frac{(c^2x^2-1)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	121
default	$c^3\left(\frac{a\left(-\frac{d}{3c\,x^3}-\frac{e}{cx}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{3c\,x^3}-\frac{\operatorname{arccsc}(cx)e}{cx}-\frac{(c^2x^2-1)\left(2c^4d\,x^2+9c^2e\,x^2+c^2d\right)}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	121

input
```
int((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arccsc(c*x)*e/x-1/3*arccsc(c*x)*d/x^3/c^3
-1/9/c^6*(c^2*x^2-1)*(2*c^4*d*x^2+9*c^2*e*x^2+c^2*d)/((c^2*x^2-1)/c^2/x^2)
^(1/2)/x^4)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^4}\,dx = -\frac{9\,aex^2+3\,ad+3\,(3\,bex^2+bd)\operatorname{arccsc}(cx)+\sqrt{c^2x^2-1}((2\,bc^2d+9\,be)x^2+bd)}{9\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*arccsc(c*x) + sqrt(c^2*x^2 -
 1)*((2*b*c^2*d + 9*b*e)*x^2 + b*d))/x^3
```

3.80.6 Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.44

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{ad}{3x^3}-\frac{ae}{x}-bce\sqrt{1-\frac{1}{c^2x^2}}-\frac{bd\operatorname{acsc}(cx)}{3x^3}-\frac{be\operatorname{acsc}(cx)}{x} - \frac{bd\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*acsc(c*x))/x**4,x)
```

output
```
-a*d/(3*x**3) - a*e/x - b*c*e*sqrt(1 - 1/(c**2*x**2)) - b*d*acsc(c*x)/(3*x
**3) - b*e*acsc(c*x)/x - b*d*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) +
 c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2
*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$= -\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)be + \frac{1}{9}\,bd\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arccsc}(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

output
```
-(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*e + 1/9*b*d*((c^4*(-1/(c^2*x
^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - a*
e/x - 1/3*a*d/x^3
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$= \frac{1}{9}\left(bc^2d\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,bc^2d\sqrt{-\frac{1}{c^2x^2}+1} - \frac{3\,bcd\left(\frac{1}{c^2x^2}-1\right)\arcsin\left(\frac{1}{cx}\right)}{x} - \frac{3\,bcd\arcsin\left(\frac{1}{cx}\right)}{x} - 9\,be\sqrt{-\right.$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

output
```
1/9*(b*c^2*d*(-1/(c^2*x^2) + 1)^(3/2) - 3*b*c^2*d*sqrt(-1/(c^2*x^2) + 1) -
 3*b*c*d*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 3*b*c*d*arcsin(1/(c*x))/x -
 9*b*e*sqrt(-1/(c^2*x^2) + 1) - 9*b*e*arcsin(1/(c*x))/(c*x) - 9*a*e/(c*x)
- 3*a*d/(c*x^3))*c
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^4, x)
```

3.81 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx$

3.81.1 Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{2bc^3(12c^2d+25e)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{bc(12c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d(a+b\csc^{-1}(cx))}{5x^5} - \frac{e(a+b\csc^{-1}(cx))}{3x^3}$$

output
```
-1/5*d*(a+b*arccsc(c*x))/x^5-1/3*e*(a+b*arccsc(c*x))/x^3-2/225*b*c^3*(12*c
^2*d+25*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/25*b*c*d*(c^2*x^2-1)^(1/2)/
x^4/(c^2*x^2)^(1/2)-1/225*b*c*(12*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x
^2)^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{15a(3d+5ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(25ex^2(1+2c^2x^2)+3d(3+4c^2x^2+8c^4x^4))+15b(3d+5ex^2)\csc^{-1}(cx}{225x^5}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

output
```
-1/225*(15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(25*e*x^2*(1 +
2*c^2*x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) + 15*b*(3*d + 5*e*x^2)*ArcCs
c[c*x])/x^5
```

3.81.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5762, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^6}\,dx$$

$$\Big\downarrow \; 5762$$

$$\frac{bcx\int -\frac{5ex^2+3d}{15x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\int \frac{5ex^2+3d}{x^6\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \; 359$$

$$-\frac{bcx\left(\frac{1}{5}(12c^2d+25e)\int \frac{1}{x^4\sqrt{c^2x^2-1}}dx+\frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \; 245$$

$$-\frac{bcx\left(\frac{1}{5}(12c^2d+25e)\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)+\frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \; 242$$

$$-\frac{d(a+b\csc^{-1}(cx))}{5x^5}-\frac{e(a+b\csc^{-1}(cx))}{3x^3}-\frac{bcx\left(\frac{1}{5}\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)(12c^2d+25e)+\frac{3d\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

output
```
-1/15*(b*c*x*((3*d*Sqrt[-1 + c^2*x^2])/(5*x^5) + ((12*c^2*d + 25*e)*(Sqrt[
-1 + c^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-1 + c^2*x^2])/(3*x)))/5))/Sqrt[c^2*x^
2] - (d*(a + b*ArcCsc[c*x]))/(5*x^5) - (e*(a + b*ArcCsc[c*x]))/(3*x^3)
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.81.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

method	result
parts	$a\left(-\frac{d}{5x^5}-\frac{e}{3x^3}\right)+b\,c^5\left(-\frac{\operatorname{arccsc}(cx)d}{5x^5c^5}-\frac{\operatorname{arccsc}(cx)e}{3c^5x^3}-\frac{(c^2x^2-1)(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9}{225c^8\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^6}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{5c^3x^5}-\frac{\operatorname{arccsc}(cx)e}{3c^3x^3}-\frac{(c^2x^2-1)(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9c^2d)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$
default	$c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{5c^3x^5}-\frac{\operatorname{arccsc}(cx)e}{3c^3x^3}-\frac{(c^2x^2-1)(24c^6d\,x^4+50c^4e\,x^4+12c^4d\,x^2+25c^2e\,x^2+9c^2d)}{225\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$

input
```
int((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/5*d/x^5-1/3*e/x^3)+b*c^5*(-1/5*arccsc(c*x)*d/x^5/c^5-1/3/c^5*arccsc(
c*x)*e/x^3-1/225/c^8*(c^2*x^2-1)*(24*c^6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+2
5*c^2*e*x^2+9*c^2*d)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{75\,aex^2+45\,ad+15\,(5\,bex^2+3\,bd)\,\mathrm{arccsc}\,(cx)+(2\,(12\,bc^4d+25\,bc^2e)x^4+(12\,bc^2d+25\,be)x^2+9\,bd)}{225\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*arccsc(c*x) + (2*(12*
b*c^4*d + 25*b*c^2*e)*x^4 + (12*b*c^2*d + 25*b*e)*x^2 + 9*b*d)*sqrt(c^2*x^
2 - 1))/x^5
```

3.81.6 Sympy [A] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{ad}{5x^5}-\frac{ae}{3x^3}-\frac{bd\,\mathrm{acsc}\,(cx)}{5x^5}-\frac{be\,\mathrm{acsc}\,(cx)}{3x^3} - \frac{bd\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x}+\frac{4c^3\sqrt{c^2x^2-1}}{15x^3}+\frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1\\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x}+\frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3}+\frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c} - \frac{be\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)*(a+b*acsc(c*x))/x**6,x)
```

output
```
-a*d/(5*x**5) - a*e/(3*x**3) - b*d*acsc(c*x)/(5*x**5) - b*e*acsc(c*x)/(3*x
**3) - b*d*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2
*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1)
, (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(1
5*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - b*e*Piecewise(
(2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c*
*2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2
+ 1)/(3*x**3), True))/(3*c)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{1}{75}bd\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right) + \frac{1}{9}be\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right)-\frac{ae}{3x^3}-\frac{ad}{5x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/75*b*d*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/
2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 1/9*b*e*((c^
4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*
x)/x^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{1}{225}\left(9bc^4d\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-30bc^4d\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45bc^3d\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+45\right.$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/225*(9*b*c^4*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 30*b*c^4*d*
(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/
x + 45*b*c^4*d*sqrt(-1/(c^2*x^2) + 1) + 90*b*c^3*d*(1/(c^2*x^2) - 1)*arcsi
n(1/(c*x))/x - 25*b*c^2*e*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d*arcsin(1/(
c*x))/x + 75*b*c^2*e*sqrt(-1/(c^2*x^2) + 1) + 75*b*c*e*(1/(c^2*x^2) - 1)*a
rcsin(1/(c*x))/x + 75*b*c*e*arcsin(1/(c*x))/x + 75*a*e/(c*x^3) + 45*a*d/(c
*x^5))*c
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^6, x)
```

3.82 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx$

3.82.1 Optimal result

Integrand size = 19, antiderivative size = 197

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx = &-\frac{8bc^5(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675\sqrt{c^2x^2}}\\ &-\frac{bcd\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}}-\frac{bc(30c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}}\\ &-\frac{4bc^3(30c^2d+49e)\sqrt{-1+c^2x^2}}{3675x^2\sqrt{c^2x^2}}\\ &-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}\end{aligned}$$

output
```
-1/7*d*(a+b*arccsc(c*x))/x^7-1/5*e*(a+b*arccsc(c*x))/x^5-8/3675*b*c^5*(30*
c^2*d+49*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/49*b*c*d*(c^2*x^2-1)^(1/2)
/x^6/(c^2*x^2)^(1/2)-1/1225*b*c*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2
*x^2)^(1/2)-4/3675*b*c^3*(30*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(
1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{105a(5d+7ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(49ex^2(3+4c^2x^2+8c^4x^4)+15d(5+6c^2x^2+8c^4x^4+16c^6x^6))+105}{3675x^7}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

output
```
-1/3675*(105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(49*e*x^2*(3
+ 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6))
+ 105*b*(5*d + 7*e*x^2)*ArcCsc[c*x])/x^7
```

3.82.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5762, 27, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^8}\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int -\frac{7ex^2+5d}{35x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{7ex^2+5d}{x^8\sqrt{c^2x^2-1}}dx}{35\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{359}$$

$$-\frac{bcx\left(\frac{1}{7}(30c^2d+49e)\int \frac{1}{x^6\sqrt{c^2x^2-1}}dx+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{245}$$

$$-\frac{bcx\left(\frac{1}{7}(30c^2d+49e)\left(\frac{4}{5}c^2\int\frac{1}{x^4\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{ 245}$$

$$-\frac{bcx\left(\frac{1}{7}(30c^2d+49e)\left(\frac{4}{5}c^2\left(\frac{2}{3}c^2\int\frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)+\frac{\sqrt{c^2x^2-1}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{d(a+b\csc^{-1}(cx))}{7x^7}-\frac{e(a+b\csc^{-1}(cx))}{5x^5}-\frac{bcx\left(\frac{1}{7}\left(\frac{\sqrt{c^2x^2-1}}{5x^5}+\frac{4}{5}c^2\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)\right)(30c^2d+49e)+\frac{5d\sqrt{c^2x^2-1}}{7x^7}\right)}{35\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

output
```
-1/35*(b*c*x*((5*d*Sqrt[-1 + c^2*x^2])/(7*x^7) + ((30*c^2*d + 49*e)*(Sqrt[
-1 + c^2*x^2]/(5*x^5) + (4*c^2*(Sqrt[-1 + c^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-
1 + c^2*x^2])/(3*x)))/5))/7))/Sqrt[c^2*x^2] - (d*(a + b*ArcCsc[c*x]))/(7*x
^7) - (e*(a + b*ArcCsc[c*x]))/(5*x^5)
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.82.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.74

method	result
parts	$a\left(-\frac{d}{7x^7}-\frac{e}{5x^5}\right)+b\,c^7\left(-\frac{\operatorname{arccsc}(cx)d}{7x^7c^7}-\frac{\operatorname{arccsc}(cx)e}{5c^7x^5}-\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e}{3675c^{10}\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^8}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsc}(cx)e}{5c^5x^5}-\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsc}(cx)e}{5c^5x^5}-\frac{(c^2x^2-1)(240c^8d\,x^6+392c^6e\,x^6+120c^6d\,x^4+196c^4e\,x^4+90c^4d\,x}{3675\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$

input
```
int((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7*d/x^7-1/5*e/x^5)+b*c^7*(-1/7*arccsc(c*x)*d/x^7/c^7-1/5/c^7*arccsc(
c*x)*e/x^5-1/3675/c^10*(c^2*x^2-1)*(240*c^8*d*x^6+392*c^6*e*x^6+120*c^6*d*
x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^2+75*c^2*d)/((c^2*x^2-1)/c^2/x^
2)^(1/2)/x^8)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.55

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{735\,aex^2 + 525\,ad + 105\,(7\,bex^2 + 5\,bd)\,\mathrm{arccsc}\,(cx) + (8\,(30\,bc^6d + 49\,bc^4e)x^6 + 4\,(30\,bc^4d + 49\,bc^2e)x^4}{3675\,x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*arccsc(c*x) + (8*
(30*b*c^6*d + 49*b*c^4*e)*x^6 + 4*(30*b*c^4*d + 49*b*c^2*e)*x^4 + 3*(30*b*
c^2*d + 49*b*e)*x^2 + 75*b*d)*sqrt(c^2*x^2 - 1))/x^7
```

3.82.6 Sympy [A] (verification not implemented)

Time = 29.20 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx$$
$$= -\frac{ad}{7x^7} - \frac{ae}{5x^5} - \frac{bd\,\mathrm{acsc}\,(cx)}{7x^7} - \frac{be\,\mathrm{acsc}\,(cx)}{5x^5} - \frac{bd\left(\begin{cases} \frac{16c^7\sqrt{c^2x^2-1}}{35x} + \frac{8c^5\sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3\sqrt{c^2x^2-1}}{35x^5} + \frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for}\ |c^2x^2| > 1 \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5} + \frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise} \end{cases}\right)}{7c} - \frac{be\left(\begin{cases} \frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for}\ |c^2x^2| > 1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise} \end{cases}\right)}{5c}$$

input
```
integrate((e*x**2+d)*(a+b*acsc(c*x))/x**8,x)
```

output
```
-a*d/(7*x**7) - a*e/(5*x**5) - b*d*acsc(c*x)/(7*x**7) - b*e*acsc(c*x)/(5*x
**5) - b*d*Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**
2*x**2 - 1)/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2
*x**2 - 1)/(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/
(35*x) + 8*I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**
2 + 1)/(35*x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) - b*e*P
iecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(
15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*
sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*
c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx$$
$$= \frac{1}{245}bd\left(\frac{5c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\mathrm{arccsc}\,(cx)}{x^7}\right.$$
$$\left.-\frac{1}{75}be\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right)\right.$$
$$-\frac{ae}{5x^5}-\frac{ad}{7x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/245*b*d*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(5/
2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c -
35*arccsc(c*x)/x^7) - 1/75*b*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(
-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc(c*x
)/x^5) - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(169) = 338.

Time = 0.29 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{1}{3675}\left(75\,bc^6d\left(\frac{1}{c^2x^2}-1\right)^3\sqrt{-\frac{1}{c^2x^2}+1}+315\,bc^6d\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}+\frac{525\,bc^5d\left(\frac{1}{c^2x^2}-1\right)^3 \mathrm{a}}{x}\right.$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

output
```
-1/3675*(75*b*c^6*d*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) + 315*b*c^6
*d*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 525*b*c^5*d*(1/(c^2*x^2) -
 1)^3*arcsin(1/(c*x))/x - 525*b*c^6*d*(-1/(c^2*x^2) + 1)^(3/2) + 1575*b*c^
5*d*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 525*b*c^6*d*sqrt(-1/(c^2*x^2)
+ 1) + 147*b*c^4*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 1575*b*c^5
*d*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 490*b*c^4*e*(-1/(c^2*x^2) + 1)^(3
/2) + 525*b*c^5*d*arcsin(1/(c*x))/x + 735*b*c^3*e*(1/(c^2*x^2) - 1)^2*arcs
in(1/(c*x))/x + 735*b*c^4*e*sqrt(-1/(c^2*x^2) + 1) + 1470*b*c^3*e*(1/(c^2*
x^2) - 1)*arcsin(1/(c*x))/x + 735*b*c^3*e*arcsin(1/(c*x))/x + 735*a*e/(c*x
^5) + 525*a*d/(c*x^7))*c
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^8, x)
```

3.83 $\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

3.83.1 Optimal result

Integrand size = 19, antiderivative size = 196

$$\int x^5\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(4c^2d+3e)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(8c^2d+9e)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}+\frac{b(4c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)$$

output
```
1/6*d*x^6*(a+b*arccsc(c*x))+1/8*e*x^8*(a+b*arccsc(c*x))+1/72*b*(8*c^2*d+9*
e)*x*(c^2*x^2-1)^(3/2)/c^7/(c^2*x^2)^(1/2)+1/120*b*(4*c^2*d+9*e)*x*(c^2*x^
2-1)^(5/2)/c^7/(c^2*x^2)^(1/2)+1/56*b*e*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^
(1/2)+1/24*b*(4*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.59

$$\int x^5(d+ex^2)(a+b\csc^{-1}(cx))\,dx$$
$$=\frac{x\left(105ax^5(4d+3ex^2)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}(144e+8c^2(28d+9ex^2)+2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6))}{c^7}+105bx^5(4d+3ex^2)\operatorname{cs}\right.}{2520}$$

input
```
Integrate[x^5*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(x*(105*a*x^5*(4*d + 3*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(144*e + 8*c^2*(2
8*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))
/c^7 + 105*b*x^5*(4*d + 3*e*x^2)*ArcCsc[c*x]))/2520
```

3.83.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5762, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex^2)(a+b\csc^{-1}(cx))\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int \frac{x^5(3ex^2+4d)}{24\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{6}dx^6(a+b\csc^{-1}(cx))+\frac{1}{8}ex^8(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{x^5(3ex^2+4d)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{c^2x^2}}+\frac{1}{6}dx^6(a+b\csc^{-1}(cx))+\frac{1}{8}ex^8(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{x^4(3ex^2+4d)}{\sqrt{c^2x^2-1}}dx^2}{48\sqrt{c^2x^2}}+\frac{1}{6}dx^6(a+b\csc^{-1}(cx))+\frac{1}{8}ex^8(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \ 86$$

$$\frac{bcx\int\left(\frac{3e(c^2x^2-1)^{5/2}}{c^6}+\frac{(4dc^2+9e)(c^2x^2-1)^{3/2}}{c^6}+\frac{(8dc^2+9e)\sqrt{c^2x^2-1}}{c^6}+\frac{4dc^2+3e}{c^6\sqrt{c^2x^2-1}}\right)dx^2}{48\sqrt{c^2x^2}}+\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{6}dx^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\csc^{-1}(cx)\right)+\frac{bcx\left(\frac{2(c^2x^2-1)^{5/2}(4c^2d+9e)}{5c^8}+\frac{2(c^2x^2-1)^{3/2}(8c^2d+9e)}{3c^8}+\frac{2\sqrt{c^2x^2-1}(4c^2d+3e)}{c^8}+\frac{6e(c^2x^2-1)^{7/2}}{7c^8}\right)}{48\sqrt{c^2x^2}}$$

input
```
Int[x^5*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(b*c*x*((2*(4*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2])/c^8 + (2*(8*c^2*d + 9*e)*(-
1 + c^2*x^2)^(3/2))/(3*c^8) + (2*(4*c^2*d + 9*e)*(-1 + c^2*x^2)^(5/2))/(5*
c^8) + (6*e*(-1 + c^2*x^2)^(7/2))/(7*c^8)))/(48*Sqrt[c^2*x^2]) + (d*x^6*(a
 + b*ArcCsc[c*x]))/6 + (e*x^8*(a + b*ArcCsc[c*x]))/8
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.83.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

method	result
parts	$a\left(\frac{1}{8}e\,x^{8}+\frac{1}{6}x^{6}d\right)+\frac{b\left(\frac{c^{6}\operatorname{arccsc}(cx)e\,x^{8}}{8}+\frac{\operatorname{arccsc}(cx)d\,x^{6}c^{6}}{6}+\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72c^{2}e \cdots\right)}{2520c^{3}\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,x}\cdots\right)}{c^{6}}\cdots$
derivativedivides	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccsc}(cx)d\,c^{8}x^{6}}{6}+\frac{\operatorname{arccsc}(cx)e\,c^{8}x^{8}}{8}+\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72c^{2}e\,x^{2}+2\cdots\right)}{2520\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\cdots\right)}{c^{2}}\cdots}{c^{6}}$
default	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccsc}(cx)d\,c^{8}x^{6}}{6}+\frac{\operatorname{arccsc}(cx)e\,c^{8}x^{8}}{8}+\frac{\left(c^{2}x^{2}-1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}+54c^{4}e\,x^{4}+112c^{4}d\,x^{2}+72c^{2}e\,x^{2}+2\cdots\right)}{2520\sqrt{\frac{c^{2}x^{2}-1}{c^{2}x^{2}}}\,cx}\cdots\right)}{c^{2}}\cdots}{c^{6}}$

input
```
int(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e*x^8+1/6*x^6*d)+b/c^6*(1/8*c^6*arccsc(c*x)*e*x^8+1/6*arccsc(c*x)*d
*x^6*c^6+1/2520/c^3*(c^2*x^2-1)*(45*c^6*e*x^6+84*c^6*d*x^4+54*c^4*e*x^4+11
2*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^8ex^8+420\,ac^8dx^6+105\,(3\,bc^8ex^8+4\,bc^8dx^6)\,\mathrm{arccsc}\,(cx)+(45\,bc^6ex^6+6\,(14\,bc^6d+9\,bc^4e)x^4+22}{2520\,c^8}$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^8*e*x^8 + 420*a*c^8*d*x^6 + 105*(3*b*c^8*e*x^8 + 4*b*c^8*d
*x^6)*arccsc(c*x) + (45*b*c^6*e*x^6 + 6*(14*b*c^6*d + 9*b*c^4*e)*x^4 + 224
*b*c^2*d + 8*(14*b*c^4*d + 9*b*c^2*e)*x^2 + 144*b*e)*sqrt(c^2*x^2 - 1))/c^
8
```

3.83.6 Sympy [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.86

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{adx^6}{6}+\frac{aex^8}{8}+\frac{bdx^6\,\mathrm{acsc}\,(cx)}{6}+\frac{bex^8\,\mathrm{acsc}\,(cx)}{8}$$
$$+\frac{bd\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$
$$+\frac{be\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

input
```
integrate(x**5*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

output
```
a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*acsc(c*x)/6 + b*e*x**8*acsc(c*x)/8 + b*
d*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(
15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*s
qrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*
sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(6*c) + b*e*Piecewise((x**6*sqrt(c*
*2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/(35*c**3) + 8*x**2*sqrt(c*
*2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/(35*c**7), Abs(c**2*x**2)
> 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x**4*sqrt(-c**2*x**2 + 1)/(
35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**5) + 16*I*sqrt(-c**2*x**2
+ 1)/(35*c**7), True))/(8*c)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{8}aex^8 + \frac{1}{6}adx^6 + \frac{1}{90}\left(15\,x^6\,\mathrm{arccsc}\,(cx) + \frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bd + \frac{1}{280}\left(35\,x^8\,\mathrm{arccsc}\,(cx) + \frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} + 21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 35\,x\sqrt{-}}{c^7}\right.$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2
*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2
*x^2) + 1))/c^5)*b*d + 1/280*(35*x^8*arccsc(c*x) + (5*c^6*x^7*(-1/(c^2*x^2
) + 1)^(7/2) + 21*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x
^2) + 1)^(3/2) + 35*x*sqrt(-1/(c^2*x^2) + 1))/c^7)*b*e
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(168) = 336$.

Time = 0.43 (sec) , antiderivative size = 1244, normalized size of antiderivative = 6.35

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/645120*(315*b*e*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*arcsin(1/(c*x))/c + 3
15*a*e*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8/c + 90*b*e*x^7*(sqrt(-1/(c^2*x^2
) + 1) + 1)^7/c^2 + 1680*b*d*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(
c*x))/c + 1680*a*d*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 2520*b*e*x^6*(sq
rt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c^3 + 2520*a*e*x^6*(sqrt(-1/(c
^2*x^2) + 1) + 1)^6/c^3 + 672*b*d*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^2 +
 882*b*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^4 + 10080*b*d*x^4*(sqrt(-1/(
c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 10080*a*d*x^4*(sqrt(-1/(c^2*x^2
) + 1) + 1)^4/c^3 + 8820*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(
c*x))/c^5 + 8820*a*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^5 + 5600*b*d*x^3
*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^4 + 4410*b*e*x^3*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^3/c^6 + 25200*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))
/c^5 + 25200*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 17640*b*e*x^2*(s
qrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^7 + 17640*a*e*x^2*(sqrt(-1/
(c^2*x^2) + 1) + 1)^2/c^7 + 33600*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 +
 22050*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^8 + 33600*b*d*arcsin(1/(c*x))/
c^7 + 33600*a*d/c^7 + 22050*b*e*arcsin(1/(c*x))/c^9 + 22050*a*e/c^9 - 3360
0*b*d/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 22050*b*e/(c^10*x*(sqrt(-1/(c
^2*x^2) + 1) + 1)) + 25200*b*d*arcsin(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^2*x^2)
 + 1) + 1)^2) + 25200*a*d/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 17...
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int x^5(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx = \int x^5\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.84 $\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$

3.84.1 Optimal result

Integrand size = 19, antiderivative size = 153

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(3c^2d+2e)\,x\sqrt{-1+c^2x^2}}{12c^5\sqrt{c^2x^2}}+\frac{b(3c^2d+4e)\,x(-1+c^2x^2)^{3/2}}{36c^5\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)$$

output
```
1/4*d*x^4*(a+b*arccsc(c*x))+1/6*e*x^6*(a+b*arccsc(c*x))+1/36*b*(3*c^2*d+4*
e)*x*(c^2*x^2-1)^(3/2)/c^5/(c^2*x^2)^(1/2)+1/30*b*e*x*(c^2*x^2-1)^(5/2)/c^
5/(c^2*x^2)^(1/2)+1/12*b*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(
1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{1}{180}x\left(15ax^3\left(3d+2ex^2\right)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}\left(16e+c^2(30d+8ex^2)+3c^4(5dx^2+2ex^4)\right)}{c^5}+15bx^3\left(3d+2ex^2\right)\csc^{-1}(cx)\right)$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 - 1/(c^2*x^2)]*(16*e + c^2*(30*d
+ 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*Ar
cCsc[c*x]))/180
```

3.84.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5762, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int \frac{x^3(2ex^2+3d)}{12\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{x^3(2ex^2+3d)}{\sqrt{c^2x^2-1}}dx}{12\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{x^2(2ex^2+3d)}{\sqrt{c^2x^2-1}}dx^2}{24\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{86}$$

$$\frac{bcx\int \left(\frac{2e\left(c^2x^2-1\right)^{3/2}}{c^4}+\frac{(3dc^2+4e)\sqrt{c^2x^2-1}}{c^4}+\frac{3dc^2+2e}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{24\sqrt{c^2x^2}}+\frac{1}{4}dx^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}dx^4(a+b\csc^{-1}(cx))+\frac{1}{6}ex^6(a+b\csc^{-1}(cx))+\frac{bcx\left(\frac{2(c^2x^2-1)^{3/2}(3c^2d+4e)}{3c^6}+\frac{2\sqrt{c^2x^2-1}(3c^2d+2e)}{c^6}+\frac{4e(c^2x^2-1)^{5/2}}{5c^6}\right)}{24\sqrt{c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(b*c*x*((2*(3*c^2*d + 2*e)*Sqrt[-1 + c^2*x^2])/c^6 + (2*(3*c^2*d + 4*e)*(-
1 + c^2*x^2)^(3/2))/(3*c^6) + (4*e*(-1 + c^2*x^2)^(5/2))/(5*c^6)))/(24*Sqr
t[c^2*x^2]) + (d*x^4*(a + b*ArcCsc[c*x]))/4 + (e*x^6*(a + b*ArcCsc[c*x]))/
6
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.84.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

method	result
parts	$a\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b\left(\frac{c^4\,\operatorname{arccsc}(cx)e\,x^6}{6}+\frac{\operatorname{arccsc}(cx)c^4d\,x^4}{4}+\frac{(c^2x^2-1)(6c^4e\,x^4+15c^4d\,x^2+8c^2e\,x^2+30c^2d+16e)}{180c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}\right)}{c^4}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d(c^2e\,x^2+c^2d)^2}{2}-\frac{(c^2e\,x^2+c^2d)^3}{3}\right)}{2c^2e^2}-\frac{b\,c^4\operatorname{arccsc}(cx)d^3}{12e^2}+\frac{b\operatorname{arccsc}(cx)d\,c^4x^4}{4}+\frac{b\,c^4e\operatorname{arccsc}(cx)x^6}{6}+\frac{b\,c^3\sqrt{c^2x^2-1}\,d^3\arctan}{12e^2\sqrt{\frac{c^2x^2-}{c^2x^2}}}}{c^4}$
default	$\frac{-\frac{a\left(\frac{c^2d(c^2e\,x^2+c^2d)^2}{2}-\frac{(c^2e\,x^2+c^2d)^3}{3}\right)}{2c^2e^2}-\frac{b\,c^4\operatorname{arccsc}(cx)d^3}{12e^2}+\frac{b\operatorname{arccsc}(cx)d\,c^4x^4}{4}+\frac{b\,c^4e\operatorname{arccsc}(cx)x^6}{6}+\frac{b\,c^3\sqrt{c^2x^2-1}\,d^3\arctan}{12e^2\sqrt{\frac{c^2x^2-}{c^2x^2}}}}{c^4}$

input
```
int(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arccsc(c*x)*e*x^6+1/4*arccsc(c*x)*c
^4*d*x^4+1/180/c^3*(c^2*x^2-1)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^2*e*x^2+30*c^
2*d+16*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.69

$$\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$=\frac{30\,ac^6ex^6+45\,ac^6dx^4+15\,(2\,bc^6ex^6+3\,bc^6dx^4)\,\operatorname{arccsc}(cx)+(6\,bc^4ex^4+30\,bc^2d+(15\,bc^4d+8\,bc^2e)x^2+}{180\,c^6}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*x^4
)*arccsc(c*x) + (6*b*c^4*e*x^4 + 30*b*c^2*d + (15*b*c^4*d + 8*b*c^2*e)*x^2
 + 16*b*e)*sqrt(c^2*x^2 - 1))/c^6
```

3.84.6 Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.78

$$\int x^3 (d+ex^2) \left(a+b\csc^{-1}(cx)\right) dx$$
$$= \frac{adx^4}{4} + \frac{aex^6}{6} + \frac{bdx^4 \operatorname{acsc}(cx)}{4} + \frac{bex^6 \operatorname{acsc}(cx)}{6} + \frac{bd\left(\begin{cases} \frac{x^2\sqrt{c^2x^2-1}}{3c} + \frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2| > 1 \\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c} + \frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise} \end{cases}\right)}{4c} + \frac{be\left(\begin{cases} \frac{x^4\sqrt{c^2x^2-1}}{5c} + \frac{4x^2\sqrt{c^2x^2-1}}{15c^3} + \frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2| > 1 \\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c} + \frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3} + \frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise} \end{cases}\right)}{6c}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

output
```
a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*acsc(c*x)/4 + b*e*x**6*acsc(c*x)/6 + b*
d*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**
3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c*
*2*x**2 + 1)/(3*c**3), True))/(4*c) + b*e*Piecewise((x**4*sqrt(c**2*x**2 -
 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(
15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x*
*2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), Tr
ue))/(6*c)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

$$\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3\,x^4\operatorname{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd$$
$$+\frac{1}{90}\left(15\,x^6\operatorname{arccsc}(cx)+\frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arccsc
(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^2) + 1
)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(131) = 262.

Time = 0.34 (sec) , antiderivative size = 900, normalized size of antiderivative = 5.88

$$\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/5760*(15*b*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15*a
*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*e*x^5*(sqrt(-1/(c^2*x^2) + 1
) + 1)^5/c^2 + 90*b*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c
 + 90*a*d*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 90*b*e*x^4*(sqrt(-1/(c^2*
x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*e*x^4*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^4/c^3 + 60*b*d*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 50*b*e*x^3*(s
qrt(-1/(c^2*x^2) + 1) + 1)^3/c^4 + 360*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1
)^2*arcsin(1/(c*x))/c^3 + 360*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 +
 225*b*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 225*a*e*
x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^5 + 540*b*d*x*(sqrt(-1/(c^2*x^2) + 1)
 + 1)/c^4 + 300*b*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 + 540*b*d*arcsin(1/
(c*x))/c^5 + 540*a*d/c^5 + 300*b*e*arcsin(1/(c*x))/c^7 + 300*a*e/c^7 - 540
*b*d/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) - 300*b*e/(c^8*x*(sqrt(-1/(c^2*x
^2) + 1) + 1)) + 360*b*d*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^2) + 360*a*d/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*b*e*arcsi
n(1/(c*x))/(c^9*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 225*a*e/(c^9*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2) - 60*b*d/(c^8*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1
)^3) - 50*b*e/(c^10*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 90*b*d*arcsin(1/
(c*x))/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4) + 90*a*d/(c^9*x^4*(sqrt(-1
/(c^2*x^2) + 1) + 1)^4) + 90*b*e*arcsin(1/(c*x))/(c^11*x^4*(sqrt(-1/(c^...
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^3\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.85 $\int x(d+ex^2)\,(a+b\csc^{-1}(cx))\,dx$

3.85.1 Optimal result

Integrand size = 17, antiderivative size = 138

$$\int x(d+ex^2)\,(a+b\csc^{-1}(cx))\,dx=\frac{b(2c^2d+e)\,x\sqrt{-1+c^2x^2}}{4c^3\sqrt{c^2x^2}}+\frac{bex(-1+c^2x^2)^{3/2}}{12c^3\sqrt{c^2x^2}}+\frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{4e}+\frac{bcd^2x\arctan\left(\sqrt{-1+c^2x^2}\right)}{4e\sqrt{c^2x^2}}$$

output
```
1/4*(e*x^2+d)^2*(a+b*arccsc(c*x))/e+1/12*b*e*x*(c^2*x^2-1)^(3/2)/c^3/(c^2*
x^2)^(1/2)+1/4*b*c*d^2*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/4*b
*(2*c^2*d+e)*x*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int x(d+ex^2)\,(a+b\csc^{-1}(cx))\,dx=\frac{x\left(3ac^3x(2d+ex^2)+b\sqrt{1-\frac{1}{c^2x^2}}(2e+c^2(6d+ex^2))+3bc^3x(2d+ex^2)\csc^{-1}(cx)\right)}{12c^3}$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(x*(3*a*c^3*x*(2*d + e*x^2) + b*Sqrt[1 - 1/(c^2*x^2)]*(2*e + c^2*(6*d + e*
x^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcCsc[c*x]))/(12*c^3)
```

3.85.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5760, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5760}$$

$$\frac{bcx\int \frac{\left(ex^2+d\right)^2}{x\sqrt{c^2x^2-1}}dx}{4e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{\left(ex^2+d\right)^2}{x^2\sqrt{c^2x^2-1}}dx^2}{8e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{99}$$

$$\frac{bcx\int \left(\frac{d^2}{x^2\sqrt{c^2x^2-1}}+\frac{e^2\sqrt{c^2x^2-1}}{c^2}+\frac{e\left(2dc^2+e\right)}{c^2\sqrt{c^2x^2-1}}\right)dx^2}{8e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{4e}+\frac{bcx\left(2d^2\arctan\left(\sqrt{c^2x^2-1}\right)+\frac{2e\sqrt{c^2x^2-1}\left(2c^2d+e\right)}{c^4}+\frac{2e^2\left(c^2x^2-1\right)^{3/2}}{3c^4}\right)}{8e\sqrt{c^2x^2}}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/(4*e) + (b*c*x*((2*e*(2*c^2*d + e)*Sqr
t[-1 + c^2*x^2])/c^4 + (2*e^2*(-1 + c^2*x^2)^(3/2))/(3*c^4) + 2*d^2*ArcTan
[Sqrt[-1 + c^2*x^2]]))/(8*e*Sqrt[c^2*x^2])
```

3.85.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.85.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.57

method	result
parts	$\frac{a(e\,x^2+d)^2}{4e}+\frac{b\,\operatorname{arccsc}(cx)e\,x^4}{4}+\frac{b\,\operatorname{arccsc}(cx)x^2d}{2}+\frac{b\,d^2\operatorname{arccsc}(cx)}{4e}+\frac{b(c^2x^2-1)xe}{12c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b\sqrt{c^2x^2-1}\,d^2\arctan\left(\cdots\right.}{4ce\sqrt{\frac{c^2x^2-1}{c^2x^2}}\cdots}$
derivativedivides	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^2}{4c^2e}+\frac{b\,c^2\operatorname{arccsc}(cx)d^2}{4e}+\frac{b\,\operatorname{arccsc}(cx)d\,c^2x^2}{2}+\frac{b\,c^2e\,\operatorname{arccsc}(cx)x^4}{4}-\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{b\left(c^2x^2-1\right)\cdots}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\cdots}}{c^2}$
default	$\frac{\frac{a\left(c^2e\,x^2+c^2d\right)^2}{4c^2e}+\frac{b\,c^2\operatorname{arccsc}(cx)d^2}{4e}+\frac{b\,\operatorname{arccsc}(cx)d\,c^2x^2}{2}+\frac{b\,c^2e\,\operatorname{arccsc}(cx)x^4}{4}-\frac{bc\sqrt{c^2x^2-1}\,d^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{b\left(c^2x^2-1\right)\cdots}{2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\cdots}}{c^2}$

input
```
int(x*(e*x^2+d)*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x^2+d)^2/e+1/4*b*arccsc(c*x)*e*x^4+1/2*b*arccsc(c*x)*x^2*d+1/4*b*
d^2*arccsc(c*x)/e+1/12*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x*e-1
/4*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2*arctan(1/(c^2
*x^2-1)^(1/2))+1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d+1/6*b
/c^5*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^4ex^4+6\,ac^4dx^2+3\,(bc^4ex^4+2\,bc^4dx^2)\,\mathrm{arccsc}\,(cx)+(bc^2ex^2+6\,bc^2d+2\,be)\sqrt{c^2x^2-1}}{12\,c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2)*arcc
sc(c*x) + (b*c^2*e*x^2 + 6*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))/c^4
```

3.85.6 Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\,\mathrm{acsc}\,(cx)}{2}+\frac{bex^4\,\mathrm{acsc}\,(cx)}{4}+\frac{bd\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}+\frac{be\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c}$$

input
```
integrate(x*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

output
```
a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*acsc(c*x)/2 + b*e*x**4*acsc(c*x)/4 + b*
d*Piecewise((sqrt(c**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**
2 + 1)/c, True))/(2*c) + b*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2
*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**
2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(4*c)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.71

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{4}aex^4+\frac{1}{2}adx^2+\frac{1}{2}\left(x^2\operatorname{arccsc}(cx)+\frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd$$
$$+\frac{1}{12}\left(3x^4\operatorname{arccsc}(cx)+\frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+3x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)be$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arccsc(c*x) + x*sqrt(-1/(c^2*x^2) + 1
)/c)*b*d + 1/12*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3
*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*e
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(118) = 236.

Time = 0.33 (sec) , antiderivative size = 556, normalized size of antiderivative = 4.03

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{1}{192}\left(\frac{3bex^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4\arcsin\left(\frac{1}{cx}\right)}{c}+\frac{3aex^4\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^4}{c}+\frac{2bex^3\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)^3}{c^2}+\right.$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/192*(3*b*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c + 3*a*e*
x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 2*b*e*x^3*(sqrt(-1/(c^2*x^2) + 1) +
 1)^3/c^2 + 24*b*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c +
24*a*d*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c + 12*b*e*x^2*(sqrt(-1/(c^2*x^2
) + 1) + 1)^2*arcsin(1/(c*x))/c^3 + 12*a*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1
)^2/c^3 + 48*b*d*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 18*b*e*x*(sqrt(-1/(c
^2*x^2) + 1) + 1)/c^4 + 48*b*d*arcsin(1/(c*x))/c^3 + 48*a*d/c^3 + 18*b*e*a
rcsin(1/(c*x))/c^5 + 18*a*e/c^5 - 48*b*d/(c^4*x*(sqrt(-1/(c^2*x^2) + 1) +
1)) - 18*b*e/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 24*b*d*arcsin(1/(c*x))
/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 24*a*d/(c^5*x^2*(sqrt(-1/(c^2*
x^2) + 1) + 1)^2) + 12*b*e*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1
) + 1)^2) + 12*a*e/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 2*b*e/(c^8*x
^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3*b*e*arcsin(1/(c*x))/(c^9*x^4*(sqrt(
-1/(c^2*x^2) + 1) + 1)^4) + 3*a*e/(c^9*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4)
)*c
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.86 $\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx$

3.86.1 Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx = \frac{be\sqrt{1-\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}ibd\csc^{-1}(cx)^2+\frac{1}{2}ex^2(a+b\csc^{-1}(cx)) - bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + bd\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{1}{2}ibd\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

output
```
1/2*I*b*d*arccsc(c*x)^2+1/2*e*x^2*(a+b*arccsc(c*x))-b*d*arccsc(c*x)*ln(1-(
I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*d*arccsc(c*x)*ln(1/x)-d*(a+b*arccsc(c*x))*
ln(1/x)+1/2*I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+1/2*b*e*x*(1-1/
c^2/x^2)^(1/2)/c
```

3.86.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx = \frac{1}{2}aex^2+\frac{bex\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bex^2\csc^{-1}(cx) - bd\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)+ad\log(x) + \frac{1}{2}ibd\left(\csc^{-1}(cx)^2+\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
(a*e*x^2)/2 + (b*e*x*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcC
sc[c*x])/2 - b*d*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + a*d*Log[x] +
 (I/2)*b*d*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

3.86.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5764, 5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x}\,dx$$

$$\Big\downarrow \ 5764$$

$$-\int \left(\frac{d}{x^2}+e\right)x^3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \ 5230$$

$$\frac{b\int -\frac{ex^2-2d\log\left(\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-d\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{ex^2-2d\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(\frac{ex^2}{\sqrt{1-\frac{1}{c^2x^2}}}-\frac{2d\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-d\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)-$$
$$\frac{b\left(-icd\operatorname{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-icd\arcsin\left(\frac{1}{cx}\right)^2+2cd\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2cd\log\left(\frac{1}{x}\right)\arcsin\left(\right.\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
(e*x^2*(a + b*ArcSin[1/(c*x)]))/2 - d*(a + b*ArcSin[1/(c*x)])*Log[x^(-1)]
- (b*(-(e*Sqrt[1 - 1/(c^2*x^2)]*x) - I*c*d*ArcSin[1/(c*x)]^2 + 2*c*d*ArcSi
n[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])] - 2*c*d*ArcSin[1/(c*x)]*Log[
x^(-1)] - I*c*d*PolyLog[2, E^((2*I)*ArcSin[1/(c*x)])]))/(2*c)
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5230
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.86.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.53

method	result
parts	$\frac{ae\,x^2}{2}+ad\ln(x)+b\left(\frac{i\,\mathrm{arccsc}(cx)^2 d}{2}+\frac{e\left(c^2x^2\,\mathrm{arccsc}(cx)+xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2c^2}-d\,\mathrm{arccsc}(cx)\ln\left(1+\right.\right.$
derivativedivides	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2d\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(c^2x^2\,\mathrm{arccsc}(cx)+xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)c^2d\,\mathrm{arccsc}(cx\right.}{}$
default	$\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{ic^2d\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(c^2x^2\,\mathrm{arccsc}(cx)+xc\sqrt{\frac{c^2x^2-1}{c^2x^2}}-i\right)}{2}-\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)c^2d\,\mathrm{arccsc}(cx\right.}{}$

input
```
int((e*x^2+d)*(a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*e*x^2+a*d*ln(x)+b*(1/2*I*arccsc(c*x)^2*d+1/2*e*(c^2*x^2*arccsc(c*x)+
x*c*((c^2*x^2-1)/c^2/x^2)^(1/2)-I)/c^2-d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x
^2)^(1/2))-d*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*d*polylog(2,-I/
c/x-(1-1/c^2/x^2)^(1/2))+I*d*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2)))
```

3.86.5 Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\operatorname{arccsc}(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))/x, x)
```

3.86.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \int \frac{(a+b\operatorname{acsc}(cx))\,(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsc(c*x))/x,x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x**2)/x, x)
```

3.86.7 Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\operatorname{arccsc}(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + a*d*log(x) + 1/4*(2*I*b*c^2*d*log(-c*x + 1)*log(x) + 2*I*b*c
^2*d*log(x)^2 + 2*I*b*c^2*d*dilog(c*x) + 2*I*b*c^2*d*dilog(-c*x) + 2*(b*c^
2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^2*log(c))*e*x^2 - I*(b*e
*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*b*d*integrate(1/2*log(x)/(c^2*x
^3 - x), x))*c^2 + 4*c^2*integrate(1/2*(b*e*x^2 + 2*b*d*log(x))*sqrt(c*x +
 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) + I*b*e*log(c*x - 1) + (-I*b*c^2*e*x^2
 - 2*I*b*c^2*d*log(x))*log(c^2*x^2) + (2*I*b*c^2*d*log(x) + I*b*e)*log(c*x
 + 1) - 2*(-I*b*c^2*e*x^2 - 2*(b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)) + I*b*c^2*log(c))*d)*log(x))/c^2
```

3.86.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

3.86.9 Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x}\,dx = {} & \frac{a\,e\,x^2}{2} - a\,d\,\ln\left(\frac{1}{x}\right) - b\,d\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right) \\ & + \frac{b\,e\,x\left(\sqrt{1-\frac{1}{c^2\,x^2}}+c\,x\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{2\,c} \\ & + \frac{b\,d\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,1\mathrm{i}}{2} + \frac{b\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2\,1\mathrm{i}}{2}\end{aligned}$$

input
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x,x)
```

output
```
(b*d*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 - a*d*log(1/x) + (b*d*asin(1/
(c*x))^2*1i)/2 + (a*e*x^2)/2 - b*d*log(1 - exp(asin(1/(c*x))*2i))*asin(1/(
c*x)) + (b*e*x*((1 - 1/(c^2*x^2))^(1/2) + c*x*asin(1/(c*x))))/(2*c)
```

3.87 $\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx$

3.87.1 Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(d+ex^2)\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx = -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{1}{4}bc^2d\csc^{-1}(cx)+\frac{1}{2}ibe\csc^{-1}(cx)^2 - \frac{d(a+b\csc^{-1}(cx))}{2x^2}-be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + be\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-e\left(a+b\csc^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{1}{2}ibe\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

output
```
1/4*b*c^2*d*arccsc(c*x)+1/2*I*b*e*arccsc(c*x)^2-1/2*d*(a+b*arccsc(c*x))/x^
2-b*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*e*arccsc(c*x)*ln(1
/x)-e*(a+b*arccsc(c*x))*ln(1/x)+1/2*I*b*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(
1/2))^2)-1/4*b*c*d*(1-1/c^2/x^2)^(1/2)/x
```

3.87.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^3}\,dx = -\frac{ad}{2x^2} - \frac{bcd\sqrt{\frac{-1+c^2x^2}{c^2x^2}}}{4x} - \frac{bd\csc^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2d\arcsin\left(\frac{1}{cx}\right) - be\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + ae\log(x) + \frac{1}{2}ibe\left(\csc^{-1}(cx)^2 + \operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 - (b*c*d*Sqrt[(-1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcCs
c[c*x])/(2*x^2) + (b*c^2*d*ArcSin[1/(c*x)])/4 - b*e*ArcCsc[c*x]*Log[1 - E^
((2*I)*ArcCsc[c*x])] + a*e*Log[x] + (I/2)*b*e*(ArcCsc[c*x]^2 + PolyLog[2,
E^((2*I)*ArcCsc[c*x])])
```

3.87.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5764, 5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\csc^{-1}(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \left(\frac{d}{x^2}+e\right)x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{5230}$$

$$\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c} - \frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2x^2} - e\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 7293$$

$$\frac{b\int \left(\frac{d}{\sqrt{1-\frac{1}{c^2x^2}}x^2}+\frac{2e\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 2009$$

$$-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+$$

$$\frac{b\left(\frac{1}{2}c^3d\arcsin\left(\frac{1}{cx}\right)+ice\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)+ice\arcsin\left(\frac{1}{cx}\right)^2-2ce\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)+2c\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcSin[1/(c*x)]))/x^2 - e*(a + b*ArcSin[1/(c*x)])*Log[x^(-1
)] + (b*(-1/2*(c^2*d*Sqrt[1 - 1/(c^2*x^2)])/x + (c^3*d*ArcSin[1/(c*x)])/2
+ I*c*e*ArcSin[1/(c*x)]^2 - 2*c*e*ArcSin[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[
1/(c*x)])] + 2*c*e*ArcSin[1/(c*x)]*Log[x^(-1)] + I*c*e*PolyLog[2, E^((2*I)
*ArcSin[1/(c*x)])]))/(2*c)
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5230
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.87.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.39

method	result
parts	$-\frac{ad}{2x^2}+ae\ln(x)+b\,c^2\left(\frac{i\,\mathrm{arccsc}(cx)^2 e}{2c^2}-\frac{e\,\mathrm{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{c^2}+\frac{ie\,\mathrm{polylog}\left(2,-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2\ldots}}\right.}{c^2}\right.$
derivativedivides	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{b\left(\frac{i\,\mathrm{arccsc}(cx)^2 e}{2}-e\,\mathrm{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+ie\,\mathrm{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-e\ldots\right.}{\ldots}\right.$
default	$c^2\left(-\frac{ad}{2c^2x^2}+\frac{ae\ln(cx)}{c^2}+\frac{b\left(\frac{i\,\mathrm{arccsc}(cx)^2 e}{2}-e\,\mathrm{arccsc}(cx)\ln\left(1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}\right)+ie\,\mathrm{polylog}\left(2,\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)-e\ldots\right.}{\ldots}\right.$

input
```
int((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*d/x^2+a*e*ln(x)+b*c^2*(1/2*I/c^2*arccsc(c*x)^2*e-1/c^2*e*arccsc(c*x
)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I/c^2*e*polylog(2,-I/c/x-(1-1/c^2/x^2)^(
1/2))-1/c^2*e*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I/c^2*e*polylog(
2,I/c/x+(1-1/c^2/x^2)^(1/2))+1/4*arccsc(c*x)*d*cos(2*arccsc(c*x))-1/8*d*si
n(2*arccsc(c*x)))
```

3.87.5 Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))/x^3, x)
```

3.87.6 Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsc(c*x))/x**3,x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x**2)/x**3, x)
```

3.87.7 Maxima [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

output
```
(c^2*integrate(sqrt(c*x + 1)*sqrt(c*x - 1)*log(x)/(c^4*x^3 - c^2*x), x) +
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*log(x))*b*e + 1/4*b*d*((c^4*x*sqrt
(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-
1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)/x^2) + a*e*log(x) - 1/2*a*d/x^2
```

3.87.8 Giac [F]

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex^2)\,(a+b\csc^{-1}(cx))}{x^3}\,dx = -a\,e\,\ln\left(\frac{1}{x}\right) - \frac{a\,d}{2\,x^2} - b\,e\,\ln\left(1-\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,\mathrm{asin}\left(\frac{1}{c\,x}\right) - \frac{b\,c\,d\,\sqrt{1-\frac{1}{c^2\,x^2}}}{4\,x} - \frac{b\,c^2\,d\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\,\left(\frac{2}{c^2\,x^2}-1\right)}{4} + \frac{b\,e\,\mathrm{polylog}\left(2,\mathrm{e}^{\mathrm{asin}\left(\frac{1}{c\,x}\right)\,2\mathrm{i}}\right)\,1\mathrm{i}}{2} + \frac{b\,e\,\mathrm{asin}\left(\frac{1}{c\,x}\right)^2\,1\mathrm{i}}{2}$$

input
```
int(((d + e*x^2)*(a + b*asin(1/(c*x))))/x^3,x)
```

output
```
(b*e*polylog(2, exp(asin(1/(c*x))*2i))*1i)/2 - a*e*log(1/x) + (b*e*asin(1/
(c*x))^2*1i)/2 - (a*d)/(2*x^2) - b*e*log(1 - exp(asin(1/(c*x))*2i))*asin(1
/(c*x)) - (b*c*d*(1 - 1/(c^2*x^2))^(1/2))/(4*x) - (b*c^2*d*asin(1/(c*x))*(
2/(c^2*x^2) - 1))/4
```

3.88 $\int x^2(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx$

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 252

$$\int x^2(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx = \frac{b(280c^4d^2+252c^2de+75e^2)\,x^2\sqrt{-1+c^2x^2}}{1680c^5\sqrt{c^2x^2}} + \frac{be(84c^2d+25e)\,x^4\sqrt{-1+c^2x^2}}{840c^3\sqrt{c^2x^2}} + \frac{be^2x^6\sqrt{-1+c^2x^2}}{42c\sqrt{c^2x^2}} + \frac{1}{3}d^2x^3(a+b\csc^{-1}(cx)) + \frac{2}{5}dex^5(a+b\csc^{-1}(cx)) + \frac{1}{7}e^2x^7(a+b\csc^{-1}(cx)) + \frac{b(280c^4d^2+252c^2de+75e^2)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{1680c^6\sqrt{c^2x^2}}$$

output
```
1/3*d^2*x^3*(a+b*arccsc(c*x))+2/5*d*e*x^5*(a+b*arccsc(c*x))+1/7*e^2*x^7*(a
+b*arccsc(c*x))+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*e^2)*x*arctanh(c*x/(c
^2*x^2-1)^(1/2))/c^6/(c^2*x^2)^(1/2)+1/1680*b*(280*c^4*d^2+252*c^2*d*e+75*
e^2)*x^2*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)+1/840*b*e*(84*c^2*d+25*e)*x
^4*(c^2*x^2-1)^(1/2)/c^3/(c^2*x^2)^(1/2)+1/42*b*e^2*x^6*(c^2*x^2-1)^(1/2)/
c/(c^2*x^2)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.73

$$\int x^2\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{c^2x^2\left(16ac^5x(35d^2+42dex^2+15e^2x^4)+b\sqrt{1-\frac{1}{c^2x^2}}(75e^2+2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2+\right.}{}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[1 - 1/(c^
2*x^2)]*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2
+ 5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*
x] + b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(1 + Sqrt[1 - 1/(c^2*x^2)]
)*x])/(1680*c^7)
```

3.88.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5762, 27, 1590, 27, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int \frac{x^2\left(15e^2x^4+42dex^2+35d^2\right)}{105\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{x^2\left(15e^2x^4+42dex^2+35d^2\right)}{\sqrt{c^2x^2-1}}dx}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{1590}$$

$$\frac{bcx\left(\frac{\int \frac{3x^2\left(70c^2d^2+e\left(84dc^2+25e\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{6c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{x^2\left(70c^2d^2+e\left(84dc^2+25e\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{363}$$

$$\frac{bcx\left(\frac{\frac{\left(280c^4d^2+252c^2de+75e^2\right)\int \frac{x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{bcx\left(\frac{\frac{\left(280c^4d^2+252c^2de+75e^2\right)\left(\frac{\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{bcx\left(\frac{\frac{\left(280c^4d^2+252c^2de+75e^2\right)\left(\frac{\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}\left(84c^2d+25e\right)}{4c^2}}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\csc^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\csc^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\csc^{-1}(cx)\right)$$

3.88. $\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}d^2x^3(a+b\csc^{-1}(cx))+\frac{2}{5}dex^5(a+b\csc^{-1}(cx))+\frac{1}{7}e^2x^7(a+b\csc^{-1}(cx))+\frac{bcx\left(\frac{\frac{\left(\frac{\operatorname{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{2c^3}+\frac{x\sqrt{c^2x^2-1}}{2c^2}\right)(280c^4d^2+252c^2de+75e^2)}{4c^2}+\frac{ex^3\sqrt{c^2x^2-1}(84c^2d+25e)}{4c^2}}{2c^2}+\frac{5e^2x^5\sqrt{c^2x^2-1}}{2c^2}\right)}{105\sqrt{c^2x^2}}$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcCsc[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsc[c*x]))/5 + (e^2
*x^7*(a + b*ArcCsc[c*x]))/7 + (b*c*x*((5*e^2*x^5*Sqrt[-1 + c^2*x^2])/(2*c^
2) + ((e*(84*c^2*d + 25*e)*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + ((280*c^4*d^2
 + 252*c^2*d*e + 75*e^2)*((x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ArcTanh[(c*x)/S
qrt[-1 + c^2*x^2]]/(2*c^3)))/(4*c^2))/(2*c^2)))/(105*Sqrt[c^2*x^2])
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(222) = 444$.

Time = 0.97 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.82

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}x^3d^2\right)+\frac{b\,\mathrm{arccsc}(cx)e^2x^7}{7}+\frac{2b\,\mathrm{arccsc}(cx)de\,x^5}{5}+\frac{b\,\mathrm{arccsc}(cx)d^2x^3}{3}+\frac{b(c^2x^2-1)\cdots}{42c^3\sqrt{\frac{c^2x\cdots}{c^2\cdots}}}\cdots$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\,\mathrm{arccsc}(cx)de\,x^5}{5}+\frac{b\,c^3\,\mathrm{arccsc}(cx)e^2x^7}{7}+\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\left(c^2x^2-1\cdots\right.}{10\sqrt{\frac{c^2x\cdots}{c^2\cdots}}}\cdots}{\cdots}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)d^2c^3x^3}{3}+\frac{2b\,c^3\,\mathrm{arccsc}(cx)de\,x^5}{5}+\frac{b\,c^3\,\mathrm{arccsc}(cx)e^2x^7}{7}+\frac{b\left(c^2x^2-1\right)d^2}{6\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b\left(c^2x^2-1\cdots\right.}{10\sqrt{\frac{c^2x\cdots}{c^2\cdots}}}\cdots}{\cdots}$

input
```
int(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+1/7*b*arccsc(c*x)*e^2*x^7+2/5*b*ar
ccsc(c*x)*d*e*x^5+1/3*b*arccsc(c*x)*d^2*x^3+1/42*b/c^3*(c^2*x^2-1)/((c^2*x
^2-1)/c^2/x^2)^(1/2)*x^4*e^2+1/10*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^
(1/2)*x^2*d*e+5/168*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*x^2*e^2+
1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2+3/20*b/c^5*(c^2*x^2-
1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e+1/6*b/c^4*(c^2*x^2-1)^(1/2)/((c^2*x^2-1
)/c^2/x^2)^(1/2)/x*d^2*ln(c*x+(c^2*x^2-1)^(1/2))+5/112*b/c^7*(c^2*x^2-1)/(
(c^2*x^2-1)/c^2/x^2)^(1/2)*e^2+3/20*b/c^6*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c
^2/x^2)^(1/2)/x*d*e*ln(c*x+(c^2*x^2-1)^(1/2))+5/112*b/c^8*(c^2*x^2-1)^(1/2
)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.08

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^7e^2x^7+672\,ac^7dex^5+560\,ac^7d^2x^3+16\left(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3-35\,bc^7d^2-42\,bc^7de\right.}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(15
*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b
*c^7*d*e - 15*b*c^7*e^2)*arccsc(c*x) - 32*(35*b*c^7*d^2 + 42*b*c^7*d*e + 1
5*b*c^7*e^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (280*b*c^4*d^2 + 252*b*c^2
*d*e + 75*b*e^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (40*b*c^5*e^2*x^5 + 2*(84
*b*c^5*d*e + 25*b*c^3*e^2)*x^3 + (280*b*c^5*d^2 + 252*b*c^3*d*e + 75*b*c*e
^2)*x)*sqrt(c^2*x^2 - 1))/c^7
```

3.88.6 Sympy [A] (verification not implemented)

Time = 11.86 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.15

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\operatorname{acsc}(cx)}{3}+\frac{2bdex^5\operatorname{acsc}(cx)}{5}+\frac{be^2x^7\operatorname{acsc}(cx)}{7}$$
$$+\frac{bd^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$
$$+\frac{2bde\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\operatorname{acosh}(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\operatorname{asin}(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}$$
$$+\frac{be^2\left(\begin{cases}\frac{cx^7}{6\sqrt{c^2x^2-1}}+\frac{x^5}{24c\sqrt{c^2x^2-1}}+\frac{5x^3}{48c^3\sqrt{c^2x^2-1}}-\frac{5x}{16c^5\sqrt{c^2x^2-1}}+\frac{5\operatorname{acosh}(cx)}{16c^6} & \text{for } |c^2x^2|>1\\ -\frac{icx^7}{6\sqrt{-c^2x^2+1}}-\frac{ix^5}{24c\sqrt{-c^2x^2+1}}-\frac{5ix^3}{48c^3\sqrt{-c^2x^2+1}}+\frac{5ix}{16c^5\sqrt{-c^2x^2+1}}-\frac{5i\operatorname{asin}(cx)}{16c^6} & \text{otherwise}\end{cases}\right)}{7c}$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

output
```
a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*acsc(c*x)/3 +
 2*b*d*e*x**5*acsc(c*x)/5 + b*e**2*x**7*acsc(c*x)/7 + b*d**2*Piecewise((x*
sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*
c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(
c*x)/(2*c**2), True))/(3*c) + 2*b*d*e*Piecewise((c*x**5/(4*sqrt(c**2*x**2
- 1)) + x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1))
+ 3*acosh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**
2 + 1)) - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**
2 + 1)) - 3*I*asin(c*x)/(8*c**4), True))/(5*c) + b*e**2*Piecewise((c*x**7/
(6*sqrt(c**2*x**2 - 1)) + x**5/(24*c*sqrt(c**2*x**2 - 1)) + 5*x**3/(48*c**
3*sqrt(c**2*x**2 - 1)) - 5*x/(16*c**5*sqrt(c**2*x**2 - 1)) + 5*acosh(c*x)/
(16*c**6), Abs(c**2*x**2) > 1), (-I*c*x**7/(6*sqrt(-c**2*x**2 + 1)) - I*x*
*5/(24*c*sqrt(-c**2*x**2 + 1)) - 5*I*x**3/(48*c**3*sqrt(-c**2*x**2 + 1)) +
 5*I*x/(16*c**5*sqrt(-c**2*x**2 + 1)) - 5*I*asin(c*x)/(16*c**6), True))/(7
*c)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.60

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{7}ae^2x^7+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3+\frac{1}{12}\left(4x^3\operatorname{arccsc}(cx)+\frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2}+\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2}-\frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2+\frac{1}{40}\left(16x^5\operatorname{arccsc}(cx)-\frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2c^4\left(\frac{1}{c^2x^2}-1\right)+c^4}-\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde+\frac{1}{672}\left(96x^7\operatorname{arccsc}(cx)+\frac{\frac{2\left(15\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}-1\right)^3+3c^6\left(\frac{1}{c^2x^2}-1\right)^2+3c^6\left(\frac{1}{c^2x^2}-1\right)+c^6}+\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^6}-\frac{15\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arccsc(c*x) +
(2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2
*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40
*(16*x^5*arccsc(c*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2
) + 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(
sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c
)*b*d*e + 1/672*(96*x^7*arccsc(c*x) + (2*(15*(-1/(c^2*x^2) + 1)^(5/2) - 40
*(-1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(-1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) -
 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*lo
g(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 15*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^6
)/c)*b*e^2
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. 2(222) = 444.

Time = 5.13 (sec) , antiderivative size = 1579, normalized size of antiderivative = 6.27

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/13440*(15*b*e^2*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7*arcsin(1/(c*x))/c + 1
5*a*e^2*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7/c + 5*b*e^2*x^6*(sqrt(-1/(c^2*x
^2) + 1) + 1)^6/c^2 + 168*b*d*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(
1/(c*x))/c + 168*a*d*e*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 105*b*e^2*x^
5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c^3 + 105*a*e^2*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5/c^3 + 84*b*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^
4/c^2 + 560*b*d^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c + 5
60*a*d^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 45*b*e^2*x^4*(sqrt(-1/(c^2
*x^2) + 1) + 1)^4/c^4 + 840*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsi
n(1/(c*x))/c^3 + 840*a*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^3 + 560*b*
d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 + 315*b*e^2*x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3*arcsin(1/(c*x))/c^5 + 315*a*e^2*x^3*(sqrt(-1/(c^2*x^2) + 1
) + 1)^3/c^5 + 672*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^4 + 1680*b*d
^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 1680*a*d^2*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)/c^3 + 225*b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2
/c^6 + 1680*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 168
0*a*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^5 + 2240*b*d^2*log(sqrt(-1/(c^2*x
^2) + 1) + 1)/c^4 - 2240*b*d^2*log(1/(abs(c)*abs(x)))/c^4 + 525*b*e^2*x*(s
qrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^7 + 525*a*e^2*x*(sqrt(-1/(c^2
*x^2) + 1) + 1)/c^7 + 2016*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - ...
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \int x^2\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \,dx$$

input
```
int(x^2*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.89 $\int (d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx$

3.89.1 Optimal result

Integrand size = 18, antiderivative size = 191

$$\int (d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx = \frac{be(40c^2d+9e)\,x^2\sqrt{-1+c^2x^2}}{120c^3\sqrt{c^2x^2}} + \frac{be^2x^4\sqrt{-1+c^2x^2}}{20c\sqrt{c^2x^2}} + d^2x(a+b\csc^{-1}(cx)) + \frac{2}{3}dex^3(a+b\csc^{-1}(cx)) + \frac{1}{5}e^2x^5(a+b\csc^{-1}(cx)) + \frac{b(120c^4d^2+40c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{120c^4\sqrt{c^2x^2}}$$

output
```
d^2*x*(a+b*arccsc(c*x))+2/3*d*e*x^3*(a+b*arccsc(c*x))+1/5*e^2*x^5*(a+b*arc
csc(c*x))+1/120*b*(120*c^4*d^2+40*c^2*d*e+9*e^2)*x*arctanh(c*x/(c^2*x^2-1)
^(1/2))/c^4/(c^2*x^2)^(1/2)+1/120*b*e*(40*c^2*d+9*e)*x^2*(c^2*x^2-1)^(1/2)
/c^3/(c^2*x^2)^(1/2)+1/20*b*e^2*x^4*(c^2*x^2-1)^(1/2)/c/(c^2*x^2)^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int (d+ex^2)^2 (a+b\csc^{-1}(cx))\, dx$$
$$= \frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4)+be\sqrt{1-\frac{1}{c^2x^2}}x(9e+c^2(40d+6ex^2))\right)+8bc^5x(15d^2+10dex^2+3e^2x}{120c^5}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 - 1/(c^2*x^
2)]*x*(9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e
^2*x^4)*ArcCsc[c*x] + b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1
 - 1/(c^2*x^2)])*x])/(120*c^5)
```

3.89.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5752, 27, 1473, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^2 (a+b\csc^{-1}(cx))\, dx$$
$$\Big\downarrow \text{ 5752}$$
$$\frac{bcx\int \frac{3e^2x^4+10dex^2+15d^2}{15\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+d^2x(a+b\csc^{-1}(cx))+\frac{2}{3}dex^3(a+b\csc^{-1}(cx))+\frac{1}{5}e^2x^5(a+b\csc^{-1}(cx))$$
$$\Big\downarrow \text{ 27}$$
$$\frac{bcx\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}}+d^2x(a+b\csc^{-1}(cx))+\frac{2}{3}dex^3(a+b\csc^{-1}(cx))+\frac{1}{5}e^2x^5(a+b\csc^{-1}(cx))$$
$$\Big\downarrow \text{ 1473}$$

$$\frac{bcx\left(\frac{\int \frac{60c^2d^2+e\left(40dc^2+9e\right)x^2}{\sqrt{c^2x^2-1}}dx}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \ 299$$

$$\frac{bcx\left(\frac{\frac{\left(120c^4d^2+40c^2de+9e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \ 224$$

$$\frac{bcx\left(\frac{\frac{\left(120c^4d^2+40c^2de+9e^2\right)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}+d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \ 219$$

$$d^2x\left(a+b\csc^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\csc^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\csc^{-1}(cx)\right)+\frac{bcx\left(\frac{\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)\left(120c^4d^2+40c^2de+9e^2\right)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}\left(40c^2d+9e\right)}{2c^2}}{4c^2}+\frac{3e^2x^3\sqrt{c^2x^2-1}}{4c^2}\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
d^2*x*(a + b*ArcCsc[c*x]) + (2*d*e*x^3*(a + b*ArcCsc[c*x]))/3 + (e^2*x^5*(
a + b*ArcCsc[c*x]))/5 + (b*c*x*((3*e^2*x^3*Sqrt[-1 + c^2*x^2])/(4*c^2) + (
(e*(40*c^2*d + 9*e)*x*Sqrt[-1 + c^2*x^2])/(2*c^2) + ((120*c^4*d^2 + 40*c^2
*d*e + 9*e^2)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(2*c^3))/(4*c^2)))/(15*Sq
rt[c^2*x^2])
```

3.89. $\int \left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 5752
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCsc[c*x])   u,
x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.89.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.77

method	result
parts	$a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\,\mathrm{arccsc}(cx)e^2x^5}{5}+\frac{2b\,\mathrm{arccsc}(cx)de\,x^3}{3}+b\,\mathrm{arccsc}\left(cx\right)d^2x+\frac{b\left(c^2x^2-1\right.}{20c^3\sqrt{\frac{c^2}{}}}$
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\mathrm{arccsc}(cx)d^2cx+\frac{2bc\,\mathrm{arccsc}(cx)de\,x^3}{3}+\frac{bc\,\mathrm{arccsc}(cx)e^2x^5}{5}+\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c}$
default	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+b\,\mathrm{arccsc}(cx)d^2cx+\frac{2bc\,\mathrm{arccsc}(cx)de\,x^3}{3}+\frac{bc\,\mathrm{arccsc}(cx)e^2x^5}{5}+\frac{b\sqrt{c^2x^2-1}\,d^2\ln\left(cx+\sqrt{c^2x^2-1}\right)}{\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,cx}}{c}$

input
```
int((e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+1/5*b*arccsc(c*x)*e^2*x^5+2/3*b*arccsc(c
*x)*d*e*x^3+b*arccsc(c*x)*d^2*x+1/20*b/c^3*(c^2*x^2-1)*x^2/((c^2*x^2-1)/c^
2/x^2)^(1/2)*e^2+1/3*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e+b/c
^2*(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^
(1/2))+3/40*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2+1/3*b/c^4*(c
^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2)
)+3/40*b/c^6*(c^2*x^2-1)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*ln(c*x+(c
^2*x^2-1)^(1/2))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.24

$$\int \left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x+8\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x-15\,bc^5d^2-10\,bc^5de-3\,bc\right.}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(3*b*c^5*
e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e
- 3*b*c^5*e^2)*arccsc(c*x) - 16*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2
)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e
^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + (6*b*c^3*e^2*x^3 + (40*b*c^3*d*e + 9*b
*c*e^2)*x)*sqrt(c^2*x^2 - 1))/c^5
```

3.89.6 Sympy [A] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.86

$$
\begin{aligned}
&\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx \\
&= ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\,\mathrm{acsc}\,(cx)+\frac{2bdex^3\,\mathrm{acsc}\,(cx)}{3} \\
&\quad+\frac{be^2x^5\,\mathrm{acsc}\,(cx)}{5}+\frac{bd^2\left(\begin{cases}\mathrm{acosh}\,(cx) & \text{for } |c^2x^2|>1\\ -i\,\mathrm{asin}\,(cx) & \text{otherwise}\end{cases}\right)}{c} \\
&\quad+\frac{2bde\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\mathrm{acosh}\,(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\,\mathrm{asin}\,(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c} \\
&\quad+\frac{be^2\left(\begin{cases}\frac{cx^5}{4\sqrt{c^2x^2-1}}+\frac{x^3}{8c\sqrt{c^2x^2-1}}-\frac{3x}{8c^3\sqrt{c^2x^2-1}}+\frac{3\,\mathrm{acosh}\,(cx)}{8c^4} & \text{for } |c^2x^2|>1\\ -\frac{icx^5}{4\sqrt{-c^2x^2+1}}-\frac{ix^3}{8c\sqrt{-c^2x^2+1}}+\frac{3ix}{8c^3\sqrt{-c^2x^2+1}}-\frac{3i\,\mathrm{asin}\,(cx)}{8c^4} & \text{otherwise}\end{cases}\right)}{5c}
\end{aligned}
$$

input
```
integrate((e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

output
```
a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acsc(c*x) + 2*b*d*e*x
**3*acsc(c*x)/3 + b*e**2*x**5*acsc(c*x)/5 + b*d**2*Piecewise((acosh(c*x),
Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c + 2*b*d*e*Piecewise((x*sqrt(c
**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c**2*x**2) > 1), (-I*c*x**3
/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*x**2 + 1)) - I*asin(c*x)/(
2*c**2), True))/(3*c) + b*e**2*Piecewise((c*x**5/(4*sqrt(c**2*x**2 - 1)) +
 x**3/(8*c*sqrt(c**2*x**2 - 1)) - 3*x/(8*c**3*sqrt(c**2*x**2 - 1)) + 3*aco
sh(c*x)/(8*c**4), Abs(c**2*x**2) > 1), (-I*c*x**5/(4*sqrt(-c**2*x**2 + 1))
 - I*x**3/(8*c*sqrt(-c**2*x**2 + 1)) + 3*I*x/(8*c**3*sqrt(-c**2*x**2 + 1))
 - 3*I*asin(c*x)/(8*c**4), True))/(5*c)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.55

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{5}\,ae^2x^5 + \frac{2}{3}\,adex^3 + \frac{1}{6}\left(4\,x^3\,\mathrm{arccsc}\,(cx) + \frac{\frac{2\,\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde + \frac{1}{80}\left(16\,x^5\,\mathrm{arccsc}\,(cx) - \frac{\frac{2\left(3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\,\sqrt{-\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}-1\right)^2+2\,c^4\left(\frac{1}{c^2x^2}-1\right)+c^4} - \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\,\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be^2 + ad^2x + \frac{\left(2\,cx\,\mathrm{arccsc}\,(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bd^2}{2\,c}$$

input

```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output

```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x
^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c
^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arccsc(c
*x) - (2*(3*(-1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(-1/(c^2*x^2) + 1))/(c^4*(1/(
c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) - 3*log(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^4 + 3*log(sqrt(-1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x
 + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - log(-sqrt(-1
/(c^2*x^2) + 1) + 1))*b*d^2/c
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. 2(169) = 338.

Time = 3.50 (sec) , antiderivative size = 1033, normalized size of antiderivative = 5.41

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/960*(6*b*e^2*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5*arcsin(1/(c*x))/c + 6*a*
e^2*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c + 3*b*e^2*x^4*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^4/c^2 + 80*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*
x))/c + 80*a*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c + 30*b*e^2*x^3*(sqrt
(-1/(c^2*x^2) + 1) + 1)^3*arcsin(1/(c*x))/c^3 + 30*a*e^2*x^3*(sqrt(-1/(c^2
*x^2) + 1) + 1)^3/c^3 + 80*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^2 +
480*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c + 480*a*d^2*x*(
sqrt(-1/(c^2*x^2) + 1) + 1)/c + 24*b*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^
2/c^4 + 240*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^3 + 240
*a*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^3 + 960*b*d^2*log(sqrt(-1/(c^2*x^2
) + 1) + 1)/c^2 - 960*b*d^2*log(1/(abs(c)*abs(x)))/c^2 + 60*b*e^2*x*(sqrt(
-1/(c^2*x^2) + 1) + 1)*arcsin(1/(c*x))/c^5 + 60*a*e^2*x*(sqrt(-1/(c^2*x^2)
 + 1) + 1)/c^5 + 320*b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 - 320*b*d*e
*log(1/(abs(c)*abs(x)))/c^4 + 480*b*d^2*arcsin(1/(c*x))/(c^3*x*(sqrt(-1/(c
^2*x^2) + 1) + 1)) + 480*a*d^2/(c^3*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 72*b
*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^6 - 72*b*e^2*log(1/(abs(c)*abs(x)))
/c^6 + 240*b*d*e*arcsin(1/(c*x))/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 24
0*a*d*e/(c^5*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*b*e^2*arcsin(1/(c*x))/(c
^7*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 60*a*e^2/(c^7*x*(sqrt(-1/(c^2*x^2) +
1) + 1)) - 80*b*d*e/(c^6*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) - 24*b*e^2...
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)^2\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

output
```
int((d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.90 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^2}\,dx$

3.90.1 Optimal result

Integrand size = 21, antiderivative size = 163

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^2}\,dx = &-\frac{bcd^2\sqrt{-1+c^2x^2}}{\sqrt{c^2x^2}}+\frac{be^2x^2\sqrt{-1+c^2x^2}}{6c\sqrt{c^2x^2}}\\&-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))\\&+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))\\&+\frac{be(12c^2d+e)\,x\operatorname{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{6c^2\sqrt{c^2x^2}}\end{aligned}$$

output
```
-d^2*(a+b*arccsc(c*x))/x+2*d*e*x*(a+b*arccsc(c*x))+1/3*e^2*x^3*(a+b*arccsc
(c*x))+1/6*b*e*(12*c^2*d+e)*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/c^2/(c^2*x^2)
^(1/2)-b*c*d^2*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)+1/6*b*e^2*x^2*(c^2*x^2-1)
^(1/2)/c/(c^2*x^2)^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx$$
$$= \frac{c^2\left(b\sqrt{1-\frac{1}{c^2x^2}}x(-6c^2d^2+e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\csc^{-1}(cx)+}{6c^3x}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
(c^2*(b*Sqrt[1 - 1/(c^2*x^2)]*x*(-6*c^2*d^2 + e^2*x^2) + 2*a*c*(-3*d^2 + 6
*d*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCsc[c*x]
+ b*e*(12*c^2*d + e)*x*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/(6*c^3*x)
```

3.90.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5762, 27, 1588, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{5762}$$
$$\frac{bcx\int -\frac{-e^2x^4-6dex^2+3d^2}{3x^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))$$
$$\Big\downarrow \text{27}$$
$$-\frac{bcx\int \frac{-e^2x^4-6dex^2+3d^2}{x^2\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))$$
$$\Big\downarrow \text{1588}$$
$$-\frac{bcx\left(\int -\frac{e(ex^2+6d)}{\sqrt{c^2x^2-1}}dx+\frac{3d^2\sqrt{c^2x^2-1}}{x}\right)}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+$$
$$\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \; 25$$

$$-\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-\int \frac{e(ex^2+6d)}{\sqrt{c^2x^2-1}}dx\right)}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\int \frac{ex^2+6d}{\sqrt{c^2x^2-1}}dx\right)}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \; 299$$

$$-\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{(12c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}}dx}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \; 224$$

$$-\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{(12c^2d+e)\int \frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}}{2c^2}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \; 219$$

$$-\frac{d^2(a+b\csc^{-1}(cx))}{x}+2dex(a+b\csc^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\csc^{-1}(cx))-\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{x}-e\left(\frac{\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)(12c^2d+e)}{2c^3}+\frac{ex\sqrt{c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
-((d^2*(a + b*ArcCsc[c*x]))/x) + 2*d*e*x*(a + b*ArcCsc[c*x]) + (e^2*x^3*(a
 + b*ArcCsc[c*x]))/3 - (b*c*x*((3*d^2*Sqrt[-1 + c^2*x^2])/x - e*((e*x*Sqrt
[-1 + c^2*x^2])/(2*c^2) + ((12*c^2*d + e)*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]
])/(2*c^3))))/(3*Sqrt[c^2*x^2])
```

3.90. $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^2}\,dx$

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.90.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.53

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+\frac{b\ \mathrm{arccsc}(cx)e^2x^3}{3}+2be\ \mathrm{arccsc}\left(cx\right)xd-\frac{b\ \mathrm{arccsc}(cx)d^2}{x}+\frac{b(c^2x^2-1)e^2}{6c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\ \mathrm{arccsc}(cx)dex}{c}+\frac{b\ \mathrm{arccsc}(cx)e^2x^3}{3c}-\frac{b\ \mathrm{arccsc}(cx)d^2}{cx}-\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{2b}{}\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{2b\ \mathrm{arccsc}(cx)dex}{c}+\frac{b\ \mathrm{arccsc}(cx)e^2x^3}{3c}-\frac{b\ \mathrm{arccsc}(cx)d^2}{cx}-\frac{b(c^2x^2-1)d^2}{c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{2b}{}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+1/3*b*arccsc(c*x)*e^2*x^3+2*b*e*arccsc(c*x)*
x*d-b*arccsc(c*x)*d^2/x+1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)*
e^2-b/c*(c^2*x^2-1)/x^2/((c^2*x^2-1)/c^2/x^2)^(1/2)*d^2+2*b/c^2*(c^2*x^2-1
)^(1/2)/x/((c^2*x^2-1)/c^2/x^2)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))+1/6*b/
c^4*e^2*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*ln(c*x+(c^2*x^2-1)
^(1/2))
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \frac{2\,ac^3e^2x^4-6\,bc^4d^2x+12\,ac^3dex^2-6\,ac^3d^2+4\,(3\,bc^3d^2-6\,bc^3de-bc^3e^2)x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^4 - 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 + 4*
(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*arctan(-c*x + sqrt(c^2*x^2 - 1))
 - (12*b*c^2*d*e + b*e^2)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*e^2*x
^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^
2)*x)*arccsc(c*x) - (6*b*c^3*d^2 - b*c*e^2*x^2)*sqrt(c^2*x^2 - 1))/(c^3*x)
```

3.90.6 Sympy [A] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx$$
$$= -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} - bcd^2\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{acsc}(cx)}{x} + 2bdex\operatorname{acsc}(cx) + \frac{be^2x^3\operatorname{acsc}(cx)}{3} + \frac{2bde\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c} + \frac{be^2\left(\begin{cases}\frac{x\sqrt{c^2x^2-1}}{2c}+\frac{\operatorname{acosh}(cx)}{2c^2} & \text{for } |c^2x^2|>1\\ -\frac{icx^3}{2\sqrt{-c^2x^2+1}}+\frac{ix}{2c\sqrt{-c^2x^2+1}}-\frac{i\operatorname{asin}(cx)}{2c^2} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**2,x)
```

output
```
-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 - b*c*d**2*sqrt(1 - 1/(c**2*x**2)) -
 b*d**2*acsc(c*x)/x + 2*b*d*e*x*acsc(c*x) + b*e**2*x**3*acsc(c*x)/3 + 2*b*
d*e*Piecewise((acosh(c*x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c +
b*e**2*Piecewise((x*sqrt(c**2*x**2 - 1)/(2*c) + acosh(c*x)/(2*c**2), Abs(c
**2*x**2) > 1), (-I*c*x**3/(2*sqrt(-c**2*x**2 + 1)) + I*x/(2*c*sqrt(-c**2*
x**2 + 1)) - I*asin(c*x)/(2*c**2), True))/(3*c)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx$$
$$= \frac{1}{3}ae^2x^3 - \left(c\sqrt{-\frac{1}{c^2x^2}+1} + \frac{\operatorname{arccsc}(cx)}{x}\right)bd^2 + \frac{1}{12}\left(4x^3\operatorname{arccsc}(cx) + \frac{\frac{2\sqrt{-\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)}{c^2} - \frac{\log\left(\sqrt{-\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 + 2\,adex + \frac{\left(2\,cx\operatorname{arccsc}(cx) + \log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)bde}{c} - \frac{ad^2}{x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 - (c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d^2 + 1/12*(4
*x^3*arccsc(c*x) + (2*sqrt(-1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) - 1) + c^2)
 + log(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 - log(sqrt(-1/(c^2*x^2) + 1) - 1)/c
^2)/c)*b*e^2 + 2*a*d*e*x + (2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1)
 + 1) - log(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*d*e/c - a*d^2/x
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2502 vs. $2(145) = 290$.

Time = 2.32 (sec) , antiderivative size = 2502, normalized size of antiderivative = 15.35

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2} dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

output
```
1/24*(b*e^2*arcsin(1/(c*x))/(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c
*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)) + a*e^2/(c/(x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5)) + b*e^2/(c*x*(sqrt
(-1/(c^2*x^2) + 1) + 1)*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5
*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*b*d*e*arcsin(1/(c*x))/(x^2*(sqrt(-
1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5
*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 24*a*d*e/(x^2*(sqrt(-1/(c^2*x^2) + 1)
 + 1)^2*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x
^2) + 1) + 1)^5))) - 24*b*c*d^2/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3*(c/(x^
3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^
5))) + 4*b*e^2*arcsin(1/(c*x))/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/
(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) +
1)^5))) + 4*a*e^2/(c^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x^3*(sqrt(-1
/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5))) + 48*
b*d*e*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^
3*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^2*x^2) +
1) + 1)^5))) - 48*b*d*e*log(1/(abs(c)*abs(x)))/(c*x^3*(sqrt(-1/(c^2*x^2) +
 1) + 1)^3*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/(c^
2*x^2) + 1) + 1)^5))) - 48*b*d^2*arcsin(1/(c*x))/(x^4*(sqrt(-1/(c^2*x^2) +
 1) + 1)^4*(c/(x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 1/(c*x^5*(sqrt(-1/...
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^2, x)
```

3.91 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^4}\,dx$

3.91.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{2bcd(c^2d+9e)\sqrt{-1+c^2x^2}}{9\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{9x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3x^3} - \frac{2de(a+b\csc^{-1}(cx))}{x} + e^2x(a+b\csc^{-1}(cx)) + \frac{be^2x\,\text{arctanh}\left(\frac{cx}{\sqrt{-1+c^2x^2}}\right)}{\sqrt{c^2x^2}}$$

output
```
-1/3*d^2*(a+b*arccsc(c*x))/x^3-2*d*e*(a+b*arccsc(c*x))/x+e^2*x*(a+b*arccsc
(c*x))+b*e^2*x*arctanh(c*x/(c^2*x^2-1)^(1/2))/(c^2*x^2)^(1/2)-2/9*b*c*d*(c
^2*d+9*e)*(c^2*x^2-1)^(1/2)/(c^2*x^2)^(1/2)-1/9*b*c*d^2*(c^2*x^2-1)^(1/2)/
x^2/(c^2*x^2)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+18ex^2)+3a(d^2+6dex^2-3e^2x^4)}{9x^3} - \frac{b(d^2+6dex^2-3e^2x^4)\csc^{-1}(cx)}{3x^3} + \frac{be^2\log\left(\left(1+\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{c}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```

output
```
-1/9*(b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 + 18*e*x^2) + 3*a*(d^
2 + 6*d*e*x^2 - 3*e^2*x^4))/x^3 - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCsc[
c*x])/(3*x^3) + (b*e^2*Log[(1 + Sqrt[1 - 1/(c^2*x^2)])*x])/c
```

3.91.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5762, 27, 1588, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4} dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int -\frac{-3e^2x^4+6dex^2+d^2}{3x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{-3e^2x^4+6dex^2+d^2}{x^4\sqrt{c^2x^2-1}}dx}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{1588}$$

$$-\frac{bcx\left(\frac{1}{3}\int \frac{2d(dc^2+9e)-9e^2x^2}{x^2\sqrt{c^2x^2-1}}dx+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{358}$$

$$-\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-9e^2\int \frac{1}{\sqrt{c^2x^2-1}}dx\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{224}$$

$$-\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-9e^2\int\frac{1}{1-\frac{c^2x^2}{c^2x^2-1}}d\frac{x}{\sqrt{c^2x^2-1}}\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))$$

$\downarrow$ 219

$$-\frac{d^2(a+b\csc^{-1}(cx))}{3x^3}-\frac{2de(a+b\csc^{-1}(cx))}{x}+e^2x(a+b\csc^{-1}(cx))-\frac{bcx\left(\frac{1}{3}\left(\frac{2d\sqrt{c^2x^2-1}(c^2d+9e)}{x}-\frac{9e^2\text{arctanh}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)+\frac{d^2\sqrt{c^2x^2-1}}{3x^3}\right)}{3\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcCsc[c*x]))/x^3 - (2*d*e*(a + b*ArcCsc[c*x]))/x + e^2*x
*(a + b*ArcCsc[c*x]) - (b*c*x*((d^2*Sqrt[-1 + c^2*x^2])/(3*x^3) + ((2*d*(c
^2*d + 9*e)*Sqrt[-1 + c^2*x^2])/x - (9*e^2*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2
]])/c)/3))/(3*Sqrt[c^2*x^2])
```

3.91.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], 
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.91.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.46

method	result
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\,\mathrm{arccsc}\,(cx)\,e^2x-\frac{2b\,\mathrm{arccsc}(cx)de}{x}-\frac{b\,\mathrm{arccsc}(cx)d^2}{3x^3}-\frac{2bc(c^2x^2-1)d^2}{9\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x^2}-$
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)e^2x}{c^3}-\frac{b\,\mathrm{arccsc}(cx)d^2}{3c^3x^3}-\frac{2b\,\mathrm{arccsc}(cx)de}{c^3x}-\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\,\mathrm{arccsc}(cx)e^2x}{c^3}-\frac{b\,\mathrm{arccsc}(cx)d^2}{3c^3x^3}-\frac{2b\,\mathrm{arccsc}(cx)de}{c^3x}-\frac{2b(c^2x^2-1)d^2}{9c^2x^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*arccsc(c*x)*e^2*x-2*b*arccsc(c*x)*d*e/x-1/
3*b*arccsc(c*x)*d^2/x^3-2/9*b*c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^
2*d^2-2*b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^2*d*e+b/c^2*(c^2*x^2
-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*e^2*ln(c*x+(c^2*x^2-1)^(1/2))-1/9*
b/c*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^4*d^2
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$= \frac{9\,ace^2x^4 - 9\,be^2x^3\log\left(-cx+\sqrt{c^2x^2-1}\right) - 18\,acdex^2 + 6\,(bcd^2+6\,bcde-3\,bce^2)x^3\arctan\left(-cx+\sqrt{c^2x\right.}}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/9*(9*a*c*e^2*x^4 - 9*b*e^2*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - 18*a*c*d*
e*x^2 + 6*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*arctan(-c*x + sqrt(c^2*x^2
 - 1)) - 3*a*c*d^2 - 2*(b*c^4*d^2 + 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 -
6*b*c*d*e*x^2 - b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*arccsc(c*
x) - (b*c*d^2 + 2*(b*c^3*d^2 + 9*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx = -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x - 2bcde\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\operatorname{acsc}(cx)}{3x^3} - \frac{2bde\operatorname{acsc}(cx)}{x} + be^2x\operatorname{acsc}(cx) - \frac{bd^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x}+\frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1\\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x}+\frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c} + \frac{be^2\left(\begin{cases}\operatorname{acosh}(cx) & \text{for } |c^2x^2|>1\\ -i\operatorname{asin}(cx) & \text{otherwise}\end{cases}\right)}{c}$$

input
```
integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**4,x)
```

output
```
-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - 2*b*c*d*e*sqrt(1 - 1/(c**2*x**2)
) - b*d**2*acsc(c*x)/(3*x**3) - 2*b*d*e*acsc(c*x)/x + b*e**2*x*acsc(c*x) -
 b*d**2*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1
)/(3*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*
c*sqrt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c) + b*e**2*Piecewise((acosh(c*
x), Abs(c**2*x**2) > 1), (-I*asin(c*x), True))/c
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$= -2\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)bde + ae^2x + \frac{1}{9}bd^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arccsc}(cx)}{x^3}\right) + \frac{\left(2cx\operatorname{arccsc}(cx)+\log\left(\sqrt{-\frac{1}{c^2x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{1}{c^2x^2}+1}+1\right)\right)be^2}{2c} - \frac{2ade}{x} - \frac{ad^2}{3x^3}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

output
```
-2*(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*
((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccs
c(c*x)/x^3) + 1/2*(2*c*x*arccsc(c*x) + log(sqrt(-1/(c^2*x^2) + 1) + 1) - l
og(-sqrt(-1/(c^2*x^2) + 1) + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

3.91.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4288 vs. $2(139) = 278$.

Time = 98.89 (sec) , antiderivative size = 4288, normalized size of antiderivative = 27.31

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

output
```
-1/18*(4*b*c^3*d^2/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2
) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)))
- 9*b*e^2*arcsin(1/(c*x))/(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(
sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5
) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)) - 9*a*e^2/(c/(x*(sqrt(-1/(
c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^
5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1
)^7)) + 36*b*c*d*e/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2
) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt
(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7)))
- 18*b*e^2*log(sqrt(-1/(c^2*x^2) + 1) + 1)/(c*x*(sqrt(-1/(c^2*x^2) + 1) +
1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/(c^5*x^7*(sqrt(-1
/(c^2*x^2) + 1) + 1)^7))) + 18*b*e^2*log(1/(abs(c)*abs(x)))/(c*x*(sqrt(-1/
(c^2*x^2) + 1) + 1)*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c*x^3*(sqrt(-
1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5) + 1/
(c^5*x^7*(sqrt(-1/(c^2*x^2) + 1) + 1)^7))) + 72*b*d*e*arcsin(1/(c*x))/(x^2
*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*(c/(x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 3/(c
*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3) + 3/(c^3*x^5*(sqrt(-1/(c^2*x^2) + ...
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^4, x)
```

3.91. $\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x^4}\,dx$

3.92 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^6}\,dx$

3.92.1 Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{bc(24c^4d^2+100c^2de+225e^2)\sqrt{-1+c^2x^2}}{225\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{25x^4\sqrt{c^2x^2}} - \frac{2bcd(6c^2d+25e)\sqrt{-1+c^2x^2}}{225x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a+b\csc^{-1}(cx))}{x}$$

output
```
-1/5*d^2*(a+b*arccsc(c*x))/x^5-2/3*d*e*(a+b*arccsc(c*x))/x^3-e^2*(a+b*arcc
sc(c*x))/x-1/225*b*c*(24*c^4*d^2+100*c^2*d*e+225*e^2)*(c^2*x^2-1)^(1/2)/(c
^2*x^2)^(1/2)-1/25*b*c*d^2*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-2/225*b*c
*d*(6*c^2*d+25*e)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^2)^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6} dx = -\frac{15a(3d^2+10dex^2+15e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(225e^2x^4+50dex^2(1+2c^2x^2)+3d^2(3+4c^2x^2+8c^4x^4))}{225x^5}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6,x]
```

output
```
-1/225*(15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x^2)]
*x*(225*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*c^
4*x^4)) + 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c*x])/x^5
```

3.92.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5762, 27, 1588, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6} dx$$

$$\Big\downarrow \ 5762$$

$$\frac{bcx\int -\frac{15e^2x^4+10dex^2+3d^2}{15x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a+b\csc^{-1}(cx))}{x}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\int \frac{15e^2x^4+10dex^2+3d^2}{x^6\sqrt{c^2x^2-1}}dx}{15\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a+b\csc^{-1}(cx))}{x}$$

$$\Big\downarrow \ 1588$$

$$-\frac{bcx\left(\frac{1}{5}\int \frac{75e^2x^2+2d(6dc^2+25e)}{x^4\sqrt{c^2x^2-1}}dx+\frac{3d^2\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{5x^5} - \frac{2de(a+b\csc^{-1}(cx))}{3x^3} - \frac{e^2(a+b\csc^{-1}(cx))}{x}$$

$$\Big\downarrow \; 359$$

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(24c^4d^2+100c^2de+225e^2\right)\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{2d\sqrt{c^2x^2-1}\left(6c^2d+25e\right)}{3x^3}\right)+\frac{3d^2\sqrt{c^2x^2-1}}{5x^5}\right)}{15\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \; 242$$

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{x}-\frac{bcx\left(\frac{3d^2\sqrt{c^2x^2-1}}{5x^5}+\frac{1}{5}\left(\frac{2d\sqrt{c^2x^2-1}\left(6c^2d+25e\right)}{3x^3}+\frac{\sqrt{c^2x^2-1}\left(24c^4d^2+100c^2de+225e^2\right)}{3x}\right)\right)}{15\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^6,x]
```

output
```
-1/15*(b*c*x*((3*d^2*Sqrt[-1 + c^2*x^2])/(5*x^5) + ((2*d*(6*c^2*d + 25*e)*
Sqrt[-1 + c^2*x^2])/(3*x^3) + ((24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[-
1 + c^2*x^2])/(3*x))/5))/Sqrt[c^2*x^2] - (d^2*(a + b*ArcCsc[c*x]))/(5*x^5)
 - (2*d*e*(a + b*ArcCsc[c*x]))/(3*x^3) - (e^2*(a + b*ArcCsc[c*x]))/x
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.92.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{d^2}{5x^5}-\frac{2de}{3x^3}\right)+b\,c^5\left(-\frac{\operatorname{arccsc}(cx)e^2}{c^5x}-\frac{\operatorname{arccsc}(cx)d^2}{5x^5c^5}-\frac{2\operatorname{arccsc}(cx)de}{3c^5x^3}-\frac{(c^2x^2-1)(24c^8d^2x^4+1\cdots}{\cdots}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)e^2}{cx}-\frac{\operatorname{arccsc}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arccsc}(cx)de}{3c\,x^3}-\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2\cdots}{225\sqrt{\frac{c^2x^2-\cdots}{c^2x^2}}}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)e^2}{cx}-\frac{\operatorname{arccsc}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arccsc}(cx)de}{3c\,x^3}-\frac{(c^2x^2-1)(24c^8d^2x^4+100c^6de\,x^4+12c^6d^2\cdots}{225\sqrt{\frac{c^2x^2-\cdots}{c^2x^2}}}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-e^2/x-1/5*d^2/x^5-2/3*d*e/x^3)+b*c^5*(-1/c^5*arccsc(c*x)*e^2/x-1/5*arc
csc(c*x)*d^2/x^5/c^5-2/3/c^5*arccsc(c*x)*d*e/x^3-1/225/c^10*(c^2*x^2-1)*(2
4*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^
2+9*c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^6)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{225\,ae^2x^4 + 150\,adex^2 + 45\,ad^2 + 15\,(15\,be^2x^4 + 10\,bdex^2 + 3\,bd^2)\,\mathrm{arccsc}\,(cx) + ((24\,bc^4d^2 + 100\,bc^2de}{225\,x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b
*d*e*x^2 + 3*b*d^2)*arccsc(c*x) + ((24*b*c^4*d^2 + 100*b*c^2*d*e + 225*b*e
^2)*x^4 + 9*b*d^2 + 2*(6*b*c^2*d^2 + 25*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^5
```

3.92.6 Sympy [A] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6}\,dx$$
$$= -\frac{ad^2}{5x^5} - \frac{2ade}{3x^3} - \frac{ae^2}{x} - bce^2\sqrt{1-\frac{1}{c^2x^2}} - \frac{bd^2\,\mathrm{acsc}\,(cx)}{5x^5} - \frac{2bde\,\mathrm{acsc}\,(cx)}{3x^3}$$
$$- \frac{be^2\,\mathrm{acsc}\,(cx)}{x} - \frac{bd^2\left(\begin{cases} \frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2|>1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise} \end{cases}\right)}{5c}$$
$$- \frac{2bde\left(\begin{cases} \frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2|>1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**6,x)
```

output
```
-a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x - b*c*e**2*sqrt(1 - 1/(c**2
*x**2)) - b*d**2*acsc(c*x)/(5*x**5) - 2*b*d*e*acsc(c*x)/(3*x**3) - b*e**2*
acsc(c*x)/x - b*d**2*Piecewise((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3
*sqrt(c**2*x**2 - 1)/(15*x**3) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*
x**2) > 1), (8*I*c**5*sqrt(-c**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x*
*2 + 1)/(15*x**3) + I*c*sqrt(-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - 2*b*
d*e*Piecewise((2*c**3*sqrt(c**2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3
*x**3), Abs(c**2*x**2) > 1), (2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sq
rt(-c**2*x**2 + 1)/(3*x**3), True))/(3*c)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6}\,dx$$
$$= -\left(c\sqrt{-\frac{1}{c^2x^2}+1}+\frac{\operatorname{arccsc}(cx)}{x}\right)be^2 - \frac{1}{75}\,bd^2\left(\frac{3\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c}+\frac{15\,\operatorname{arccsc}(cx)}{x^5}\right) + \frac{2}{9}\,bde\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arccsc}(cx)}{x^3}\right)-\frac{ae^2}{x}-\frac{2\,ade}{3\,x^3}-\frac{ad^2}{5\,x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

output
```
-(c*sqrt(-1/(c^2*x^2) + 1) + arccsc(c*x)/x)*b*e^2 - 1/75*b*d^2*((3*c^6*(-1
/(c^2*x^2) + 1)^(5/2) - 10*c^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(
c^2*x^2) + 1))/c + 15*arccsc(c*x)/x^5) + 2/9*b*d*e*((c^4*(-1/(c^2*x^2) + 1
)^(3/2) - 3*c^4*sqrt(-1/(c^2*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - a*e^2/x -
 2/3*a*d*e/x^3 - 1/5*a*d^2/x^5
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6} dx =$$
$$-\frac{1}{225}\left(9\,bc^4d^2\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1}-30\,bc^4d^2\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+\frac{45\,bc^3d^2\left(\frac{1}{c^2x^2}-1\right)^2\arcsin\left(\frac{1}{cx}\right)}{x}+\right.$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/225*(9*b*c^4*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) - 30*b*c^4*
d^2*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*d^2*(1/(c^2*x^2) - 1)^2*arcsin(1/(
c*x))/x + 45*b*c^4*d^2*sqrt(-1/(c^2*x^2) + 1) + 90*b*c^3*d^2*(1/(c^2*x^2)
- 1)*arcsin(1/(c*x))/x - 50*b*c^2*d*e*(-1/(c^2*x^2) + 1)^(3/2) + 45*b*c^3*
d^2*arcsin(1/(c*x))/x + 150*b*c^2*d*e*sqrt(-1/(c^2*x^2) + 1) + 150*b*c*d*e
*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x + 150*b*c*d*e*arcsin(1/(c*x))/x + 225
*b*e^2*sqrt(-1/(c^2*x^2) + 1) + 225*b*e^2*arcsin(1/(c*x))/(c*x) + 225*a*e^
2/(c*x) + 150*a*d*e/(c*x^3) + 45*a*d^2/(c*x^5))*c
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^6} dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6} dx$$

input
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^6, x)
```

3.93 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^8}\,dx$

3.93.1 Optimal result

Integrand size = 21, antiderivative size = 241

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^8}\,dx = -\frac{2bc^3(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025\sqrt{c^2x^2}} - \frac{bcd^2\sqrt{-1+c^2x^2}}{49x^6\sqrt{c^2x^2}} - \frac{2bcd(15c^2d+49e)\sqrt{-1+c^2x^2}}{1225x^4\sqrt{c^2x^2}} - \frac{bc(360c^4d^2+1176c^2de+1225e^2)\sqrt{-1+c^2x^2}}{11025x^2\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{7x^7} - \frac{2de(a+b\csc^{-1}(cx))}{5x^5} - \frac{e^2(a+b\csc^{-1}(cx))}{3x^3}$$

output
```
-1/7*d^2*(a+b*arccsc(c*x))/x^7-2/5*d*e*(a+b*arccsc(c*x))/x^5-1/3*e^2*(a+b*
arccsc(c*x))/x^3-2/11025*b*c^3*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^
2-1)^(1/2)/(c^2*x^2)^(1/2)-1/49*b*c*d^2*(c^2*x^2-1)^(1/2)/x^6/(c^2*x^2)^(1
/2)-2/1225*b*c*d*(15*c^2*d+49*e)*(c^2*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-1/1
1025*b*c*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*(c^2*x^2-1)^(1/2)/x^2/(c^2*x^
2)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8} \, dx =$$
$$-\frac{105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(1225e^2x^4(1+2c^2x^2)+294dex^2(3+4c^2x^2+8c^4x^4)+}{11025x^7}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^8,x]
```

output
```
-1/11025*(105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 - 1/(c^2*x
^2)]*x*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*
x^4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) + 105*b*(15*d^2 +
42*d*e*x^2 + 35*e^2*x^4)*ArcCsc[c*x])/x^7
```

3.93.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5762, 27, 1588, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8} \, dx$$
$$\Big\downarrow \ 5762$$
$$\frac{bcx\int -\frac{35e^2x^4+42dex^2+15d^2}{105x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{7x^7}-\frac{2de(a+b\csc^{-1}(cx))}{5x^5}-\frac{e^2(a+b\csc^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \ 27$$
$$-\frac{bcx\int \frac{35e^2x^4+42dex^2+15d^2}{x^8\sqrt{c^2x^2-1}}dx}{105\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{7x^7}-\frac{2de(a+b\csc^{-1}(cx))}{5x^5}-\frac{e^2(a+b\csc^{-1}(cx))}{3x^3}$$
$$\Big\downarrow \ 1588$$

$$-\frac{bcx\left(\frac{1}{7}\int \frac{245e^2x^2+6d(15dc^2+49e)}{x^6\sqrt{c^2x^2-1}}dx+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}\right)}{105\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}$$

$\Big\downarrow$ 359

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(360c^4d^2+1176c^2de+1225e^2\right)\int \frac{1}{x^4\sqrt{c^2x^2-1}}dx+\frac{6d\sqrt{c^2x^2-1}(15c^2d+49e)}{5x^5}\right)+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}\right)}{105\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}$$

$\Big\downarrow$ 245

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(360c^4d^2+1176c^2de+1225e^2\right)\left(\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)+\frac{6d\sqrt{c^2x^2-1}(15c^2d+49e)}{5x^5}\right)+\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}\right)}{105\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}$$

$\Big\downarrow$ 242

$$-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\csc^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\csc^{-1}(cx)\right)}{3x^3}-\frac{bcx\left(\frac{15d^2\sqrt{c^2x^2-1}}{7x^7}+\frac{1}{7}\left(\frac{6d\sqrt{c^2x^2-1}(15c^2d+49e)}{5x^5}+\frac{1}{5}\left(\frac{2c^2\sqrt{c^2x^2-1}}{3x}+\frac{\sqrt{c^2x^2-1}}{3x^3}\right)\left(360c^4d^2+1176c^2de+1225e^2\right)\right)\right)}{105\sqrt{c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^8,x]
```

output
```
-1/105*(b*c*x*((15*d^2*Sqrt[-1 + c^2*x^2])/(7*x^7) + ((6*d*(15*c^2*d + 49*
e)*Sqrt[-1 + c^2*x^2])/(5*x^5) + ((360*c^4*d^2 + 1176*c^2*d*e + 1225*e^2)*
(Sqrt[-1 + c^2*x^2]/(3*x^3) + (2*c^2*Sqrt[-1 + c^2*x^2])/(3*x)))/5)/7))/Sq
rt[c^2*x^2] - (d^2*(a + b*ArcCsc[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcCsc[c*x
]))/(5*x^5) - (e^2*(a + b*ArcCsc[c*x]))/(3*x^3)
```

3.93. $\int \frac{\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)}{x^8}\,dx$

3.93.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.93. $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^8}\,dx$

3.93.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

method	result
parts	$a\left(-\frac{d^2}{7x^7}-\frac{2de}{5x^5}-\frac{e^2}{3x^3}\right)+b\,c^7\left(-\frac{\operatorname{arccsc}(cx)d^2}{7x^7c^7}-\frac{2\operatorname{arccsc}(cx)de}{5c^7x^5}-\frac{\operatorname{arccsc}(cx)e^2}{3c^7x^3}-\frac{(c^2x^2-1)\left(720c^{10}d^2x^6\right.}{}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arccsc}(cx)de}{5c^3x^5}-\frac{(c^2x^2-1)\left(720c^{10}d^2x^6+2352c^8de\,x^6\right.}{}\right.}{}\right.$
default	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{d^2}{7c^3x^7}-\frac{2de}{5c^3x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsc}(cx)e^2}{3c^3x^3}-\frac{\operatorname{arccsc}(cx)d^2}{7c^3x^7}-\frac{2\operatorname{arccsc}(cx)de}{5c^3x^5}-\frac{(c^2x^2-1)\left(720c^{10}d^2x^6+2352c^8de\,x^6\right.}{}\right.}{}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7*d^2/x^7-2/5*d*e/x^5-1/3/x^3*e^2)+b*c^7*(-1/7*arccsc(c*x)*d^2/x^7/c
^7-2/5/c^7*arccsc(c*x)*d*e/x^5-1/3/c^7*arccsc(c*x)/x^3*e^2-1/11025/c^12*(c
^2*x^2-1)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*
x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*
c^4*d^2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x^8)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\,(35\,be^2x^4+42\,bdex^2+15\,bd^2)\operatorname{arccsc}(cx)+(2\,(360\,bc^6d^2+}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
 + 42*b*d*e*x^2 + 15*b*d^2)*arccsc(c*x) + (2*(360*b*c^6*d^2 + 1176*b*c^4*d
*e + 1225*b*c^2*e^2)*x^6 + (360*b*c^4*d^2 + 1176*b*c^2*d*e + 1225*b*e^2)*x
^4 + 225*b*d^2 + 18*(15*b*c^2*d^2 + 49*b*d*e)*x^2)*sqrt(c^2*x^2 - 1))/x^7
```

3.93.6 Sympy [A] (verification not implemented)

Time = 30.57 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8}\,dx$$
$$= -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bd^2\operatorname{acsc}(cx)}{7x^7} - \frac{2bde\operatorname{acsc}(cx)}{5x^5} - \frac{be^2\operatorname{acsc}(cx)}{3x^3}$$
$$- \frac{bd^2\left(\begin{cases}\frac{16c^7\sqrt{c^2x^2-1}}{35x} + \frac{8c^5\sqrt{c^2x^2-1}}{35x^3} + \frac{6c^3\sqrt{c^2x^2-1}}{35x^5} + \frac{c\sqrt{c^2x^2-1}}{7x^7} & \text{for } |c^2x^2| > 1 \\ \frac{16ic^7\sqrt{-c^2x^2+1}}{35x} + \frac{8ic^5\sqrt{-c^2x^2+1}}{35x^3} + \frac{6ic^3\sqrt{-c^2x^2+1}}{35x^5} + \frac{ic\sqrt{-c^2x^2+1}}{7x^7} & \text{otherwise}\end{cases}\right)}{7c}$$
$$- \frac{2bde\left(\begin{cases}\frac{8c^5\sqrt{c^2x^2-1}}{15x} + \frac{4c^3\sqrt{c^2x^2-1}}{15x^3} + \frac{c\sqrt{c^2x^2-1}}{5x^5} & \text{for } |c^2x^2| > 1 \\ \frac{8ic^5\sqrt{-c^2x^2+1}}{15x} + \frac{4ic^3\sqrt{-c^2x^2+1}}{15x^3} + \frac{ic\sqrt{-c^2x^2+1}}{5x^5} & \text{otherwise}\end{cases}\right)}{5c}$$
$$- \frac{be^2\left(\begin{cases}\frac{2c^3\sqrt{c^2x^2-1}}{3x} + \frac{c\sqrt{c^2x^2-1}}{3x^3} & \text{for } |c^2x^2| > 1 \\ \frac{2ic^3\sqrt{-c^2x^2+1}}{3x} + \frac{ic\sqrt{-c^2x^2+1}}{3x^3} & \text{otherwise}\end{cases}\right)}{3c}$$

input
```
integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**8,x)
```

output
```
-a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*d**2*acsc(c*x)/(
7*x**7) - 2*b*d*e*acsc(c*x)/(5*x**5) - b*e**2*acsc(c*x)/(3*x**3) - b*d**2*
Piecewise((16*c**7*sqrt(c**2*x**2 - 1)/(35*x) + 8*c**5*sqrt(c**2*x**2 - 1)
/(35*x**3) + 6*c**3*sqrt(c**2*x**2 - 1)/(35*x**5) + c*sqrt(c**2*x**2 - 1)/
(7*x**7), Abs(c**2*x**2) > 1), (16*I*c**7*sqrt(-c**2*x**2 + 1)/(35*x) + 8*
I*c**5*sqrt(-c**2*x**2 + 1)/(35*x**3) + 6*I*c**3*sqrt(-c**2*x**2 + 1)/(35*
x**5) + I*c*sqrt(-c**2*x**2 + 1)/(7*x**7), True))/(7*c) - 2*b*d*e*Piecewis
e((8*c**5*sqrt(c**2*x**2 - 1)/(15*x) + 4*c**3*sqrt(c**2*x**2 - 1)/(15*x**3
) + c*sqrt(c**2*x**2 - 1)/(5*x**5), Abs(c**2*x**2) > 1), (8*I*c**5*sqrt(-c
**2*x**2 + 1)/(15*x) + 4*I*c**3*sqrt(-c**2*x**2 + 1)/(15*x**3) + I*c*sqrt(
-c**2*x**2 + 1)/(5*x**5), True))/(5*c) - b*e**2*Piecewise((2*c**3*sqrt(c**
2*x**2 - 1)/(3*x) + c*sqrt(c**2*x**2 - 1)/(3*x**3), Abs(c**2*x**2) > 1), (
2*I*c**3*sqrt(-c**2*x**2 + 1)/(3*x) + I*c*sqrt(-c**2*x**2 + 1)/(3*x**3), T
rue))/(3*c)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8}\,dx$$
$$= \frac{1}{245}bd^2\left(\frac{5c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} - 21c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35c^8\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 35c^8\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{35\,\mathrm{arccsc}\,(cx}{x^7}\right.$$
$$- \frac{2}{75}bde\left(\frac{3c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10c^6\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15c^6\sqrt{-\frac{1}{c^2x^2}+1}}{c} + \frac{15\,\mathrm{arccsc}\,(cx)}{x^5}\right)$$
$$+ \frac{1}{9}be^2\left(\frac{c^4\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3c^4\sqrt{-\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\mathrm{arccsc}\,(cx)}{x^3}\right) - \frac{ae^2}{3x^3} - \frac{2ade}{5x^5} - \frac{ad^2}{7x^7}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/245*b*d^2*((5*c^8*(-1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(-1/(c^2*x^2) + 1)^(
5/2) + 35*c^8*(-1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(-1/(c^2*x^2) + 1))/c
- 35*arccsc(c*x)/x^7) - 2/75*b*d*e*((3*c^6*(-1/(c^2*x^2) + 1)^(5/2) - 10*c
^6*(-1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(-1/(c^2*x^2) + 1))/c + 15*arccsc
(c*x)/x^5) + 1/9*b*e^2*((c^4*(-1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(-1/(c^2
*x^2) + 1))/c - 3*arccsc(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a
*d^2/x^7
```

3.93.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(211) = 422$.

Time = 0.30 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^8}\,dx =$$
$$-\frac{1}{11025}\left(225\,bc^6d^2\left(\frac{1}{c^2x^2}-1\right)^3\sqrt{-\frac{1}{c^2x^2}+1} + 945\,bc^6d^2\left(\frac{1}{c^2x^2}-1\right)^2\sqrt{-\frac{1}{c^2x^2}+1} + \frac{1575\,bc^5d^2\left(\frac{1}{c^2x^2}\right.}{x}\right.$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

output
```
-1/11025*(225*b*c^6*d^2*(1/(c^2*x^2) - 1)^3*sqrt(-1/(c^2*x^2) + 1) + 945*b
*c^6*d^2*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2) + 1) + 1575*b*c^5*d^2*(1/(c
^2*x^2) - 1)^3*arcsin(1/(c*x))/x - 1575*b*c^6*d^2*(-1/(c^2*x^2) + 1)^(3/2)
 + 4725*b*c^5*d^2*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 1575*b*c^6*d^2*s
qrt(-1/(c^2*x^2) + 1) + 882*b*c^4*d*e*(1/(c^2*x^2) - 1)^2*sqrt(-1/(c^2*x^2
) + 1) + 4725*b*c^5*d^2*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 2940*b*c^4*d
*e*(-1/(c^2*x^2) + 1)^(3/2) + 1575*b*c^5*d^2*arcsin(1/(c*x))/x + 4410*b*c^
3*d*e*(1/(c^2*x^2) - 1)^2*arcsin(1/(c*x))/x + 4410*b*c^4*d*e*sqrt(-1/(c^2*
x^2) + 1) + 8820*b*c^3*d*e*(1/(c^2*x^2) - 1)*arcsin(1/(c*x))/x - 1225*b*c^
2*e^2*(-1/(c^2*x^2) + 1)^(3/2) + 4410*b*c^3*d*e*arcsin(1/(c*x))/x + 3675*b
*c^2*e^2*sqrt(-1/(c^2*x^2) + 1) + 3675*b*c*e^2*(1/(c^2*x^2) - 1)*arcsin(1/
(c*x))/x + 3675*b*c*e^2*arcsin(1/(c*x))/x + 3675*a*e^2/(c*x^3) + 4410*a*d*
e/(c*x^5) + 1575*a*d^2/(c*x^7))*c
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\,(a+b\csc^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^8, x)
```

3.94 $\int x^3(d+ex^2)^2(a+b\csc^{-1}(cx))\,dx$

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 242

$$\begin{aligned}\int x^3(d+ex^2)^2(a+b\csc^{-1}(cx))\,dx &= \frac{b(6c^4d^2+8c^2de+3e^2)\,x\sqrt{-1+c^2x^2}}{24c^7\sqrt{c^2x^2}}+\frac{b(6c^4d^2+16c^2de+9e^2)\,x(-1+c^2x^2)^{3/2}}{72c^7\sqrt{c^2x^2}}\\&+\frac{be(8c^2d+9e)\,x(-1+c^2x^2)^{5/2}}{120c^7\sqrt{c^2x^2}}+\frac{be^2x(-1+c^2x^2)^{7/2}}{56c^7\sqrt{c^2x^2}}\\&+\frac{1}{4}d^2x^4(a+b\csc^{-1}(cx))+\frac{1}{3}dex^6(a+b\csc^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\csc^{-1}(cx))\end{aligned}$$

output
```
1/4*d^2*x^4*(a+b*arccsc(c*x))+1/3*d*e*x^6*(a+b*arccsc(c*x))+1/8*e^2*x^8*(a
+b*arccsc(c*x))+1/72*b*(6*c^4*d^2+16*c^2*d*e+9*e^2)*x*(c^2*x^2-1)^(3/2)/c^
7/(c^2*x^2)^(1/2)+1/120*b*e*(8*c^2*d+9*e)*x*(c^2*x^2-1)^(5/2)/c^7/(c^2*x^2
)^(1/2)+1/56*b*e^2*x*(c^2*x^2-1)^(7/2)/c^7/(c^2*x^2)^(1/2)+1/24*b*(6*c^4*d
^2+8*c^2*d*e+3*e^2)*x*(c^2*x^2-1)^(1/2)/c^7/(c^2*x^2)^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.66

$$\int x^3(d+ex^2)^2(a+b\csc^{-1}(cx))\,dx$$
$$=\frac{x\left(105ax^3(6d^2+8dex^2+3e^2x^4)+\frac{b\sqrt{1-\frac{1}{c^2x^2}}(144e^2+8c^2e(56d+9ex^2)+c^4(420d^2+224dex^2+54e^2x^4)+3c^6(70d^2x^2+56dex^4+15e^2x^6))}{c^7}+105bx^3(6d^2+8dex^2+3e^2x^4)\csc^{-1}(cx)\right)}{2520}$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(
144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x
^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6*d^
2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCsc[c*x]))/2520
```

3.94.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5762, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)^2(a+b\csc^{-1}(cx))\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int\frac{x^3(3e^2x^4+8dex^2+6d^2)}{24\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4(a+b\csc^{-1}(cx))+\frac{1}{3}dex^6(a+b\csc^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int\frac{x^3(3e^2x^4+8dex^2+6d^2)}{\sqrt{c^2x^2-1}}dx}{24\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4(a+b\csc^{-1}(cx))+\frac{1}{3}dex^6(a+b\csc^{-1}(cx))+\frac{1}{8}e^2x^8(a+b\csc^{-1}(cx))$$

$$\Big\downarrow \text{1578}$$

$$\frac{bcx\int \frac{x^2(3e^2x^4+8dex^2+6d^2)}{\sqrt{c^2x^2-1}}dx^2}{48\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{1195}$$

$$\frac{bcx\int \left(\frac{3e^2(c^2x^2-1)^{5/2}}{c^6}+\frac{e(8dc^2+9e)(c^2x^2-1)^{3/2}}{c^6}+\frac{(6d^2c^4+16dec^2+9e^2)\sqrt{c^2x^2-1}}{c^6}+\frac{6d^2c^4+8dec^2+3e^2}{c^6\sqrt{c^2x^2-1}}\right)dx^2}{48\sqrt{c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}d^2x^4\left(a+b\csc^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\csc^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\csc^{-1}(cx)\right)+\frac{bcx\left(\frac{2e(c^2x^2-1)^{5/2}(8c^2d+9e)}{5c^8}+\frac{6e^2(c^2x^2-1)^{7/2}}{7c^8}+\frac{2(c^2x^2-1)^{3/2}(6c^4d^2+16c^2de+9e^2)}{3c^8}+\frac{2\sqrt{c^2x^2-1}(6c^4d^2+8c^2de+3e^2)}{c^8}\right)}{48\sqrt{c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(b*c*x*((2*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*Sqrt[-1 + c^2*x^2])/c^8 + (2*(6
*c^4*d^2 + 16*c^2*d*e + 9*e^2)*(-1 + c^2*x^2)^(3/2))/(3*c^8) + (2*e*(8*c^2
*d + 9*e)*(-1 + c^2*x^2)^(5/2))/(5*c^8) + (6*e^2*(-1 + c^2*x^2)^(7/2))/(7*
c^8)))/(48*Sqrt[c^2*x^2]) + (d^2*x^4*(a + b*ArcCsc[c*x]))/4 + (d*e*x^6*(a
+ b*ArcCsc[c*x]))/3 + (e^2*x^8*(a + b*ArcCsc[c*x]))/8
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.94.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.82

method	result
parts	$a\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}x^4d^2\right)+\frac{b\left(\frac{c^4\operatorname{arccsc}(cx)e^2x^8}{8}+\frac{c^4\operatorname{arccsc}(cx)de\,x^6}{3}+\frac{\operatorname{arccsc}(cx)d^2x^4c^4}{4}+\frac{\left(c^2x^2-1\right)\left(45c^6e^2x^6 \cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^2d\left(c^2e\,x^2+c^2d\right)^3}{3}-\frac{\left(c^2e\,x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{b\,c^4\operatorname{arccsc}(cx)d^4}{24e^2}+\frac{b\operatorname{arccsc}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\operatorname{arccsc}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\operatorname{arccsc}(cx)x^8}{8}\cdots}{\cdots}$
default	$\frac{-\frac{a\left(\frac{c^2d\left(c^2e\,x^2+c^2d\right)^3}{3}-\frac{\left(c^2e\,x^2+c^2d\right)^4}{4}\right)}{2c^4e^2}-\frac{b\,c^4\operatorname{arccsc}(cx)d^4}{24e^2}+\frac{b\operatorname{arccsc}(cx)d^2c^4x^4}{4}+\frac{b\,c^4e\operatorname{arccsc}(cx)d\,x^6}{3}+\frac{b\,c^4e^2\operatorname{arccsc}(cx)x^8}{8}\cdots}{\cdots}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*x^4*d^2)+b/c^4*(1/8*c^4*arccsc(c*x)*e^2*x^8
+1/3*c^4*arccsc(c*x)*d*e*x^6+1/4*arccsc(c*x)*d^2*x^4*c^4+1/2520/c^5*(c^2*x
^2-1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2+54*c^4*e^2*x^4+224*c
^4*d*e*x^2+420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e+144*e^2)/((c^2*x^2-1)/c^
2/x^2)^(1/2)/x)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.77

$$\int x^3(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^8e^2x^8+840\,ac^8dex^6+630\,ac^8d^2x^4+105\left(3\,bc^8e^2x^8+8\,bc^8dex^6+6\,bc^8d^2x^4\right)\operatorname{arccsc}(cx)+\left(45\,bc^6e^2\right.}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 + 630*a*c^8*d^2*x^4 + 105*(3
*b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4)*arccsc(c*x) + (45*b*c^
6*e^2*x^6 + 420*b*c^4*d^2 + 448*b*c^2*d*e + 6*(28*b*c^6*d*e + 9*b*c^4*e^2)
*x^4 + 144*b*e^2 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^2)*s
qrt(c^2*x^2 - 1))/c^8
```

3.94.6 Sympy [A] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.04

$$\int x^3 (d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx = \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{acsc}(cx)}{4} + \frac{bdex^6\operatorname{acsc}(cx)}{3} + \frac{be^2x^8\operatorname{acsc}(cx)}{8} + \frac{bd^2\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{4c} + \frac{bde\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{3c} + \frac{be^2\left(\begin{cases}\frac{x^6\sqrt{c^2x^2-1}}{7c}+\frac{6x^4\sqrt{c^2x^2-1}}{35c^3}+\frac{8x^2\sqrt{c^2x^2-1}}{35c^5}+\frac{16\sqrt{c^2x^2-1}}{35c^7} & \text{for } |c^2x^2|>1\\ \frac{ix^6\sqrt{-c^2x^2+1}}{7c}+\frac{6ix^4\sqrt{-c^2x^2+1}}{35c^3}+\frac{8ix^2\sqrt{-c^2x^2+1}}{35c^5}+\frac{16i\sqrt{-c^2x^2+1}}{35c^7} & \text{otherwise}\end{cases}\right)}{8c}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

output
```
a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*acsc(c*x)/4 + b
*d*e*x**6*acsc(c*x)/3 + b*e**2*x**8*acsc(c*x)/8 + b*d**2*Piecewise((x**2*s
qrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 - 1)/(3*c**3), Abs(c**2*x**2)
> 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*I*sqrt(-c**2*x**2 + 1)/(3*c**
3), True))/(4*c) + b*d*e*Piecewise((x**4*sqrt(c**2*x**2 - 1)/(5*c) + 4*x**
2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**2*x**2 - 1)/(15*c**5), Abs(c**
2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(5*c) + 4*I*x**2*sqrt(-c**2*x**
2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/(15*c**5), True))/(3*c) + b*e*
*2*Piecewise((x**6*sqrt(c**2*x**2 - 1)/(7*c) + 6*x**4*sqrt(c**2*x**2 - 1)/
(35*c**3) + 8*x**2*sqrt(c**2*x**2 - 1)/(35*c**5) + 16*sqrt(c**2*x**2 - 1)/
(35*c**7), Abs(c**2*x**2) > 1), (I*x**6*sqrt(-c**2*x**2 + 1)/(7*c) + 6*I*x
**4*sqrt(-c**2*x**2 + 1)/(35*c**3) + 8*I*x**2*sqrt(-c**2*x**2 + 1)/(35*c**
5) + 16*I*sqrt(-c**2*x**2 + 1)/(35*c**7), True))/(8*c)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05

$$\int x^3(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{12}\left(3\,x^4\,\mathrm{arccsc}\,(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2 + \frac{1}{45}\left(15\,x^6\,\mathrm{arccsc}\,(cx) + \frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)bde + \frac{1}{280}\left(35\,x^8\,\mathrm{arccsc}\,(cx) + \frac{5\,c^6x^7\left(-\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} + 21\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 35\,x\sqrt{-}}{c^7}\right.$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arccsc(c*x) +
(c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d^2
 + 1/45*(15*x^6*arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2
*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*d*e +
1/280*(35*x^8*arccsc(c*x) + (5*c^6*x^7*(-1/(c^2*x^2) + 1)^(7/2) + 21*c^4*x
^5*(-1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(-1/(c^2*x^2) + 1)^(3/2) + 35*x*s
qrt(-1/(c^2*x^2) + 1))/c^7)*b*e^2
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1706 vs. 2(212) = 424.

Time = 0.45 (sec) , antiderivative size = 1706, normalized size of antiderivative = 7.05

$$\int x^3(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/645120*(315*b*e^2*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8*arcsin(1/(c*x))/c +
 315*a*e^2*x^8*(sqrt(-1/(c^2*x^2) + 1) + 1)^8/c + 90*b*e^2*x^7*(sqrt(-1/(c
^2*x^2) + 1) + 1)^7/c^2 + 3360*b*d*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*ar
csin(1/(c*x))/c + 3360*a*d*e*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 2520*b
*e^2*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c^3 + 2520*a*e^2*x
^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c^3 + 1344*b*d*e*x^5*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^5/c^2 + 10080*b*d^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/
(c*x))/c + 10080*a*d^2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 882*b*e^2*x^
5*(sqrt(-1/(c^2*x^2) + 1) + 1)^5/c^4 + 20160*b*d*e*x^4*(sqrt(-1/(c^2*x^2)
+ 1) + 1)^4*arcsin(1/(c*x))/c^3 + 20160*a*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1)
+ 1)^4/c^3 + 6720*b*d^2*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^2 + 8820*b*e^
2*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^5 + 8820*a*e^2*x^4*
(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c^5 + 11200*b*d*e*x^3*(sqrt(-1/(c^2*x^2) +
1) + 1)^3/c^4 + 40320*b*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c
*x))/c^3 + 40320*a*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 4410*b*e^2
*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/c^6 + 50400*b*d*e*x^2*(sqrt(-1/(c^2*x^
2) + 1) + 1)^2*arcsin(1/(c*x))/c^5 + 50400*a*d*e*x^2*(sqrt(-1/(c^2*x^2) +
1) + 1)^2/c^5 + 60480*b*d^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 17640*b*e
^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c^7 + 17640*a*e^2*x^
2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^7 + 67200*b*d*e*x*(sqrt(-1/(c^2*x^2)...
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^3\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.95 $\int x(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx$

3.95.1 Optimal result

Integrand size = 19, antiderivative size = 195

$$\int x(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx = \frac{b(3c^4d^2+3c^2de+e^2)\,x\sqrt{-1+c^2x^2}}{6c^5\sqrt{c^2x^2}} + \frac{be(3c^2d+2e)\,x(-1+c^2x^2)^{3/2}}{18c^5\sqrt{c^2x^2}} + \frac{be^2x(-1+c^2x^2)^{5/2}}{30c^5\sqrt{c^2x^2}} + \frac{(d+ex^2)^3 (a+b\csc^{-1}(cx))}{6e} + \frac{bcd^3x\arctan\left(\sqrt{-1+c^2x^2}\right)}{6e\sqrt{c^2x^2}}$$

output

```
1/6*(e*x^2+d)^3*(a+b*arccsc(c*x))/e+1/18*b*e*(3*c^2*d+2*e)*x*(c^2*x^2-1)^(
3/2)/c^5/(c^2*x^2)^(1/2)+1/30*b*e^2*x*(c^2*x^2-1)^(5/2)/c^5/(c^2*x^2)^(1/2
)+1/6*b*c*d^3*x*arctan((c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/6*b*(3*c^4*d
^2+3*c^2*d*e+e^2)*x*(c^2*x^2-1)^(1/2)/c^5/(c^2*x^2)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.64

$$\int x(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right) + \frac{b\sqrt{1-\frac{1}{c^2x^2}}(8e^2+2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4))}{c^5} + 15bx\left(3d^2+3dex^2+e^2x^4\right)\csc^{-1}(cx)\right)$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 - 1/(c^2*x^2)]*(8*e^2
 + 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 +
 15*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsc[c*x]))/90
```

3.95.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5760, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 5760}$$

$$\frac{bcx\int\frac{(ex^2+d)^3}{x\sqrt{c^2x^2-1}}dx}{6e\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\left(a+b\csc^{-1}(cx)\right)}{6e}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{bcx\int\frac{(ex^2+d)^3}{x^2\sqrt{c^2x^2-1}}dx^2}{12e\sqrt{c^2x^2}} + \frac{(d+ex^2)^3\left(a+b\csc^{-1}(cx)\right)}{6e}$$

$$\Big\downarrow \text{99}$$

$$\frac{bcx\int\left(\frac{d^3}{x^2\sqrt{c^2x^2-1}}+\frac{e^3\left(c^2x^2-1\right)^{3/2}}{c^4}+\frac{e^2\left(3dc^2+2e\right)\sqrt{c^2x^2-1}}{c^4}+\frac{e\left(3d^2c^4+3dec^2+e^2\right)}{c^4\sqrt{c^2x^2-1}}\right)dx^2}{12e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)}{6e}+\frac{bcx\left(2d^3\arctan\left(\sqrt{c^2x^2-1}\right)+\frac{2e^2\left(c^2x^2-1\right)^{3/2}\left(3c^2d+2e\right)}{3c^6}+\frac{2e^3\left(c^2x^2-1\right)^{5/2}}{5c^6}+\frac{2e\sqrt{c^2x^2-1}\left(3c^4d^2+3c^2de+e^2\right)}{c^6}\right)}{12e\sqrt{c^2x^2}}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcCsc[c*x]))/(6*e) + (b*c*x*((2*e*(3*c^4*d^2 + 3*c^
2*d*e + e^2)*Sqrt[-1 + c^2*x^2])/c^6 + (2*e^2*(3*c^2*d + 2*e)*(-1 + c^2*x^
2)^(3/2))/(3*c^6) + (2*e^3*(-1 + c^2*x^2)^(5/2))/(5*c^6) + 2*d^3*ArcTan[Sq
rt[-1 + c^2*x^2]]))/(12*e*Sqrt[c^2*x^2])
```

3.95.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(169) = 338$.

Time = 0.97 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

method	result
parts	$\frac{a(e x^2+d)^3}{6e} + \frac{b \operatorname{arccsc}(cx)e^2x^6}{6} + \frac{b \operatorname{arccsc}(cx)de x^4}{2} + \frac{b \operatorname{arccsc}(cx)d^2x^2}{2} + \frac{b d^3 \operatorname{arccsc}(cx)}{6e} + \frac{b(c^2x^2-1)x^3e^2}{30c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}} +$
derivativedivides	$\frac{\frac{a\left(c^2 e x^2+c^2 d\right)^3}{6c^4 e} + \frac{b c^2 \operatorname{arccsc}(cx)d^3}{6e} + \frac{b \operatorname{arccsc}(cx)d^2c^2x^2}{2} + \frac{b c^2 e \operatorname{arccsc}(cx)d x^4}{2} + \frac{b c^2 e^2 \operatorname{arccsc}(cx)x^6}{6} - \frac{bc\sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{}{\sqrt{c}}\right.}{6e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}}{c^2}$
default	$\frac{\frac{a\left(c^2 e x^2+c^2 d\right)^3}{6c^4 e} + \frac{b c^2 \operatorname{arccsc}(cx)d^3}{6e} + \frac{b \operatorname{arccsc}(cx)d^2c^2x^2}{2} + \frac{b c^2 e \operatorname{arccsc}(cx)d x^4}{2} + \frac{b c^2 e^2 \operatorname{arccsc}(cx)x^6}{6} - \frac{bc\sqrt{c^2x^2-1}\, d^3 \arctan\left(\frac{}{\sqrt{c}}\right.}{6e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\, x}}{c^2}$

input
```
int(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*(e*x^2+d)^3/e+1/6*b*arccsc(c*x)*e^2*x^6+1/2*b*arccsc(c*x)*d*e*x^4+1/
2*b*arccsc(c*x)*d^2*x^2+1/6*b*d^3*arccsc(c*x)/e+1/30*b/c^3*(c^2*x^2-1)/((c
^2*x^2-1)/c^2/x^2)^(1/2)*x^3*e^2+1/6*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^
2)^(1/2)*x*d*e-1/6*b/c/e*(c^2*x^2-1)^(1/2)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d
^3*arctan(1/(c^2*x^2-1)^(1/2))+2/45*b/c^5*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2
)^(1/2)*x*e^2+1/2*b/c^3*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d^2+1/3*
b/c^5*e*(c^2*x^2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x*d+4/45*b/c^7*e^2*(c^2*x^
2-1)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.78

$$\int x(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{15\,ac^6e^2x^6+45\,ac^6dex^4+45\,ac^6d^2x^2+15\,(bc^6e^2x^6+3\,bc^6dex^4+3\,bc^6d^2x^2)\,\mathrm{arccsc}\,(cx)+(3\,bc^4e^2x^4+45}{90\,c^6}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 + 45*a*c^6*d^2*x^2 + 15*(b*c^6*e
^2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2)*arccsc(c*x) + (3*b*c^4*e^2*x^4
 + 45*b*c^4*d^2 + 30*b*c^2*d*e + 8*b*e^2 + (15*b*c^4*d*e + 4*b*c^2*e^2)*x^
2)*sqrt(c^2*x^2 - 1))/c^6
```

3.95.6 Sympy [A] (verification not implemented)

Time = 2.78 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.81

$$\int x(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$=\frac{ad^2x^2}{2}+\frac{adex^4}{2}+\frac{ae^2x^6}{6}+\frac{bd^2x^2\,\mathrm{acsc}\,(cx)}{2}+\frac{bdex^4\,\mathrm{acsc}\,(cx)}{2}$$
$$+\frac{be^2x^6\,\mathrm{acsc}\,(cx)}{6}+\frac{bd^2\left(\begin{cases}\frac{\sqrt{c^2x^2-1}}{c} & \text{for } |c^2x^2|>1\\ \frac{i\sqrt{-c^2x^2+1}}{c} & \text{otherwise}\end{cases}\right)}{2c}$$
$$+\frac{bde\left(\begin{cases}\frac{x^2\sqrt{c^2x^2-1}}{3c}+\frac{2\sqrt{c^2x^2-1}}{3c^3} & \text{for } |c^2x^2|>1\\ \frac{ix^2\sqrt{-c^2x^2+1}}{3c}+\frac{2i\sqrt{-c^2x^2+1}}{3c^3} & \text{otherwise}\end{cases}\right)}{2c}$$
$$+\frac{be^2\left(\begin{cases}\frac{x^4\sqrt{c^2x^2-1}}{5c}+\frac{4x^2\sqrt{c^2x^2-1}}{15c^3}+\frac{8\sqrt{c^2x^2-1}}{15c^5} & \text{for } |c^2x^2|>1\\ \frac{ix^4\sqrt{-c^2x^2+1}}{5c}+\frac{4ix^2\sqrt{-c^2x^2+1}}{15c^3}+\frac{8i\sqrt{-c^2x^2+1}}{15c^5} & \text{otherwise}\end{cases}\right)}{6c}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

output
```
a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*acsc(c*x)/2 + b
*d*e*x**4*acsc(c*x)/2 + b*e**2*x**6*acsc(c*x)/6 + b*d**2*Piecewise((sqrt(c
**2*x**2 - 1)/c, Abs(c**2*x**2) > 1), (I*sqrt(-c**2*x**2 + 1)/c, True))/(2
*c) + b*d*e*Piecewise((x**2*sqrt(c**2*x**2 - 1)/(3*c) + 2*sqrt(c**2*x**2 -
 1)/(3*c**3), Abs(c**2*x**2) > 1), (I*x**2*sqrt(-c**2*x**2 + 1)/(3*c) + 2*
I*sqrt(-c**2*x**2 + 1)/(3*c**3), True))/(2*c) + b*e**2*Piecewise((x**4*sqr
t(c**2*x**2 - 1)/(5*c) + 4*x**2*sqrt(c**2*x**2 - 1)/(15*c**3) + 8*sqrt(c**
2*x**2 - 1)/(15*c**5), Abs(c**2*x**2) > 1), (I*x**4*sqrt(-c**2*x**2 + 1)/(
5*c) + 4*I*x**2*sqrt(-c**2*x**2 + 1)/(15*c**3) + 8*I*sqrt(-c**2*x**2 + 1)/
(15*c**5), True))/(6*c)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97

$$\int x(d+ex^2)^2\left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= \frac{1}{6}\,ae^2x^6 + \frac{1}{2}\,adex^4 + \frac{1}{2}\,ad^2x^2 + \frac{1}{2}\left(x^2\,\mathrm{arccsc}\,(cx) + \frac{x\sqrt{-\frac{1}{c^2x^2}+1}}{c}\right)bd^2$$
$$+ \frac{1}{6}\left(3\,x^4\,\mathrm{arccsc}\,(cx) + \frac{c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 3\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^3}\right)bde$$
$$+ \frac{1}{90}\left(15\,x^6\,\mathrm{arccsc}\,(cx) + \frac{3\,c^4x^5\left(-\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 10\,c^2x^3\left(-\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{-\frac{1}{c^2x^2}+1}}{c^5}\right)be^2$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arccsc(c*x) + x*s
qrt(-1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arccsc(c*x) + (c^2*x^3*(-1/(c^
2*x^2) + 1)^(3/2) + 3*x*sqrt(-1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*
arccsc(c*x) + (3*c^4*x^5*(-1/(c^2*x^2) + 1)^(5/2) + 10*c^2*x^3*(-1/(c^2*x^
2) + 1)^(3/2) + 15*x*sqrt(-1/(c^2*x^2) + 1))/c^5)*b*e^2
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1160 vs. $2(169) = 338$.

Time = 0.39 (sec) , antiderivative size = 1160, normalized size of antiderivative = 5.95

$$\int x(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
1/5760*(15*b*e^2*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6*arcsin(1/(c*x))/c + 15
*a*e^2*x^6*(sqrt(-1/(c^2*x^2) + 1) + 1)^6/c + 6*b*e^2*x^5*(sqrt(-1/(c^2*x^
2) + 1) + 1)^5/c^2 + 180*b*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1
/(c*x))/c + 180*a*d*e*x^4*(sqrt(-1/(c^2*x^2) + 1) + 1)^4/c + 90*b*e^2*x^4*
(sqrt(-1/(c^2*x^2) + 1) + 1)^4*arcsin(1/(c*x))/c^3 + 90*a*e^2*x^4*(sqrt(-1
/(c^2*x^2) + 1) + 1)^4/c^3 + 120*b*d*e*x^3*(sqrt(-1/(c^2*x^2) + 1) + 1)^3/
c^2 + 720*b*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(1/(c*x))/c + 720
*a*d^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c + 50*b*e^2*x^3*(sqrt(-1/(c^2*x
^2) + 1) + 1)^3/c^4 + 720*b*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2*arcsin(
1/(c*x))/c^3 + 720*a*d*e*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2/c^3 + 1440*b*d
^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^2 + 225*b*e^2*x^2*(sqrt(-1/(c^2*x^2) +
 1) + 1)^2*arcsin(1/(c*x))/c^5 + 225*a*e^2*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1
)^2/c^5 + 1080*b*d*e*x*(sqrt(-1/(c^2*x^2) + 1) + 1)/c^4 + 1440*b*d^2*arcsi
n(1/(c*x))/c^3 + 1440*a*d^2/c^3 + 300*b*e^2*x*(sqrt(-1/(c^2*x^2) + 1) + 1)
/c^6 + 1080*b*d*e*arcsin(1/(c*x))/c^5 + 1080*a*d*e/c^5 - 1440*b*d^2/(c^4*x
*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 300*b*e^2*arcsin(1/(c*x))/c^7 + 300*a*e^2
/c^7 - 1080*b*d*e/(c^6*x*(sqrt(-1/(c^2*x^2) + 1) + 1)) + 720*b*d^2*arcsin(
1/(c*x))/(c^5*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2) + 720*a*d^2/(c^5*x^2*(sq
rt(-1/(c^2*x^2) + 1) + 1)^2) - 300*b*e^2/(c^8*x*(sqrt(-1/(c^2*x^2) + 1) +
1)) + 720*b*d*e*arcsin(1/(c*x))/(c^7*x^2*(sqrt(-1/(c^2*x^2) + 1) + 1)^2...
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^2 (a+b\csc^{-1}(cx))\,dx = \int x\,(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.96 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx$

3.96.1 Optimal result

Integrand size = 21, antiderivative size = 186

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx &= \frac{be(6c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x}{6c^3}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}x^3}{12c}\\ &\quad+\frac{1}{2}ibd^2\csc^{-1}(cx)^2+dex^2(a+b\csc^{-1}(cx))\\ &\quad+\frac{1}{4}e^2x^4(a+b\csc^{-1}(cx))\\ &\quad-bd^2\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right)\\ &\quad+bd^2\csc^{-1}(cx)\log\left(\frac{1}{x}\right)-d^2(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right)\\ &\quad+\frac{1}{2}ibd^2\operatorname{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\end{aligned}$$

output
```
1/2*I*b*d^2*arccsc(c*x)^2+d*e*x^2*(a+b*arccsc(c*x))+1/4*e^2*x^4*(a+b*arccs
c(c*x))-b*d^2*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)+b*d^2*arccsc
(c*x)*ln(1/x)-d^2*(a+b*arccsc(c*x))*ln(1/x)+1/2*I*b*d^2*polylog(2,(I/c/x+(
1-1/c^2/x^2)^(1/2))^2)+1/6*b*e*(6*c^2*d+e)*x*(1-1/c^2/x^2)^(1/2)/c^3+1/12*
b*e^2*x^3*(1-1/c^2/x^2)^(1/2)/c
```

3.96.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx = adex^2+\frac{1}{4}ae^2x^4+\frac{bdex\left(\sqrt{1-\frac{1}{c^2x^2}}+cx\csc^{-1}(cx)\right)}{c}+\frac{be^2x\left(\sqrt{1-\frac{1}{c^2x^2}}(2+c^2x^2)+3c^3x^3\csc^{-1}(cx)\right)}{12c^3}+ad^2\log(x)+\frac{1}{2}ibd^2\left(\csc^{-1}(cx)\left(\csc^{-1}(cx)+2i\log\left(1-e^{2i\csc^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
a*d*e*x^2 + (a*e^2*x^4)/4 + (b*d*e*x*(Sqrt[1 - 1/(c^2*x^2)] + c*x*ArcCsc[c
*x]))/c + (b*e^2*x*(Sqrt[1 - 1/(c^2*x^2)]*(2 + c^2*x^2) + 3*c^3*x^3*ArcCsc
[c*x]))/(12*c^3) + a*d^2*Log[x] + (I/2)*b*d^2*(ArcCsc[c*x]*(ArcCsc[c*x] +
(2*I)*Log[1 - E^((2*I)*ArcCsc[c*x])]) + PolyLog[2, E^((2*I)*ArcCsc[c*x])])
```

3.96.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5764, 5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \left(\frac{d}{x^2}+e\right)^2x^5\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{5230}$$

$$\frac{b\int -\frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{4\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 7293$$

$$-\frac{b\int \left(\frac{e\left(\frac{4d}{x^2}+e\right)x^4}{\sqrt{1-\frac{1}{c^2x^2}}}-\frac{4d^2\log\left(\frac{1}{x}\right)}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 2009$$

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)-\frac{b\left(-2icd^2\operatorname{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2icd^2\arcsin\left(\frac{1}{cx}\right)^2+4cd^2\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-4cd^2\log\left(\frac{1}{x}\right)\text{ar}\right.}{4c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
d*e*x^2*(a + b*ArcSin[1/(c*x)]) + (e^2*x^4*(a + b*ArcSin[1/(c*x)]))/4 - d^
2*(a + b*ArcSin[1/(c*x)])*Log[x^(-1)] - (b*((-2*e*(6*d + e/c^2)*Sqrt[1 - 1
/(c^2*x^2)]*x)/3 - (e^2*Sqrt[1 - 1/(c^2*x^2)]*x^3)/3 - (2*I)*c*d^2*ArcSin[
1/(c*x)]^2 + 4*c*d^2*ArcSin[1/(c*x)]*Log[1 - E^((2*I)*ArcSin[1/(c*x)])] -
4*c*d^2*ArcSin[1/(c*x)]*Log[x^(-1)] - (2*I)*c*d^2*PolyLog[2, E^((2*I)*ArcS
in[1/(c*x)])]))/(4*c)
```

3.96. $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x}\,dx$

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5230
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.96.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.55

method	result
parts	$a\left(\frac{e^2x^4}{4}+de\,x^2+d^2\ln\left(x\right)\right)+b\left(\frac{id^2\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(12c^4d\,\mathrm{arccsc}(cx)x^2+3\,\mathrm{arccsc}(cx)e\,c^4x^4+12\sqrt{\frac{c^2x^2}{c^2x}}\right.}{}\right.$
derivativedivides	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+\frac{b\left(\frac{ic^4d^2\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(12c^4d\,\mathrm{arccsc}(cx)x^2+3\,\mathrm{arccsc}(cx)e\,c^4x^4+12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3d\right.}{12}\right.}{}$
default	$ade\,x^2+\frac{a\,e^2x^4}{4}+a\,d^2\ln\left(cx\right)+\frac{b\left(\frac{ic^4d^2\,\mathrm{arccsc}(cx)^2}{2}+\frac{e\left(12c^4d\,\mathrm{arccsc}(cx)x^2+3\,\mathrm{arccsc}(cx)e\,c^4x^4+12\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3d\right.}{12}\right.}{}$

input
```
int((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^2*x^4+d*e*x^2+d^2*ln(x))+b*(1/2*I*d^2*arccsc(c*x)^2+1/12/c^4*e*(1
2*c^4*d*arccsc(c*x)*x^2+3*arccsc(c*x)*e*c^4*x^4+12*((c^2*x^2-1)/c^2/x^2)^(
1/2)*c^3*d*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-12*I*c^2*d+2*((c^2*x^2-
1)/c^2/x^2)^(1/2)*e*c*x-2*I*e)-d^2*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1
/2))-d^2*arccsc(c*x)*ln(1-I/c/x-(1-1/c^2/x^2)^(1/2))+I*d^2*polylog(2,-I/c/
x-(1-1/c^2/x^2)^(1/2))+I*d^2*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2)))
```

3.96.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccsc(c*x))/x, x)
```

3.96.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^2}{x}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x,x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x, x)
```

3.96.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + 1/8*(4*I*b*c^4*d^2*log(-c*x + 1
)*log(x) + 4*I*b*c^4*d^2*log(x)^2 + 4*I*b*c^4*d^2*dilog(c*x) + 4*I*b*c^4*d
^2*dilog(-c*x) + 2*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^
4*log(c))*e^2*x^4 - I*(b*e^2*(x^2/c^2 + log(c*x + 1)/c^4 + log(c*x - 1)/c^
4) + 4*b*d*e*(log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 32*b*d^2*integrate(1/
4*log(x)/(c^2*x^3 - x), x))*c^4 + 8*c^4*integrate(1/4*(b*e^2*x^4 + 4*b*d*e
*x^2 + 4*b*d^2*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/(c^2*x^3 - x), x) + (I*
b*c^2*e^2 + 8*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^4*log
(c))*d*e)*x^2 + (-I*b*c^4*e^2*x^4 - 4*I*b*c^4*d*e*x^2 - 4*I*b*c^4*d^2*log(
x))*log(c^2*x^2) + (4*I*b*c^4*d^2*log(x) + 4*I*b*c^2*d*e + I*b*e^2)*log(c*
x + 1) + (4*I*b*c^2*d*e + I*b*e^2)*log(c*x - 1) - 2*(-I*b*c^4*e^2*x^4 - 4*
I*b*c^4*d*e*x^2 - 4*(b*c^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c
^4*log(c))*d^2)*log(x))/c^4
```

3.96.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x, x)
```

3.97 $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^3}\,dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 189

$$\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^3}\,dx = -\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1-\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{4}bc^2d^2\csc^{-1}(cx)+ibde\csc^{-1}(cx)^2 - \frac{d^2(a+b\csc^{-1}(cx))}{2x^2}+\frac{1}{2}e^2x^2(a+b\csc^{-1}(cx)) - 2bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + 2bde\csc^{-1}(cx)\log\left(\frac{1}{x}\right) - 2de(a+b\csc^{-1}(cx))\log\left(\frac{1}{x}\right) + ibde\,\text{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)$$

output
```
1/4*b*c^2*d^2*arccsc(c*x)+I*b*d*e*arccsc(c*x)^2-1/2*d^2*(a+b*arccsc(c*x))/
x^2+1/2*e^2*x^2*(a+b*arccsc(c*x))-2*b*d*e*arccsc(c*x)*ln(1-(I/c/x+(1-1/c^2
/x^2)^(1/2))^2)+2*b*d*e*arccsc(c*x)*ln(1/x)-2*d*e*(a+b*arccsc(c*x))*ln(1/x
)+I*b*d*e*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)-1/4*b*c*d^2*(1-1/c^2/x^
2)^(1/2)/x+1/2*b*e^2*x*(1-1/c^2/x^2)^(1/2)/c
```

3.97.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} dx$$
$$= \frac{1}{4}\left(-\frac{2ad^2}{x^2} + 2ae^2x^2 - \frac{2bd^2\csc^{-1}(cx)}{x^2} + \frac{2be^2x\left(\sqrt{1-\frac{1}{c^2x^2}} + cx\csc^{-1}(cx)\right)}{c} - \frac{bd^2\left(-1+c^2x^2+c^2x^2\sqrt{-1+c^2x^2}\arctan\left(\sqrt{-1+c^2x^2}\right)\right)}{c\sqrt{1-\frac{1}{c^2x^2}}x^3} - 8bde\csc^{-1}(cx)\log\left(1-e^{2i\csc^{-1}(cx)}\right) + 8ade\log(x) + 4ibde\left(\csc^{-1}(cx)^2 + \text{PolyLog}\left(2, e^{2i\csc^{-1}(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^3,x]
```

output
```
((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcCsc[c*x])/x^2 + (2*b*e^2*x*(Sq
rt[1 - 1/(c^2*x^2)] + c*x*ArcCsc[c*x]))/c - (b*d^2*(-1 + c^2*x^2 + c^2*x^2
*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]]))/(c*Sqrt[1 - 1/(c^2*x^2)]*
x^3) - 8*b*d*e*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] + 8*a*d*e*Log[x]
 + (4*I)*b*d*e*(ArcCsc[c*x]^2 + PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/4
```

3.97.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5764, 5230, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} dx$$
$$\Big\downarrow \text{ 5764}$$

$$-\int \left(\frac{d}{x^2}+e\right)^2 x^3 \left(a+b \arcsin\left(\frac{1}{cx}\right)\right) d\frac{1}{x}$$

$$\Big\downarrow \ 5230$$

$$\frac{b\int -\frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{2\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-\frac{d^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 7293$$

$$-\frac{b\int \left(-\frac{d^2}{\sqrt{1-\frac{1}{c^2x^2}}x^2}-\frac{4e\log\left(\frac{1}{x}\right)d}{\sqrt{1-\frac{1}{c^2x^2}}}+\frac{e^2x^2}{\sqrt{1-\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 2009$$

$$-\frac{d^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)-\frac{b\left(-\frac{1}{2}c^3d^2\arcsin\left(\frac{1}{cx}\right)-2icde\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)-2icde\arcsin\left(\frac{1}{cx}\right)^2+4cde\arcsin\left(\frac{1}{cx}\right)\log\left(1-e^{2i\arcsin\left(\right.}\right.\right.}{2c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsc[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(a + b*ArcSin[1/(c*x)]))/x^2 + (e^2*x^2*(a + b*ArcSin[1/(c*x)]))
/2 - 2*d*e*(a + b*ArcSin[1/(c*x)])*Log[x^(-1)] - (b*((c^2*d^2*Sqrt[1 - 1/(
c^2*x^2)])/(2*x) - e^2*Sqrt[1 - 1/(c^2*x^2)]*x - (c^3*d^2*ArcSin[1/(c*x)])
/2 - (2*I)*c*d*e*ArcSin[1/(c*x)]^2 + 4*c*d*e*ArcSin[1/(c*x)]*Log[1 - E^((2
*I)*ArcSin[1/(c*x)])] - 4*c*d*e*ArcSin[1/(c*x)]*Log[x^(-1)] - (2*I)*c*d*e*
PolyLog[2, E^((2*I)*ArcSin[1/(c*x)])]))/(2*c)
```

3.97. $\int \frac{(d+ex^2)^2(a+b\csc^{-1}(cx))}{x^3}\,dx$

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5230
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Simp
[(a + b*ArcSin[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1 -
c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e,
0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.97.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.45

method	result
parts	$a\left(\frac{e^2x^2}{2}-\frac{d^2}{2x^2}+2de\ln\left(x\right)\right)+ibde\operatorname{arccsc}\left(cx\right)^2-\frac{bc\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4x}+\frac{b\,c^2d^2\operatorname{arccsc}(cx)}{4}-\frac{b\operatorname{arccsc}(cx}{2x^2}$
derivativedivides	$c^2\left(\frac{a\,x^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{ibde\operatorname{arccsc}(cx)^2}{c^2}-\frac{b\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\operatorname{arccsc}(cx)d^2}{4}-\frac{b\operatorname{arccsc}(cx)d^2}{2c^2x^2}+\right.$
default	$c^2\left(\frac{a\,x^2e^2}{2c^2}+\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{ibde\operatorname{arccsc}(cx)^2}{c^2}-\frac{b\,d^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{4cx}+\frac{b\operatorname{arccsc}(cx)d^2}{4}-\frac{b\operatorname{arccsc}(cx)d^2}{2c^2x^2}+\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^2*x^2-1/2*d^2/x^2+2*d*e*ln(x))+I*b*d*e*arccsc(c*x)^2-1/4*b*c*d^2/
x*((c^2*x^2-1)/c^2/x^2)^(1/2)+1/4*b*c^2*d^2*arccsc(c*x)-1/2*b*arccsc(c*x)*
d^2/x^2+1/2*b*e^2*arccsc(c*x)*x^2+1/2*b/c*e^2*((c^2*x^2-1)/c^2/x^2)^(1/2)*
x-1/2*I*b/c^2*e^2-2*b*d*e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+2*I*
b*d*e*polylog(2,-I/c/x-(1-1/c^2/x^2)^(1/2))-2*b*d*e*arccsc(c*x)*ln(1-I/c/x
-(1-1/c^2/x^2)^(1/2))+2*I*b*d*e*polylog(2,I/c/x+(1-1/c^2/x^2)^(1/2))
```

3.97.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arccsc}\,(cx)+a)}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccsc(c*x))/x^3, x)
```

3.97.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^2}{x^3} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsc(c*x))/x**3,x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x**2)**2/x**3, x)
```

3.97.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arccsc}\,(cx)+a)}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*e^2*x^2 + 1/4*b*d^2*((c^4*x*sqrt(-1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*
x^2) - 1) - 1) - c^3*arctan(c*x*sqrt(-1/(c^2*x^2) + 1)))/c - 2*arccsc(c*x)
/x^2) + 2*a*d*e*log(x) - 1/2*a*d^2/x^2 + 1/4*(4*I*b*c^2*d*e*log(-c*x + 1)*
log(x) + 4*I*b*c^2*d*e*log(x)^2 + 4*I*b*c^2*d*e*dilog(c*x) + 4*I*b*c^2*d*e
*dilog(-c*x) + 2*(b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^2*
log(c))*e^2*x^2 + I*b*e^2*log(c*x - 1) - I*(b*e^2*(log(c*x + 1)/c^2 + log(
c*x - 1)/c^2) + 16*b*d*e*integrate(1/2*log(x)/(c^2*x^3 - x), x))*c^2 + 4*c
^2*integrate(1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*sqrt(c*x + 1)*sqrt(c*x - 1)/
(c^2*x^3 - x), x) + (-I*b*c^2*e^2*x^2 - 4*I*b*c^2*d*e*log(x))*log(c^2*x^2)
 + (4*I*b*c^2*d*e*log(x) + I*b*e^2)*log(c*x + 1) - 2*(-I*b*c^2*e^2*x^2 - 4
*(b*c^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + I*b*c^2*log(c))*d*e)*log
(x))/c^2
```

3.97.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)/x^3, x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asin(1/(c*x))))/x^3, x)
```

3.98 $\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 565

$$\begin{aligned}\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx &= \frac{x(a+b\csc^{-1}(cx))}{e}+\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce} \\ &\quad -\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &\quad +\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &\quad -\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &\quad +\frac{\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &\quad -\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &\quad +\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &\quad -\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}} \\ &\quad +\frac{ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^{3/2}}\end{aligned}$$

output
```
x*(a+b*arccsc(c*x))/e+b*arctanh((1-1/c^2/x^2)^(1/2))/c/e-1/2*(a+b*arccsc(c
*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+
b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/
2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-
(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*
I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

3.98.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1260 vs. 2(565) = 1130.

Time = 1.82 (sec) , antiderivative size = 1260, normalized size of antiderivative = 2.23

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

output
```
((I/4)*((-4*I)*a*c*Sqrt[e]*x - (4*I)*b*c*Sqrt[e]*x*ArcCsc[c*x] + (4*I)*a*c
*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(P
i + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1
+ (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(P
i + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + b*c*Sqrt[d]*Pi*Log[1 + (Sqrt[e]
- Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*c*Sqrt[d]*ArcCsc[c
*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4
*b*c*Sqrt[d]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sq
rt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - b*c*Sqrt[d]*Pi*L
og[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*c
*Sqrt[d]*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 - b*c*Sqrt[d]*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] + 2*b*c*Sqrt[d]*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 + (I*Sqrt[e]
)/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(
I*ArcCsc[c*x]))] + b*c*Sqrt[d]*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*S
qrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*c*Sqrt[d]*ArcCsc[c*x]*Log[1 + (Sqrt[e]...
```

3.98.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \frac{x^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{5232}$$

$$-\int \left(\frac{x^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}+\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}+\frac{x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^{3/2}}-\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^{3/2}}+\frac{b\,\text{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce}$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

output
```
(x*(a + b*ArcSin[1/(c*x)]))/e + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e) -
 (Sqrt[-d]*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*
x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcSin[
1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2
*d + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sq
rt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) +
(Sqrt[-d]*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x
)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^(3/2)) - ((I/2)*b*Sqrt[-d]*PolyLog
[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/
e^(3/2) + ((I/2)*b*Sqrt[-d]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)])
)/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(3/2) - ((I/2)*b*Sqrt[-d]*PolyLog[2, ((-
I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(3/2)
 + ((I/2)*b*Sqrt[-d]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt
[e] + Sqrt[c^2*d + e])])/e^(3/2)
```

3.98. $\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

3.98.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.98.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 44.19 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.73

method	result
parts	$\frac{ax}{e} - \frac{ad\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + \frac{b\left(\frac{c^3\operatorname{arccsc}(cx)x}{e} - \frac{c^2\ln\left(-1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e} + \frac{c^2\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e} - \frac{c^4 d\left(\sum\_{\_R1=\text{RootOf}(\ldots}\right.}{\ldots}\right.}{\ldots}$
derivativedivides	$\frac{\frac{a c^3 x}{e} - \frac{a c^3 d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + b c^2\left(\frac{cx\operatorname{arccsc}(cx)}{e} - \frac{c^2 d\left(\sum_{\_R1=\text{RootOf}\left(c^2 d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)} \frac{\left(\_R1^2 c^2 d - c^2 d - 4e\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$
default	$\frac{\frac{a c^3 x}{e} - \frac{a c^3 d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e\sqrt{de}} + b c^2\left(\frac{cx\operatorname{arccsc}(cx)}{e} - \frac{c^2 d\left(\sum_{\_R1=\text{RootOf}\left(c^2 d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)} \frac{\left(\_R1^2 c^2 d - c^2 d - 4e\ldots\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/e*x-a*d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(c^3*arccsc(c*x)/e*x
-c^2/e*ln(-1+I/c/x+(1-1/c^2/x^2)^(1/2))+c^2/e*ln(1+I/c/x+(1-1/c^2/x^2)^(1/
2))-1/8*c^4/e^2*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d-c^2*d-2*e)*
(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
+1/8*c^4/e^2*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c^2*d-c^2*d-2*
e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x
-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*
d)))
```

3.98. $\int \frac{x^2(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

3.98.5 Fricas [F]

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{ex^2 + d} \, dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsc(c*x) + a*x^2)/(e*x^2 + d), x)
```

3.98.6 Sympy [F]

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2} \, dx = \int \frac{x^2(a + b \operatorname{acsc}(cx))}{d + ex^2} \, dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2), x)
```

3.98.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.98.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \int \frac{x^2\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2 + d}\,dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

3.99 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$

3.99.1 Optimal result

Integrand size = 19, antiderivative size = 507

$$\begin{aligned}\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx &= \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} \\ &- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e}\end{aligned}$$

output
```
-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e+1/2*(a+b*arccsc(c
*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))/e+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsc(
c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(
1/2)))/e+1/2*I*b*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e-1/2*I*b*polylo
g(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/e-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
```

3.99.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1123 vs. $2(507) = 1014$.

Time = 0.45 (sec) , antiderivative size = 1123, normalized size of antiderivative = 2.21

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$$
$$= \frac{ib\pi^2 - 4ib\pi\csc^{-1}(cx) + 8ib\csc^{-1}(cx)^2 - 16ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}(\pi+2\csc^{-1}(cx))\right)}{\sqrt{c^2d+e}}\right) - }{}$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

output
```
(I*b*Pi^2 - (4*I)*b*Pi*ArcCsc[c*x] + (8*I)*b*ArcCsc[c*x]^2 - (16*I)*b*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sq
rt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqr
t[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Co
t[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (S
qrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt
[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] - 2*b*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt
[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqr
t[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))...
```

3.99.3 Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \frac{x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{5232}$$

$$-\int \left(\frac{x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}-\frac{\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e}+\frac{ib\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

output
```
((a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqr
t[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sq
rt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a +
b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] +
 Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]
*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) - ((a + b*ArcS
in[1/(c*x)])*Log[1 - E^((2*I)*ArcSin[1/(c*x)])])/e - ((I/2)*b*PolyLog[2, (
(-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e - (
(I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/e - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)])
)/(Sqrt[e] + Sqrt[c^2*d + e])])/e - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I
*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e + ((I/2)*b*PolyLog[2, E
^((2*I)*ArcSin[1/(c*x)])])/e
```

3.99. $\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.99.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.78

method	result
parts	$\frac{a\ln(ex^2+d)}{2e} - \frac{ib\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)} \frac{\left(\_R1^2c^2d-c^2d-4e\right)\left(i\,\mathrm{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{\cdots}}{\_R1 \cdots}\right.\right.}{\_R1^2c^2d-c \cdots}\right.}{4e}$
derivativedivides	$\frac{ac^2\ln\left(c^2ex^2+c^2d\right)}{2e}+bc^2\left(-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{\mathrm{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{i\left(\_R \cdots\right.}{\cdots}\right.$
default	$\frac{ac^2\ln\left(c^2ex^2+c^2d\right)}{2e}+bc^2\left(-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{\mathrm{arccsc}(cx)\ln\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}+\frac{i\,\mathrm{dilog}\left(1+\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e}-\frac{i\left(\_R \cdots\right.}{\cdots}\right.$

input

```
int(x*(a+b*arccsc(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output

```
1/2*a/e*ln(e*x^2+d)-1/4*I*b/e*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2
*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-
I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2
+c^2*d))-b/e*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))+I*b/e*dilog(1+I/c
/x+(1-1/c^2/x^2)^(1/2))-I*b/e*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I*b*c^2
*d/e*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1
-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=Ro
otOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

3.99. $\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

3.99.5 Fricas [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arccsc(c*x) + a*x)/(e*x^2 + d), x)
```

3.99.6 Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x(a+b\,\mathrm{acsc}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*acsc(c*x))/(d + e*x**2), x)
```

3.99.7 Maxima [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(x*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/
2*a*log(e*x^2 + d)/e
```

3.99.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

3.100 $\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx$

3.100.1 Optimal result

Integrand size = 18, antiderivative size = 529

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = &-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &+\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/
2)/e^(1/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccsc(c*x))
*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2))
)/(-d)^(1/2)/e^(1/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,I
*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(1/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*I*b*polylog(2,I*c*(I/c
/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(1/2)/e
^(1/2)
```

3.100.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1068 vs. $2(529) = 1058$.

Time = 0.45 (sec) , antiderivative size = 1068, normalized size of antiderivative = 2.02

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = -\frac{i\left(4ia\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+8ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{c^2d+e}}\right)-8ib\arcsin\left(\frac{\sqrt{1+\cdots}}{\cdots}\right.\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2),x]
```

output
```
((-1/4*I)*((4*I)*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 - (
I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(P
i + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcC
sc[c*x])/4])/Sqrt[c^2*d + e]] + b*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] - b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(
I*ArcCsc[c*x]))] + 2*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] - b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] + 2*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo
g[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Lo
g[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*Arc
Csc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))
] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e]
 + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + b*Pi*Log[Sqrt[e] -...
```

3.100.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5754, 5172, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}dx$$

$$\Big\downarrow \text{5754}$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{5172}$$

$$-\int \left(\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\sqrt{e}-\frac{\sqrt{-d}}{x}\right)}+\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\frac{\sqrt{-d}}{x}+\sqrt{e}\right)}\right)d\frac{1}{x}$$

$$\downarrow \text{2009}$$

$$-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x^2),x]
```

output
```
-1/2*((a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) + ((a + b*ArcSin[1/(c*x)
])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]
)])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*
E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e])
+ ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(S
qrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-
I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-
d]*Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e]) - ((I/2)*b*PolyLog[2, ((-I)*
c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*
Sqrt[e]) + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[
e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*Sqrt[e])
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5172
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

3.100. $\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx$

rule 5754
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

3.100.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.99 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.51

method	result
parts	$\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{bc\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\operatorname{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_}{}\right.}{\_R1\left(\_R1^2c^2d-c^2d-2e\right)}\right.}{2}$
derivativedivides	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\operatorname{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\right.}{\_R1\left(\_R1^2c^2d-c^2d-\right.}\right.}{2}\right.}{}$
default	$\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}+bc^2\left(-\frac{\left(\sum_{\_R1=\operatorname{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\operatorname{arccsc}(cx)\ln\left(\frac{\_R1-\frac{i}{cx}-\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\right.}{\_R1\left(\_R1^2c^2d-c^2d-\right.}\right.}{2}\right.}{}$

input
```
int((a+b*arccsc(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/2*b*c*sum(1/_R1/(_R1^2*c^2*d-c^2*d
-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/
c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c
^2*d))-1/2*b*c*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/
x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R
1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

3.100.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e*x^2 + d), x)
```

3.100.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{d+ex^2}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*acsc(c*x))/(d + e*x**2), x)
```

3.100.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.100.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{d+ex^2}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{e\,x^2+d}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x^2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x^2), x)
```

3.101 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)}\,dx$

3.101.1 Optimal result

Integrand size = 21, antiderivative size = 479

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x\,(d+ex^2)}\,dx &= \frac{i(a+b\csc^{-1}(cx))^2}{2bd} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} \\ &\quad - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} \\ &\quad - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} \\ &\quad - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} \\ &\quad + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} \\ &\quad + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d}\end{aligned}$$

output
```
1/2*I*(a+b*arccsc(c*x))^2/b/d-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsc(c*x
))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2
)))/d-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,
-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d+1
/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*
d+e)^(1/2)))/d+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d
```

3.101.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1089 vs. 2(479) = 958.

Time = 0.38 (sec) , antiderivative size = 1089, normalized size of antiderivative = 2.27

$$\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)}\,dx =$$

$$-\frac{ib\pi^2-4ib\pi\csc^{-1}(cx)+4ib\csc^{-1}(cx)^2-16ib\arcsin\left(\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(-ic\sqrt{d}+\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2\csc^{-1}(cx)\right)\right)}{\sqrt{c^2d+e}}\right)}{}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

output
```
-1/8*(I*b*Pi^2 - (4*I)*b*Pi*ArcCsc[c*x] + (4*I)*b*ArcCsc[c*x]^2 - (16*I)*b
*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d]
 + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSi
n[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e
])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e]
 - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1
 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin
[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d
+ e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e
] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 +
(I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^
2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^
(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c
*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c...
```

3.101.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)}dx$$

$$\Big\downarrow \; 5764$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)x}d\frac{1}{x}$$

$$\Big\downarrow \; 5232$$

$$-\int \left(\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2d\left(\frac{\sqrt{-d}}{x}+\sqrt{e}\right)}-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2d\left(\sqrt{e}-\frac{\sqrt{-d}}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}+\frac{i\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)^2}{2bd}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d}$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)),x]
```

output
```
((I/2)*(a + b*ArcSin[1/(c*x)])^2)/(b*d) - ((a + b*ArcSin[1/(c*x)])*Log[1 -
 (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d)
- ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*
Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a
+ b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e]
 + Sqrt[c^2*d + e])])/(2*d) + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*Ar
cSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d + ((I/2)*b*PolyLog[2, (I*c
*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d + ((I/2)*
b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d
 + e])])/d + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/d
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.101.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.11 (sec) , antiderivative size = 1934, normalized size of antiderivative = 4.04

method	result	size
parts	Expression too large to display	1934
derivativedivides	Expression too large to display	1961
default	Expression too large to display	1961

input
```
int((a+b*arccsc(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a/d*ln(x)-1/2*a/d*ln(e*x^2+d)+b*(I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arccs
c(c*x)^2*e/d^3/c^4+1/4*I*(e*(c^2*d+e))^(1/2)/e/(c^2*d+e)*arccsc(c*x)^2*c^2
-1/8*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)
*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2
)+2*e))/d/e/(c^2*d+e)+1/4*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d
+e))^(1/2)*e+2*e^2)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(
c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d/e/(c^2*d+e)+1/4*I*(c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c
^2*d+e))^(1/2)+2*e))/c^2/d^2+((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^
2*d+e))^(1/2)*e+2*e^2)*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2
*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/c^4/(c^2*d+e)/d^3+1/2*I/d*arccsc(c*
x)^2-(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*e*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(
1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/d^3/c^4+1/8*I*(e*(c
^2*d+e))^(1/2)/e/(c^2*d+e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))*c^2+((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e
+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(
c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/c^2/(c^2*d+e)/d^2-1/4*(e*(c^
2*d+e))^(1/2)/e/(c^2*d+e)*c^2*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^
(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))+1/4*I*(e*(c^2*d+e))^(1/2)/d/(c
^2*d+e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*...
```

3.101.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{b\operatorname{arccsc}\left(cx\right)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e*x^3 + d*x), x)
```

3.101.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{a+b\operatorname{acsc}\left(cx\right)}{x\left(d+ex^2\right)}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*acsc(c*x))/(x*(d + e*x**2)), x)
```

3.101.7 Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{b\operatorname{arccsc}\left(cx\right)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1))/(e*x^3 + d*x), x)
```

3.101.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.102 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx$

3.102.1 Optimal result

Integrand size = 21, antiderivative size = 572

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx &= -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\csc^{-1}(cx)}{dx}\\
&-\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\
&+\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\
&-\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\
&+\frac{\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\
&-\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\
&+\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\
&-\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\\
&+\frac{ib\sqrt{e}\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}\end{aligned}$$

output
```
-a/d/x-b*arccsc(c*x)/d/x-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+
b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/
2)+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x
+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(
3/2)+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/
2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d
+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-b*c*(1-1/c^2/x^2)^(1/2)/d
```

3.102.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1241 vs. 2(572) = 1144.

Time = 1.81 (sec) , antiderivative size = 1241, normalized size of antiderivative = 2.17

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
-(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*(-((c*Sqr
t[1 - 1/(c^2*x^2)]*x + ArcCsc[c*x])/(d*x)) + (Sqrt[e]*(Pi^2 - 4*Pi*ArcCsc[
c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c
^2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCs
c[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/S
qrt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
 + (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[e]
 + (I*Sqrt[d])/x] + 8*PolyLog[2, (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] + 8*PolyLog[2, -((Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]
*E^(I*ArcCsc[c*x])))] + 4*PolyLog[2, E^((2*I)*ArcCsc[c*x])]))/(16*d^(3/2))
 - (Sqrt[e]*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1
+ (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(P
i + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (4*I)*Pi*Log[1 + (-Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 ...
```

3.102.3 Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{5232}$$

$$-\int \left(\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{d}-\frac{e\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$-\frac{\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{a}{dx}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2(-d)^{3/2}}-\frac{b\arcsin\left(\frac{1}{cx}\right)}{dx}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d}$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
-((b*c*Sqrt[1 - 1/(c^2*x^2)])/d) - a/(d*x) - (b*ArcSin[1/(c*x)])/(d*x) - (
Sqrt[e]*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]
))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcSin[1
/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*
d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*S
qrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)
) + (Sqrt[e]*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(
c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - ((I/2)*b*Sqrt[e]*Po
lyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e
])])/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/
(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(3/2) - ((I/2)*b*Sqrt[e]*PolyL
og[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])]
)/(-d)^(3/2) + ((I/2)*b*Sqrt[e]*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*
x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(3/2)
```

3.102. $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.102.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 43.16 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.58

method	result
parts	$-\frac{a}{dx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{bc\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}(cx)}{dx}+\frac{bce\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)} i\,\mathrm{ar}\right.}{}$
derivativedivides	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\right.}{}\right.$
default	$c\left(-\frac{a}{dcx}-\frac{ae\arctan\left(\frac{ex}{\sqrt{de}}\right)}{cd\sqrt{de}}-\frac{b\sqrt{\frac{c^2x^2-1}{c^2x^2}}}{d}-\frac{b\,\mathrm{arccsc}(cx)}{dcx}+\frac{be\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\right.}{}\right.$

input
```
int((a+b*arccsc(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d/x-a*e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-b*c/d*((c^2*x^2-1)/c^2/x^
2)^(1/2)-b*arccsc(c*x)/d/x+1/2*b*c*e/d*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(
I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-
1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+
1/2*b*c*e/d*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(
1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=R
ootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

3.102. $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx$

3.102.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e*x^4 + d*x^2), x)
```

3.102.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx = \int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^2(d+ex^2)}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)), x)
```

3.102.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.102.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x^2(d + ex^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x^2(d + ex^2)}\,dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,(e\,x^2 + d)}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

3.103 $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 628

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{b\sqrt{1-\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d(a+b\csc^{-1}(cx))}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2(a+b\csc^{-1}(cx))}{2e^2} - \frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}}$$
$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$- \frac{d(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{d(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$+ \frac{2d(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}$$
$$+ \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3} + \frac{ibd\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3}$$
$$+ \frac{ibd\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{e^3} - \frac{ibd\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{e^3}$$

output
```
1/2*d*(a+b*arccsc(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arccsc(c*x))/e^2+2*d*(a
+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^3-d*(a+b*arccsc(c*x)
)*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)
))/e^3-d*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsc(c*x
))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2
)))/e^3-I*b*d*polylog(2,(I/c/x+(1-1/c^2/x^2)^(1/2))^2)/e^3+I*b*d*polylog(2
,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^
3+I*b*d*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2
*d+e)^(1/2)))/e^3+I*b*d*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+I*b*d*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3-1/2*b*d*arctan((c^2*d+e
)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)+1/2*b*x*(
1-1/c^2/x^2)^(1/2)/c/e^2
```

3.103.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1480 vs. $2(628) = 1256$.

Time = 4.22 (sec) , antiderivative size = 1480, normalized size of antiderivative = 2.36

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(I*d*P
i^2 - (2*e*Sqrt[1 - 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsc[c*x] - 2*e*x^2*A
rcCsc[c*x] + (d^(3/2)*ArcCsc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcC
sc[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (8*I)*d*ArcCsc[c*x]^2 - 2*d*ArcSin[1/(c
*x)] - (16*I)*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(
((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] -
 (16*I)*d*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*S
qrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*d*Pi*L
og[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*Ar
cCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x])
)] - 8*d*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e
] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 + (-Sqr
t[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*L
og[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*d*A
rcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[
c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 - (Sqrt[e] + Sqr
t[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*d*ArcCsc[c*x]*Log[1 - (Sq
rt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*d*ArcSin[Sqrt[
1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/
(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*d*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d +...
```

3.103.3 Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 5764}$$

$$-\int \frac{x^3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5232}$$

$$-\int \left(\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)x^3}{e^2}-\frac{2d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)x}{e^3}+\frac{2d^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right)x}+\frac{d^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{e^3}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{e^3}+\frac{2d\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^3}+\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2e^2}+\frac{ibd\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}+\frac{ibd\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{e^3}-\frac{ibd\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{e^3}-\frac{bd\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{5/2}\sqrt{c^2d+e}}+\frac{bx\sqrt{1-\frac{1}{c^2x^2}}}{2ce^2}$$

input
```
Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
(b*Sqrt[1 - 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcSin[1/(c*x)]))/(2*e^2
*(e + d/x^2)) + (x^2*(a + b*ArcSin[1/(c*x)]))/(2*e^2) - (b*d*ArcTan[Sqrt[c
^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]
) - (d*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)])
)/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcSin[1/(c*x)])*Log[1 + (
I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3 - (d
*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/e^3 - (d*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sq
rt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (2*d*(a
+ b*ArcSin[1/(c*x)])*Log[1 - E^((2*I)*ArcSin[1/(c*x)])])/e^3 + (I*b*d*Poly
Log[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])
])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1
/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + (I*b*d*PolyLog[2, (I*c*Sqrt[
-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - (I*b*d*Poly
Log[2, E^((2*I)*ArcSin[1/(c*x)])])/e^3
```

3.103. $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.103.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.69 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.02

method	result
parts	$\frac{a x^2}{2e^2} - \frac{a d^2}{2e^3(e x^2+d)} - \frac{a d \ln(e x^2+d)}{e^3} + \frac{b \left(\frac{c^4 \left(2c^4 d \operatorname{arccsc}(cx) x^2 + \operatorname{arccsc}(cx) e c^4 x^4 + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\, c^3 d x + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\, e c^3 x^3 - \ldots\right.}{2\left(c^2 e x^2 + c^2 d\right) e^2} \ldots \right.}{\ldots}$
derivativedivides	$\frac{\frac{a c^6 x^2}{2e^2} - \frac{a c^8 d^2}{2e^3\left(c^2 e x^2 + c^2 d\right)} - \frac{a c^6 d \ln\left(c^2 e x^2 + c^2 d\right)}{e^3} + b c^4 \left(\frac{2c^4 d \operatorname{arccsc}(cx) x^2 + \operatorname{arccsc}(cx) e c^4 x^4 + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\, c^3 d x + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\, e c^3 x \ldots}{2\left(c^2 e x^2 + c^2 d\right) e^2} \ldots \right.}{\ldots}$
default	$\frac{\frac{a c^6 x^2}{2e^2} - \frac{a c^8 d^2}{2e^3\left(c^2 e x^2 + c^2 d\right)} - \frac{a c^6 d \ln\left(c^2 e x^2 + c^2 d\right)}{e^3} + b c^4 \left(\frac{2c^4 d \operatorname{arccsc}(cx) x^2 + \operatorname{arccsc}(cx) e c^4 x^4 + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\, c^3 d x + \sqrt{\frac{c^2x^2-1}{c^2x^2}}\, e c^3 x \ldots}{2\left(c^2 e x^2 + c^2 d\right) e^2} \ldots \right.}{\ldots}$

input
```
int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

3.103. $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\, dx$

output
```
1/2*a*x^2/e^2-1/2*a*d^2/e^3/(e*x^2+d)-a*d/e^3*ln(e*x^2+d)+b/c^6*(1/2*c^4*(
2*c^4*d*arccsc(c*x)*x^2+arccsc(c*x)*e*c^4*x^4+((c^2*x^2-1)/c^2/x^2)^(1/2)*
c^3*d*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e*c^3*x^3-I*c^2*d-I*e*c^2*x^2)/(c^2*e*
x^2+c^2*d)/e^2-2*I/e^3*d*c^6*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/2*I/e^3*
d^2*c^8*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x
-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1
=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*I*(e*(c^2*d+e))^(1/2)/(
c^2*d+e)/e^3*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*
e)/(c^2*d*e+e^2)^(1/2))*d*c^6+2*I/e^3*d*c^6*dilog(I/c/x+(1-1/c^2/x^2)^(1/2
))+1/2*I/e^3*d*c^6*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*
arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/
c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+2/
e^3*d*c^6*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2)))
```

3.103.5 Fricas [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^5*arccsc(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.103.6 Sympy [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^5(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**5*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

3.103.7 Maxima [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*inte
grate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d
^2), x)
```

3.103.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.104 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2} \, dx$

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 593

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{-a-b\csc^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{3/2}\sqrt{c^2d+e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} - \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^2} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^2}$$

output
```
1/2*(-a-b*arccsc(c*x))/e/(e+d/x^2)-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/
x^2)^(1/2))^2)/e^2+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1+I
*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2+1
/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*I*b*polylog(2,(I/
c/x+(1-1/c^2/x^2)^(1/2))^2)/e^2-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I*b*polylog(2,I*c*(
I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^2-1/2*I
*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e
)^(1/2)))/e^2-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(c^2*d+e)^(1/2)))/e^2+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(
1-1/c^2/x^2)^(1/2))/e^(3/2)/(c^2*d+e)^(1/2)
```

3.104.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1442 vs. 2(593) = 1186.

Time = 1.83 (sec) , antiderivative size = 1442, normalized size of antiderivative = 2.43

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
(I*b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsc[c*x] + (2*b*Sqrt[d]*Ar
cCsc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsc[c*x])/(Sqrt[d] +
I*Sqrt[e]*x) + (8*I)*b*ArcCsc[c*x]^2 - 4*b*ArcSin[1/(c*x)] - (16*I)*b*ArcS
in[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sq
rt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - (16*I)*b*ArcSin[Sqr
t[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Co
t[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] - 2*b*Pi*Log[1 + (Sqrt[e] - Sq
rt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (S
qrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt
[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])
/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + S
qrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sq
rt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[
d]*E^(I*ArcCsc[c*x]))] - 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqr
t[d]*E^(I*ArcCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d +
 e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*S
qrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcC
sc[c*x]))] - 2*b*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*Ar
cCsc[c*x]))] + 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*S...
```

3.104.3 Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a + b\csc^{-1}(cx))}{(d + ex^2)^2} \, dx$$

$$\Big\downarrow \; 5764$$

$$-\int \frac{x \left(a + b\arcsin\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2} + e\right)^2} d\frac{1}{x}$$

$$\Big\downarrow \; 5232$$

$$-\int \left(\frac{x \left(a + b\arcsin\left(\frac{1}{cx}\right)\right)}{e^2} - \frac{d \left(a + b\arcsin\left(\frac{1}{cx}\right)\right)}{e^2 \left(\frac{d}{x^2} + e\right) x} - \frac{d \left(a + b\arcsin\left(\frac{1}{cx}\right)\right)}{e \left(\frac{d}{x^2} + e\right)^2 x}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}-\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{2e\left(\frac{d}{x^2}+e\right)}-\frac{\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^2}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^2}+\frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{2e^2}+\frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2e^{3/2}\sqrt{c^2d+e}}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcSin[1/(c*x)])/(e*(e + d/x^2)) + (b*ArcTan[Sqrt[c^2*d + e]/(
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(2*e^(3/2)*Sqrt[c^2*d + e]) + ((a + b
*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] -
Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d
]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + ((a + b*A
rcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(2*e^2) + ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*
E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) - ((a + b*Arc
Sin[1/(c*x)])*Log[1 - E^((2*I)*ArcSin[1/(c*x)])])/e^2 - ((I/2)*b*PolyLog[2
, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^
2 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sq
rt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/
(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 - ((I/2)*b*PolyLog[2, (I*c*Sqrt
[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^2 + ((I/2)*b*P
olyLog[2, E^((2*I)*ArcSin[1/(c*x)])])/e^2
```

3.104. $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.104.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.55 (sec) , antiderivative size = 524, normalized size of antiderivative = 0.88

method	result
parts	$\frac{ad}{2e^2(e\,x^2+d)}+\frac{a\ln(e\,x^2+d)}{2e^2}-\frac{b\,c^2x^2\,\mathrm{arccsc}(cx)}{2(c^2e\,x^2+c^2d)e}-\frac{ib\,c^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{\left(\_R1^2\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+b\,c^4\left(-\frac{c^2x^2\,\mathrm{arccsc}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e^2}-\frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(\right.\right.}\right.}{}\right.}{}$
default	$\frac{\frac{a\,c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+b\,c^4\left(-\frac{c^2x^2\,\mathrm{arccsc}(cx)}{2\left(c^2e\,x^2+c^2d\right)e}-\frac{i\,\mathrm{dilog}\left(\frac{i}{cx}+\sqrt{1-\frac{1}{c^2x^2}}\right)}{e^2}-\frac{ic^2d\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(\right.\right.}\right.}{}\right.}{}$

input
```
int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*d/e^2/(e*x^2+d)+1/2*a/e^2*ln(e*x^2+d)-1/2*b*c^2*x^2*arccsc(c*x)/(c^2
*e*x^2+c^2*d)/e-1/4*I*b*c^2/e^2*d*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I
*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1
/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1
/2*I*b*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/e^2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/
c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c^2*d*e+e^2)^(1/2))-I*b/e^2*dilog(I/c/x+(1
-1/c^2/x^2)^(1/2))-1/4*I*b/e^2*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^
2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1
-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^
2+c^2*d))+I*b/e^2*dilog(1+I/c/x+(1-1/c^2/x^2)^(1/2))-b/e^2*arccsc(c*x)*ln(
1+I/c/x+(1-1/c^2/x^2)^(1/2))
```

3.104. $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

3.104.5 Fricas [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arccsc(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.104.7 Maxima [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.104.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.105 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.105.1 Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{-a-b\csc^{-1}(cx)}{2e\left(d+ex^2\right)} - \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{2de\sqrt{c^2x^2}} + \frac{bcx\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}\sqrt{c^2x^2}}$$

output
```
1/2*(-a-b*arccsc(c*x))/e/(e*x^2+d)-1/2*b*c*x*arctan((c^2*x^2-1)^(1/2))/d/e
/(c^2*x^2)^(1/2)+1/2*b*c*x*arctan(e^(1/2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2
))/d/e^(1/2)/(c^2*d+e)^(1/2)/(c^2*x^2)^(1/2)
```

3.105.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.13

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{\frac{2a}{d+ex^2}+\frac{2b\csc^{-1}(cx)}{d+ex^2}-\frac{2b\arcsin\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}\log\left(\frac{4ide-4cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2d-e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d-e}}+\frac{b\sqrt{e}\log\left(\frac{4i\left(-de+cd\sqrt{e}\left(ic\sqrt{d}+\sqrt{-c^2d-\cdots}\right.\right.}{b\sqrt{-c^2d-e}\left(\sqrt{d}+i\cdots\right.}\right.}{d\sqrt{-c^2d-e}}\cdots}{4e}$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*((2*a)/(d + e*x^2) + (2*b*ArcCsc[c*x])/(d + e*x^2) - (2*b*ArcSin[1/(c
*x)])/d + (b*Sqrt[e]*Log[((4*I)*d*e - 4*c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(
c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)/(b*Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*S
qrt[e]*x))])/(d*Sqrt[-(c^2*d) - e]) + (b*Sqrt[e]*Log[((4*I)*(-(d*e) + c*d*
Sqrt[e]*(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*Sq
rt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/(d*Sqrt[-(c^2*d) - e]))/e
```

3.105.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5760, 354, 97, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{5760}$$

$$-\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}(ex^2+d)}dx}{2e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{354}$$

$$-\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}(ex^2+d)}dx^2}{4e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{97}$$

$$-\frac{bcx\left(\frac{\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx^2}{d}-\frac{e\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d}\right)}{4e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{bcx\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}+\frac{1}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}-\frac{2e\int \frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}\right)}{4e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{a+b\csc^{-1}(cx)}{2e(d+ex^2)}-\frac{bcx\left(\frac{2\arctan\left(\sqrt{c^2x^2-1}\right)}{d}-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{d\sqrt{c^2d+e}}\right)}{4e\sqrt{c^2x^2}}$$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcCsc[c*x])/(e*(d + e*x^2)) - (b*c*x*((2*ArcTan[Sqrt[-1 + c^2
*x^2]])/d - (2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sqrt[c^2*d + e]
])/(d*Sqrt[c^2*d + e])))/(4*e*Sqrt[c^2*x^2])
```

3.105.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(112) = 224$.

Time = 8.61 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a}{2e(e\,x^2+d)}+\frac{b\left(-\frac{c^4\operatorname{arccsc}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{c\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(\frac{2\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e-2\sqrt{-c^2de}\cdots}{cex+\sqrt{-c^2de}}\cdots\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,xd\sqrt{-\frac{c^2d+\cdots}{e}}}\cdots\right)}{c^2}$
derivativedivides	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\operatorname{arccsc}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(\frac{2\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e-2\sqrt{-c^2\cdots}}{cex+\sqrt{-c^2de}}\cdots\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3xd\sqrt{-\cdots}}\cdots\right)}{c^2}$
default	$\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\operatorname{arccsc}(cx)}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{\sqrt{c^2x^2-1}\left(2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}-\ln\left(\frac{2\sqrt{c^2x^2-1}\sqrt{-\frac{c^2d+e}{e}}\,e-2\sqrt{-c^2\cdots}}{cex+\sqrt{-c^2de}}\cdots\right.\right.}{4e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,c^3xd\sqrt{-\cdots}}\cdots\right)}{c^2}$

input
```
int(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arccsc(c*x)+1/4*c/e
*(c^2*x^2-1)^(1/2)*(2*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)-ln(
2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x
+(-c^2*d*e)^(1/2)))-ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*
d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2))))/((c^2*x^2-1)/c^2/x^2)^(1/2)/
x/d/(-(c^2*d+e)/e)^(1/2))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.87

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$= \left[-\frac{2\,ac^2d^2+2\,ade+\sqrt{-c^2de-e^2}(bex^2+bd)\log\left(\frac{c^2ex^2-c^2d-2\sqrt{-c^2de-e^2}\sqrt{c^2x^2-1}-2\,e}{ex^2+d}\right)+2\,(bc^2d^2+bde)\,\mathrm{arcc}}{4\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x}\right.$$

$$-\frac{ac^2d^2+ade-\sqrt{c^2de+e^2}(bex^2+bd)\arctan\left(\frac{\sqrt{c^2de+e^2}\sqrt{c^2x^2-1}}{c^2d+e}\right)+(bc^2d^2+bde)\,\mathrm{arccsc}\,(cx)+2\,(bc^2d^2+}{2\,(c^2d^3e+d^2e^2+(c^2d^2e^2+de^3)x^2)}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c^2*d^2 + 2*a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*log((c
^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2
+ d)) + 2*(b*c^2*d^2 + b*d*e)*arccsc(c*x) + 4*(b*c^2*d^2 + b*d*e + (b*c^2*
d*e + b*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 +
 (c^2*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 + a*d*e - sqrt(c^2*d*e + e^2)
*(b*e*x^2 + b*d)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^2 - 1)/(c^2*d + e))
 + (b*c^2*d^2 + b*d*e)*arccsc(c*x) + 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b
*e^2)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d
^2*e^2 + d*e^3)*x^2)]
```

3.105.6 Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

3.105.7 Maxima [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(c^2*e^2*x^2 + c^2*d*e)*integrate(1/2*x*e^(1/2*log(c*x + 1) + 1/2*
log(c*x - 1))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e + (c^2*e^2*x^4 + (c
^2*d*e - e^2)*x^2 - d*e)*e^(log(c*x + 1) + log(c*x - 1))), x) + arctan2(1,
 sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^2*x^2 + d*e) - 1/2*a/(e^2*x^2 + d*e)
```

3.105.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.106 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^2}\,dx$

3.106.1 Optimal result

Integrand size = 21, antiderivative size = 566

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx =& -\frac{e(a+b\csc^{-1}(cx))}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{i\left(a+b\csc^{-1}(cx)\right)^2}{2bd^2}\\
&+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d+e}}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\\
&+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}\\
&+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^2}\end{aligned}$$

output
```
-1/2*e*(a+b*arccsc(c*x))/d^2/(e+d/x^2)+1/2*I*(a+b*arccsc(c*x))^2/b/d^2-1/2
*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsc(c*x))*l
n(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/
d^2-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*b*polylog(2,I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^2+1/2*I*
b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)
^(1/2)))/d^2+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(c^2*d+e)^(1/2)))/d^2+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1
-1/c^2/x^2)^(1/2))*e^(1/2)/d^2/(c^2*d+e)^(1/2)
```

3.106.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1408 vs. $2(566) = 1132$.

Time = 1.27 (sec) , antiderivative size = 1408, normalized size of antiderivative = 2.49

$$\int \frac{a + b\csc^{-1}(cx)}{x\left(d + ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
((-I)*b*Pi^2 + (4*a*d)/(d + e*x^2) + (4*I)*b*Pi*ArcCsc[c*x] + (2*b*Sqrt[d]
*ArcCsc[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsc[c*x])/(Sqrt[d]
 + I*Sqrt[e]*x) - (4*I)*b*ArcCsc[c*x]^2 - 4*b*ArcSin[1/(c*x)] + (16*I)*b*A
rcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] +
 Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (16*I)*b*ArcSin[
Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])
*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + 2*b*Pi*Log[1 + (Sqrt[e] -
 Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcCsc[c*x]*Log[1 +
 (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[S
qrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d +
e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 2*b*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
 + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcCsc[c*x]*Log[1 + (-Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + 8*b*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))] + 2*b*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))] - 4*b*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - 8*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*A
rcCsc[c*x]))] + 2*b*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I
*ArcCsc[c*x]))] - 4*b*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(...
```

3.106.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

$$\Big\downarrow \text{ 5764}$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2 x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5232}$$

$$-\int \left(\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)x}-\frac{e\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^2 x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}-\frac{e\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2d^2\left(\frac{d}{x^2}+e\right)}+\frac{i\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)^2}{2bd^2}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}+\frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^2}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{2d^2\sqrt{c^2d+e}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
-1/2*(e*(a + b*ArcSin[1/(c*x)]))/(d^2*(e + d/x^2)) + ((I/2)*(a + b*ArcSin[
1/(c*x)])^2)/(b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1
 - 1/(c^2*x^2)]*x)])/(2*d^2*Sqrt[c^2*d + e]) - ((a + b*ArcSin[1/(c*x)])*Lo
g[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(
2*d^2) - ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x
)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSin[1/(c*x)])*Log[
1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*
d^2) - ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]
))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2) + ((I/2)*b*PolyLog[2, ((-I)*c*Sqr
t[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*
PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e]
)])/d^2 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/d^2 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*Arc
Sin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^2
```

3.106. $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^2}\,dx$

3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.106.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.16 (sec) , antiderivative size = 2071, normalized size of antiderivative = 3.66

method	result	size
parts	Expression too large to display	2071
derivativedivides	Expression too large to display	2120
default	Expression too large to display	2120

input
```
int((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*ln(x)+1/2*a/d/(e*x^2+d)-1/2*a/d^2*ln(e*x^2+d)+b*(1/2*I/d^2*sum((_R1^
2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-
(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=
RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*I*arccsc(c*x)^2/d^2-1/2*
(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2
/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*arccsc(c*x)/c^2/d^3-1/2*(e*(c^2*d+e))^
(1/2)/(c^2*d+e)/d^2*arccsc(c*x)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(
c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e))+1/4*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d^2*
polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d+2*(e*(c^2*d+e))^(1/2)
+2*e))+1/2*I*(e*(c^2*d+e))^(1/2)/(c^2*d+e)/d^2*arccsc(c*x)^2+1/4*I*(c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(
c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/c^2/d^3-1/2*I*(e*(c^2*d+e))^(1/2)/(c^2*d
+e)/d^2*arctanh(1/4*(2*c^2*d*(I/c/x+(1-1/c^2/x^2)^(1/2))^2-2*c^2*d-4*e)/(c
^2*d*e+e^2)^(1/2))+1/2*I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arccsc(c*x)^2/c
^2/d^3-1/4*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+
2*e^2)*arccsc(c*x)^2/(c^2*d+e)/d^2/e+I*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*a
rccsc(c*x)^2*e/d^4/c^4-I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+
e))^(1/2)*e+2*e^2)*arccsc(c*x)^2/c^2/d^3/(c^2*d+e)-1/8*I*((e*(c^2*d+e))^(1
/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*polylog(2,d*c^2*(I/c/x+
(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))/(c^2*d+e)/d^2...
```

3.106.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x\left(d + ex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.106.7 Maxima [F]

$$\int \frac{a + b\csc^{-1}(cx)}{x\left(d + ex^2\right)^2}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\left(ex^2 + d\right)^2 x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrat
e(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x
)
```

3.106.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x\left(d + ex^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.107 $\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.107.1 Optimal result

Integrand size = 21, antiderivative size = 803

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\csc^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\csc^{-1}(cx))}{e^2}$$
$$+ \frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2} + \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$+ \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{c^2d+e}}$$
$$- \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+ \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+ \frac{3\sqrt{-d}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+ \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$- \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$
$$+ \frac{3ib\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{5/2}}$$

output
```
x*(a+b*arccsc(c*x))/e^2+b*arctanh((1-1/c^2/x^2)^(1/2))/c/e^2-3/4*(a+b*arcc
sc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4
*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c
/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e
^(5/2)-3/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*polylog(2,I*c*(I/c/x+(1-1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-
3/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^
2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-1/4*d*(
a+b*arccsc(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arccsc(c*x))/e^2
/(d/x+(-d)^(1/2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(
1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d+e)^(1/2)+1/4*
b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/
x^2)^(1/2))*d^(1/2)/e^2/(c^2*d+e)^(1/2)
```

3.107.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1634 vs. 2(803) = 1606.

Time = 6.05 (sec) , antiderivative size = 1634, normalized size of antiderivative = 2.03

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a*x)/e^2 + (a*d*x)/(2*e^2*(d + e*x^2)) - (3*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/
Sqrt[d]])/(2*e^(5/2)) + b*(-1/4*(d*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e] +
e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*
((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^
2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^2 - (
d*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x)]/Sqrt[e]
- Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e
]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*Sqrt[e]*x))]
/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(4*e^2) + (3*Sqrt[d]*(Pi^2 - 4*Pi*ArcCsc[c
*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2
]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^
2*d + e]] + (4*I)*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*A
rcCsc[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*S
qrt[d]*E^(I*ArcCsc[c*x]))] + (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))] + (4*I)*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc
[c*x]))] - (8*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))] - (16*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
+ (8*I)*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])] - (4*I)*Pi*Log[Sqrt[...
```

3.107.3 Rubi [A] (verified)

Time = 2.94 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \frac{x^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{5232}$$

$$-\int \left(\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)x^2}{e^2}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^2}\right)d\frac{1}{x}$$

$\Big\downarrow$ 2009

$$\frac{x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^2}-\frac{3\sqrt{-d}\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}+\frac{3\sqrt{-d}\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}+\frac{3\sqrt{-d}\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{b\,\mathrm{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{ce^2}+\frac{b\sqrt{d}\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{dc^2+e}}+\frac{b\sqrt{d}\,\mathrm{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4e^2\sqrt{dc^2+e}}-\frac{3ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}+\frac{3ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4e^{5/2}}-\frac{3ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}+\frac{3ib\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

3.107. $\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

output
```
-1/4*(d*(a + b*ArcSin[1/(c*x)]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a +
b*ArcSin[1/(c*x)]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcSin[1/
(c*x)]))/e^2 + (b*ArcTanh[Sqrt[1 - 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*Arc
Tanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/
(c^2*x^2)])])/(4*e^2*Sqrt[c^2*d + e]) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[
-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*e^2
*Sqrt[c^2*d + e]) - (3*Sqrt[-d]*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[
-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*
Sqrt[-d]*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)
]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSin[
1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2
*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*
Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2))
- (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2,
(I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^(5/2)
 - (((3*I)/4)*b*Sqrt[-d]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)])
)/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2) + (((3*I)/4)*b*Sqrt[-d]*PolyLog[2,
 (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^(5/2
)
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.107. $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

3.107.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 44.66 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.20

method	result	size
parts	Expression too large to display	965
derivativedivides	Expression too large to display	987
default	Expression too large to display	987

input
```
int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/e^2*x-1/e^2*d*(-1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
))))+b/c^5*(1/2*x*c^5*arccsc(c*x)*(2*c^2*e*x^2+3*c^2*d)/e^2/(c^2*e*x^2+c^2
*d)-1/e^2*c^4*ln(-1+I/c/x+(1-1/c^2/x^2)^(1/2))+1/e^2*c^4*ln(1+I/c/x+(1-1/c
^2/x^2)^(1/2))-3/16/e^3*c^6*d*sum((_R1^2*c^2*d-c^2*d-4*e)/_R1/(_R1^2*c^2*d
-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((
_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*
_Z^2+c^2*d))+3/16/e^3*c^6*d*sum((_R1^2*c^2*d+4*_R1^2*e-c^2*d)/_R1/(_R1^2*c
^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dil
og((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4
*e)*_Z^2+c^2*d))-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2
*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(
I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/e
^2/(c^2*d+e)/d^2-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2
*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctanh(c*d*
(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e
^2/(c^2*d+e)/d^2+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2
*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*
d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^2/e^2+1/2*((c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctanh(c*d*(I/c/
x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^2...
```

3.107.5 Fricas [F]

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(b \operatorname{arccsc}(cx) + a)x^4}{(ex^2 + d)^2} dx$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccsc(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.107.6 Sympy [F]

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{x^4(a + b \operatorname{acsc}(cx))}{(d + ex^2)^2} dx$$

input
```
integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**4*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

3.107.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.107.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.108 $\int \frac{x^2 (a+b \csc^{-1}(cx))}{(d+ex^2)^2} dx$

3.108.1 Optimal result

Integrand size = 21, antiderivative size = 765

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{c^2d+e}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

output
```
-1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1+I*c*
(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/
(-d)^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccsc(c*x))
*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2))
)/e^(3/2)/(-d)^(1/2)-1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,I
*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(3/
2)/(-d)^(1/2)-1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*I*b*polylog(2,I*c*(I/c
/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(3/2)/(-d)
^(1/2)+1/4*(a+b*arccsc(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arccsc(
c*x))/e/(d/x+(-d)^(1/2)*e^(1/2))-1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x
)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d+e)^(1/2)
-1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1
/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d+e)^(1/2)
```

3.108.2 Mathematica [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 1482, normalized size of antiderivative = 1.94

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d]
+ b*((2*ArcCsc[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsc[c*x])/(I*Sqrt[d]
+ Sqrt[e]*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan
[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]]
)/Sqrt[d] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]])/Sqrt
[d] - (I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x
]))])/Sqrt[d] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*
Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*
ArcCsc[c*x]))])/Sqrt[d] + (I*Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sq
rt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c*x]*Log[1 + (-Sqrt[e]
+ Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + ((4*I)*ArcSin
[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d
 + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] + (I*Pi*Log[1 - (Sqrt[e] +
Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - ((2*I)*ArcCsc[c
*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sq
rt[d] - ((4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (
Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[d] - (I*Pi
*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sq...
```

3.108.3 Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5172, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{5172}$$

$$-\int \left(-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{2e\left(-\frac{d^2}{x^2}-ed\right)}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$-\frac{\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+\frac{\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}-\frac{\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+\frac{\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4\sqrt{-d}e^{3/2}}+\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{dc^2+e}}-\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}e\sqrt{dc^2+e}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a + b*ArcSin[1/(c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSin[1/(
c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt
[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*S
qrt[c^2*d + e]) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqr
t[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*Sqrt[d]*e*Sqrt[c^2*d + e]) - ((a
+ b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e]
 - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcSin[1/(c*x)])*Log[
1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*
Sqrt[-d]*e^(3/2)) - ((a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*Ar
cSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a +
 b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e]
+ Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, ((-I)*c*Sq
rt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3
/2)) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] -
 Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*b*PolyLog[2, ((-I)*c*Sqrt[
-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(Sqrt[-d]*e^(3/2)
) + ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sq
rt[c^2*d + e])])/(Sqrt[-d]*e^(3/2))
```

3.108. $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5172
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.108.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 39.56 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.10

method	result
parts	$-\frac{ax}{2e(e x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+\frac{b\left(-\frac{c^5\operatorname{arccsc}(cx)x}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\arctan\left(\ldots\right.}{2c d^3 e}\ldots\right.}{\ldots}$
derivativedivides	$\frac{-\frac{a c^5 x}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{a c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b c^4\left(-\frac{cx\operatorname{arccsc}(cx)}{2\left(c^2 e x^2+c^2 d\right)e}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\operatorname{arcta}\ldots}{2e c^5 d^3}\ldots\right.}{\ldots}$
default	$\frac{-\frac{a c^5 x}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{a c^3\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}+b c^4\left(-\frac{cx\operatorname{arccsc}(cx)}{2\left(c^2 e x^2+c^2 d\right)e}-\frac{\sqrt{-\left(c^2d-2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)\operatorname{arcta}\ldots}{2e c^5 d^3}\ldots\right.}{\ldots}$

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

3.108. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

output
```
-1/2*a/e*x/(e*x^2+d)+1/2*a/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c^3*(-1
/2*c^5*arccsc(c*x)/e*x/(c^2*e*x^2+c^2*d)-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2
)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(I/c/x+(1-1/c
^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/c/d^3/e+1/2*(
-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c
^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2
))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d^3/c-1/2*((c
^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)
*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2))/c/d^3/e+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(
c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctanh(c*d
*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/
(c^2*d+e)/e/d^3/c-1/4/e*c^4*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*
x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^
(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/4/e*c^4*s
um(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^
(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_
Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d)))
```

3.108.5 Fricas [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsc(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.108.6 Sympy [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^2(a+b\operatorname{acsc}(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

3.108.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.108.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.109 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx$

3.109.1 Optimal result

Integrand size = 18, antiderivative size = 762

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx &= \frac{-a-b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\csc^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}+\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d+e}}\\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

output
```
1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccsc(c*x))*ln(1+I*c*(
I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2
)/e^(1/2)+1/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccsc(c*x))*
ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))
/(-d)^(3/2)/e^(1/2)+1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,I*
c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(
3/2)/e^(1/2)+1/4*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*I*b*polylog(2,I*c*(I/c/
x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^
(1/2)+1/4*(-a-b*arccsc(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arccsc(c
*x))/d/(d/x+(-d)^(1/2)*e^(1/2))+1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)
/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)+1/
4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^
2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(1/2)
```

3.109.2 Mathematica [A] (warning: unable to verify)

Time = 2.06 (sec) , antiderivative size = 1477, normalized size of antiderivative = 1.94

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^2,x]
```

output
```
((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
 + (b*((2*Sqrt[d]*ArcCsc[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (2*Sqrt[d]*A
rcCsc[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + (8*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*
Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c
*x])/4])/Sqrt[c^2*d + e]])/Sqrt[e] - (8*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi + 2*ArcCsc[c*x])/4
])/Sqrt[c^2*d + e]])/Sqrt[e] - (I*Pi*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(
c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + ((2*I)*ArcCsc[c*x]*Log[1 + (Sqrt[
e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((4*I)*Arc
Sin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (Sqrt[e] - Sqrt[c^2
*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] + (I*Pi*Log[1 + (-Sqrt[e]
 + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((2*I)*ArcCs
c[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))]
)/Sqrt[e] + ((4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1
 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/Sqrt[e] +
(I*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))])/
Sqrt[e] - ((2*I)*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d
]*E^(I*ArcCsc[c*x]))])/Sqrt[e] - ((4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc
[c*x]))])/Sqrt[e] - (I*Pi*Log[1 + (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d...
```

3.109.3 Rubi [A] (verified)

Time = 2.70 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5754, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{5754}$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{5232}$$

$$-\int \left(\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)}-\frac{e\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^2}\right)d\frac{1}{x}$$

$$\downarrow 2009$$

$$\frac{\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{4d\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{dc^2+e}}+\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{dc^2+e}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^2,x]
```

output
```
-1/4*(a + b*ArcSin[1/(c*x)])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcSin[
1/(c*x)])/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*S
qrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*
Sqrt[c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sq
rt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d + e]) + ((a +
 b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e]
- Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSin[1/(c*x)])*Log
[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4
*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I
*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) -
((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqr
t[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((I/4)*b*PolyLog[2, ((-
I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(
3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(
Sqrt[e] - Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e]) + ((I/4)*b*PolyLog[2, ((
-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^
(3/2)*Sqrt[e]) - ((I/4)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/
(Sqrt[e] + Sqrt[c^2*d + e])])/((-d)^(3/2)*Sqrt[e])
```

3.109. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx$

3.109.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5754
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

3.109.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 33.95 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.09

method	result
parts	$\frac{ax}{2d(e\,x^2+d)}+\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+\frac{b\left(\frac{c^3\,\mathrm{arccsc}(cx)x}{2d\left(c^2e\,x^2+c^2d\right)}-\frac{c^2\left(\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\,\mathrm{arccsc}(cx)\ln}{}\right.}{4d}\right.}{}$
derivativedivides	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\mathrm{arccsc}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\,\mathrm{arccsc}(cx)\ln\left(\right.}{}}{4d\,c}\right.}{}$
default	$\frac{\frac{a\,c^3x}{2d\left(c^2e\,x^2+c^2d\right)}+\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}+b\,c^4\left(\frac{\mathrm{arccsc}(cx)x}{2cd\left(c^2e\,x^2+c^2d\right)}-\frac{\sum_{\_R1=\mathrm{RootOf}\left(c^2d\_Z^4+\left(-2c^2d-4e\right)\_Z^2+c^2d\right)}\frac{i\,\mathrm{arccsc}(cx)\ln\left(\right.}{}}{4d\,c}\right.}{}$

input
```
int((a+b*arccsc(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

3.109. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx$

output
```
1/2*a*x/d/(e*x^2+d)+1/2*a/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b/c*(1/2*c
^3*arccsc(c*x)*x/d/(c^2*e*x^2+c^2*d)-1/4/d*c^2*sum(1/_R1/(_R1^2*c^2*d-c^2*
d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I
/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+
c^2*d))-1/4/d*c^2*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I
/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1))
,_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+1/2*(-(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*arctan(c*d*(I/
c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4
/c^3-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)
*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(I/c/x+(1-1/c^2
/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)/c
^3+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))
^(1/2)+2*e)*d)^(1/2))/d^4/c^3-1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1
/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*a
rctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*
d)^(1/2))/d^4/(c^2*d+e)/c^3)
```

3.109.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.109.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{a+b\operatorname{acsc}(cx)}{(d+ex^2)^2}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acsc(c*x))/(d + e*x**2)**2, x)
```

3.109.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.109.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^2}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^2, x)
```

3.110 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

3.110.1 Optimal result

Integrand size = 21, antiderivative size = 806

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}-\frac{a}{d^2x}-\frac{b\csc^{-1}(cx)}{d^2x}+\frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$
$$-\frac{e(a+b\csc^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b e \operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$
$$-\frac{b e \operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d+e}}$$
$$+\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$-\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$-\frac{3\sqrt{e}(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$+\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$
$$-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4(-d)^{5/2}}$$

output
```
-a/d^2/x-b*arccsc(c*x)/d^2/x+3/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4
*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arccsc(c*x))*ln(1-I*c*(I/c
/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)
^(5/2)-3/4*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*polylog(2,-I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-
d)^(5/2)-3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*I*b*polylog(2,-I*c*(I/c/x+(1
-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2
)-3/4*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arccsc(c*x))/d^2/(-d/x+(-d)^
(1/2)*e^(1/2))-1/4*e*(a+b*arccsc(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))-1/4*b*
e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/
x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1/2)-1/4*b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1
/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(1
/2)-b*c*(1-1/c^2/x^2)^(1/2)/d^2
```

3.110.2 Mathematica [A] (warning: unable to verify)

Time = 1.86 (sec) , antiderivative size = 1525, normalized size of antiderivative = 1.89

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(S
qrt[e]*x)/Sqrt[d]] + b*(-8*c*Sqrt[d]*Sqrt[1 - 1/(c^2*x^2)] - (8*Sqrt[d]*Ar
cCsc[c*x])/x - (2*Sqrt[d]*e*ArcCsc[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (2
*Sqrt[d]*e*ArcCsc[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - 24*Sqrt[e]*ArcSin[Sqrt
[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[(((-I)*c*Sqrt[d] + Sqrt[e])*
Cot[(Pi + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + 24*Sqrt[e]*ArcSin[Sqrt[1 +
 (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((I*c*Sqrt[d] + Sqrt[e])*Cot[(Pi
 + 2*ArcCsc[c*x])/4])/Sqrt[c^2*d + e]] + (3*I)*Sqrt[e]*Pi*Log[1 + (Sqrt[e]
 - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (6*I)*Sqrt[e]*ArcCsc[
c*x]*Log[1 + (Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] +
(12*I)*Sqrt[e]*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (
Sqrt[e] - Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] - (3*I)*Sqrt[e]*
Pi*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (
6*I)*Sqrt[e]*ArcCsc[c*x]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E
^(I*ArcCsc[c*x]))] - (12*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (-Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*
x]))] - (3*I)*Sqrt[e]*Pi*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*Sqrt[d]*E^
(I*ArcCsc[c*x]))] + (6*I)*Sqrt[e]*ArcCsc[c*x]*Log[1 - (Sqrt[e] + Sqrt[c^2*
d + e])/(c*Sqrt[d]*E^(I*ArcCsc[c*x]))] + (12*I)*Sqrt[e]*ArcSin[Sqrt[1 + (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (Sqrt[e] + Sqrt[c^2*d + e])/(c*...
```

3.110.3 Rubi [A] (verified)

Time = 2.79 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \; 5764$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^4}d\frac{1}{x}$$

$$\Big\downarrow \; 5232$$

$$-\int \left(\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^2}-\frac{2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)}+\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{d^2}\right)d\frac{1}{x}$$

$\Big\downarrow$ 2009

$$-\frac{a}{d^2x}-\frac{b\arcsin\left(\frac{1}{cx}\right)}{d^2x}+\frac{e\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}-\frac{be\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{dc^2+e}}+\frac{3\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}+\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{3ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{4(-d)^{5/2}}-\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{d^2}$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^2),x]
```

3.110. $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

output
```
-((b*c*Sqrt[1 - 1/(c^2*x^2)])/d^2) - a/(d^2*x) - (b*ArcSin[1/(c*x)])/(d^2*
x) + (e*(a + b*ArcSin[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a
+ b*ArcSin[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2
*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)
])])/(4*d^(5/2)*Sqrt[c^2*d + e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e]
)/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c
^2*d + e]) + (3*Sqrt[e]*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I
*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[
e]*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(S
qrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSin[1/(c
*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d +
 e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sq
rt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2))
 + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))
/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[e]*PolyLog[2
, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(-d)^
(5/2) + (((3*I)/4)*b*Sqrt[e]*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*
x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(-d)^(5/2) - (((3*I)/4)*b*Sqrt[e]*Poly
Log[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/
(-d)^(5/2)
```

3.110.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.110.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 35.46 (sec) , antiderivative size = 925, normalized size of antiderivative = 1.15

method	result	size
parts	Expression too large to display	925
derivativedivides	Expression too large to display	952
default	Expression too large to display	952

input
```
int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/d^2/x-e/d^2*(1/2*x/(e*x^2+d)+3/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
))+b*c*(1/2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x-1)*(arccsc(c*x)+I)/d^2/x/c-
1/2*(I*((c^2*x^2-1)/c^2/x^2)^(1/2)*c*x+1)/x/c*(arccsc(c*x)-I)/d^2-1/2*arcc
sc(c*x)/d^2*e*x*c/(c^2*e*x^2+c^2*d)-1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*e*arctan(c*d*(I/c/x+(1-1/c^2/
x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/c^5+1/2*(-(c
^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*
d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2)
)/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^5/c^5/(c^2*d+e)-1/2*((c^
2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*
e*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*
e)*d)^(1/2))/d^5/c^5+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*
(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctanh(
c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2
))/d^5/c^5/(c^2*d+e)+3/4*e/d^2*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc
(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^
2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+3/4*e/d^
2*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^
2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*
d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))
```

3.110.5 Fricas [F]

$$\int \frac{a + b\csc^{-1}(cx)}{x^2 (d + ex^2)^2} \, dx = \int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^2 x^2} \, dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

3.110.6 Sympy [F]

$$\int \frac{a + b\csc^{-1}(cx)}{x^2 (d + ex^2)^2} \, dx = \int \frac{a + b \operatorname{acsc}(cx)}{x^2 (d + ex^2)^2} \, dx$$

input
```
integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)**2), x)
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x^2 (d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.110.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.111 $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

3.111.1 Optimal result

Integrand size = 21, antiderivative size = 727

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bcd\sqrt{1-\frac{1}{c^2x^2}}}{8e^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\csc^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\csc^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$
$$+ \frac{b\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{2e^{5/2}\sqrt{c^2d+e}} + \frac{b(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}\,x}\right)}{8e^{5/2}(c^2d+e)^{3/2}}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$+ \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{(a+b\csc^{-1}(cx))\log\left(1-e^{2i\csc^{-1}(cx)}\right)}{e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3}$$
$$- \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2e^3} + \frac{ib\,\mathrm{PolyLog}\left(2,e^{2i\csc^{-1}(cx)}\right)}{2e^3}$$

output
```
1/4*(-a-b*arccsc(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arccsc(c*x))/e^2/(e+d/x^2)+
1/8*b*(c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))/
e^(5/2)/(c^2*d+e)^(3/2)-(a+b*arccsc(c*x))*ln(1-(I/c/x+(1-1/c^2/x^2)^(1/2))
^2)/e^3+1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(
1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arc
csc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+
e)^(1/2)))/e^3+1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*
(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^3+1/2*I*b*polylog(2,(I/c/x+(1-1/c^
2/x^2)^(1/2))^2)/e^3-1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^3-1/2*I*b*polylog(
2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e
^3-1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(
c^2*d+e)^(1/2)))/e^3+1/2*b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2
)^(1/2))/e^(5/2)/(c^2*d+e)^(1/2)+1/8*b*c*d*(1-1/c^2/x^2)^(1/2)/e^2/(c^2*d+
e)/(e+d/x^2)/x
```

3.111.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2053 vs. $2(727) = 1454$.

Time = 7.39 (sec) , antiderivative size = 2053, normalized size of antiderivative = 2.82

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*
x^2])/(2*e^3) + b*((((7*I)/16)*Sqrt[d]*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sqrt[e
] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt[e]
+ c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[
-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(
5/2) - (((7*I)/16)*Sqrt[d]*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*
(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sq
rt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*
(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/e^(5/2) - (d*((I*
c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sq
rt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/
(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*
Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^
2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(16*e^(5/2)
) - (d*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*S
qrt[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - A
rcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d
 + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)]
)*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(1
6*e^(5/2)) + ((I/16)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*Ar...
```

3.111.3 Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \frac{x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{5232}$$

$$-\int \left(\frac{x\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^3}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2x}-\frac{d\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^3x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc\sqrt{1-\frac{1}{c^2x^2}}d}{8e^2(dc^2+e)\left(\frac{d}{x^2}+e\right)x}-\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{2e^2\left(\frac{d}{x^2}+e\right)}-\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x^2}+e\right)^2}+\frac{b(dc^2+2e)\arctan\left(\frac{\sqrt{dc^2+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8e^{5/2}(dc^2+e)^{3/2}}+\frac{b\arctan\left(\frac{\sqrt{dc^2+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{2e^{5/2}\sqrt{dc^2+e}}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{2e^3}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{2e^3}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{e^3}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2e^3}+\frac{ib\,\text{PolyLog}\left(2,e^{2i\arcsin\left(\frac{1}{cx}\right)}\right)}{2e^3}$$

input
```
Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

3.111. $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

output
```
(b*c*d*Sqrt[1 - 1/(c^2*x^2)])/(8*e^2*(c^2*d + e)*(e + d/x^2)*x) - (a + b*A
rcSin[1/(c*x)])/(4*e*(e + d/x^2)^2) - (a + b*ArcSin[1/(c*x)])/(2*e^2*(e +
d/x^2)) + (b*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/
(2*e^(5/2)*Sqrt[c^2*d + e]) + (b*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*S
qrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)) + ((a + b*
ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - S
qrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]
*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*Ar
cSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqr
t[c^2*d + e])])/(2*e^3) + ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E
^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) - ((a + b*ArcS
in[1/(c*x)])*Log[1 - E^((2*I)*ArcSin[1/(c*x)])])/e^3 - ((I/2)*b*PolyLog[2,
 ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/e^3
 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqr
t[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(
c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 - ((I/2)*b*PolyLog[2, (I*c*Sqrt[
-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/e^3 + ((I/2)*b*Po
lyLog[2, E^((2*I)*ArcSin[1/(c*x)])])/e^3
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.111. $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

3.111.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.67 (sec) , antiderivative size = 1269, normalized size of antiderivative = 1.75

method	result	size
parts	Expression too large to display	1269
derivativedivides	Expression too large to display	1285
default	Expression too large to display	1285

input
```
int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/4*d^2/e^3/(e*x^2+d)^2+1/2/e^3*ln(e*x^2+d)+d/e^3/(e*x^2+d))+b/c^6*(-1
/8*c^6*(4*c^6*d^2*arccsc(c*x)*x^2+6*c^6*d*e*arccsc(c*x)*x^4-((c^2*x^2-1)/c
^2/x^2)^(1/2)*c^5*d^2*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d*e*x^3+I*c^4*d^2+
2*I*c^4*d*e*x^2+I*e^2*c^4*x^4+4*c^4*d*e*arccsc(c*x)*x^2+6*arccsc(c*x)*e^2*
c^4*x^4)/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)-1/4*I/(c^2*d+e)/e^2*c^8*d*sum((
_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2
)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d
*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))+I/(c^2*d+e)/e^2*c^6*dilog(1+I/c/x+(1-1/c
^2/x^2)^(1/2))-I/(c^2*d+e)/e^2*c^6*dilog(I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I/
(c^2*d+e)/e^2*c^6*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*a
rccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c
^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/(
c^2*d+e)/e^2*c^6*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/(c^2*d+e)/e
^3*c^8*d*arccsc(c*x)*ln(1+I/c/x+(1-1/c^2/x^2)^(1/2))-1/4*I/(c^2*d+e)/e^3*c
^8*d*sum((_R1^2*c^2*d-c^2*d-4*e)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln
((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2)
)/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/4*I/(c^2*d+e)/
e^3*c^10*d^2*sum((_R1^2-1)/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-
I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)
),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-5/8*I*(e*(c^2*d+e))...
```

3.111.5 Fricas [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arccsc(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.111.7 Maxima [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + b*integrate(x^5*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*
x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

3.111.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

input
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.112 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

3.112.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{8e(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} + \frac{x^4(a+b\csc^{-1}(cx))}{4d(d+ex^2)^2} + \frac{bc(c^2d+2e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2}(c^2d+e)^{3/2}\sqrt{c^2x^2}}$$

output
```
1/4*x^4*(a+b*arccsc(c*x))/d/(e*x^2+d)^2+1/8*b*c*(c^2*d+2*e)*x*arctan(e^(1/
2)*(c^2*x^2-1)^(1/2)/(c^2*d+e)^(1/2))/d/e^(3/2)/(c^2*d+e)^(3/2)/(c^2*x^2)^
(1/2)-1/8*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.48

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$$

$$= \frac{\frac{4ad}{(d+ex^2)^2} - \frac{8a}{d+ex^2} - \frac{2bce\sqrt{1-\frac{1}{c^2x^2}}\,x}{(c^2d+e)(d+ex^2)} - \frac{4b(d+2ex^2)\csc^{-1}(cx)}{(d+ex^2)^2} + \frac{4b\arcsin\left(\frac{1}{cx}\right)}{d} + \frac{b\sqrt{e}(c^2d+2e)\log\left(\frac{16d\sqrt{-c^2d-e}e^{3/2}\left(i\sqrt{e}+c\left(c\sqrt{d}-i\sqrt{\ldots}\right.\right.}{b\left(c^2d+2e\right)\left(\sqrt{d}+i\ldots\right.}\right.}{d(-c^2d-e)^{3/2}}}{16e^2}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

output
```
((4*a*d)/(d + e*x^2)^2 - (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 - 1/(c^2*x^2)
]*x)/((c^2*d + e)*(d + e*x^2)) - (4*b*(d + 2*e*x^2)*ArcCsc[c*x])/(d + e*x^
2)^2 + (4*b*ArcSin[1/(c*x)])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*d*Sqrt[-
(c^2*d) - e]*e^(3/2)*(I*Sqrt[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt
[1 - 1/(c^2*x^2)])*x))/(b*(c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(-(c
^2*d) - e)^(3/2)) + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(-16*d*Sqrt[-(c^2*d) - e]
*e^(3/2)*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^
2*x^2)])*x))/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(-(c^2*d) - e)
^(3/2)))/(16*e^2)
```

3.112.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5762, 27, 354, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+b \csc^{-1}(cx))}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{ 5762}$$

$$\frac{bcx \int \frac{x^3}{4d\sqrt{c^2x^2-1}(ex^2+d)^2}dx}{\sqrt{c^2x^2}}+\frac{x^4 (a+b \csc^{-1}(cx))}{4d (d+ex^2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bcx \int \frac{x^3}{\sqrt{c^2x^2-1}(ex^2+d)^2}dx}{4d\sqrt{c^2x^2}}+\frac{x^4 (a+b \csc^{-1}(cx))}{4d (d+ex^2)^2}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{bcx \int \frac{x^2}{\sqrt{c^2x^2-1}(ex^2+d)^2}dx^2}{8d\sqrt{c^2x^2}}+\frac{x^4 (a+b \csc^{-1}(cx))}{4d (d+ex^2)^2}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{bcx\left(\frac{(c^2d+2e)\int \frac{1}{\sqrt{c^2x^2-1}(ex^2+d)}dx^2}{2e(c^2d+e)}-\frac{d\sqrt{c^2x^2-1}}{e(c^2d+e)(d+ex^2)}\right)}{8d\sqrt{c^2x^2}}+\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{bcx\left(\frac{(c^2d+2e)\int \frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2e(c^2d+e)}-\frac{d\sqrt{c^2x^2-1}}{e(c^2d+e)(d+ex^2)}\right)}{8d\sqrt{c^2x^2}}+\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^4\left(a+b\csc^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bcx\left(\frac{(c^2d+2e)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{e^{3/2}(c^2d+e)^{3/2}}-\frac{d\sqrt{c^2x^2-1}}{e(c^2d+e)(d+ex^2)}\right)}{8d\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcCsc[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*x*(-((d*Sqrt[-1 + c^2
*x^2])/(e*(c^2*d + e)*(d + e*x^2))) + ((c^2*d + 2*e)*ArcTan[(Sqrt[e]*Sqrt[
-1 + c^2*x^2])/Sqrt[c^2*d + e]])/(e^(3/2)*(c^2*d + e)^(3/2))))/(8*d*Sqrt[c
^2*x^2])
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(135) = 270$.

Time = 6.54 (sec) , antiderivative size = 945, normalized size of antiderivative = 6.02

method	result
parts	$a\left(\frac{d}{4e^{2}(e\,x^{2}+d)^{2}}-\frac{1}{2e^{2}(e\,x^{2}+d)}\right)+\frac{b\left(-\frac{c^{6}\,\mathrm{arccsc}(cx)}{2e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{c^{8}\,\mathrm{arccsc}(cx)d}{4e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}-\frac{c^{3}\sqrt{c^{2}x^{2}-1}\left(4\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)\sqrt{-}\right.}{}\right.}{}$
derivativedivides	$\frac{a\,c^{6}\left(-\frac{1}{2e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{d\,c^{2}}{4e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}\right)+b\,c^{6}\left(-\frac{\mathrm{arccsc}(cx)}{2e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{\mathrm{arccsc}(cx)d\,c^{2}}{4e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}+\frac{\sqrt{c^{2}x^{2}-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^{2}x}}\right.\right.}{}\right.}{}$
default	$\frac{a\,c^{6}\left(-\frac{1}{2e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{d\,c^{2}}{4e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}\right)+b\,c^{6}\left(-\frac{\mathrm{arccsc}(cx)}{2e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)}+\frac{\mathrm{arccsc}(cx)d\,c^{2}}{4e^{2}\left(c^{2}e\,x^{2}+c^{2}d\right)^{2}}+\frac{\sqrt{c^{2}x^{2}-1}\left(-4\arctan\left(\frac{1}{\sqrt{c^{2}x}}\right.\right.}{}\right.}{}$

input
```
int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

3.112. $\int \frac{x^{3}\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^{2}\right)^{3}}\,dx$

output
```
a*(1/4*d/e^2/(e*x^2+d)^2-1/2/e^2/(e*x^2+d))+b/c^4*(-1/2*c^6*arccsc(c*x)/e^
2/(c^2*e*x^2+c^2*d)+1/4*c^8*arccsc(c*x)*d/e^2/(c^2*e*x^2+c^2*d)^2-1/16*c^3
*(c^2*x^2-1)^(1/2)/e*(4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c
^4*d*e*x^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d^2-ln(2
*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+
(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2
)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-ln(-2*((c^2*
x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*
d*e)^(1/2)))*c^4*d*e*x^2-ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(
-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+4*arctan(1/(c^2*
x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*e^2*c^2*x^2+4*arctan(1/(c^2*x^2-1)^(1/2
))*(-(c^2*d+e)/e)^(1/2)*c^2*d*e-2*(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*c
^2*d*e-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c
*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c
^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^2*d
*e-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-
e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^
2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^2*d
*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d+e)/e)^(1/2)/(c^2*d+e)/(-c*e*x
+(-c^2*d*e)^(1/2))/(c*e*x+(-c^2*d*e)^(1/2)))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(135) = 270$.

Time = 0.50 (sec) , antiderivative size = 1015, normalized size of antiderivative = 6.46

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{(d + ex^2)^3}\,dx$$

$$= \left[-\frac{4\,ac^4d^4 + 8\,ac^2d^3e + 4\,ad^2e^2 + 8\,(ac^4d^3e + 2\,ac^2d^2e^2 + ade^3)x^2 + (bc^2d^3 + (bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e}{} \right.$$

$$-\frac{2\,ac^4d^4 + 4\,ac^2d^3e + 2\,ad^2e^2 + 4\,(ac^4d^3e + 2\,ac^2d^2e^2 + ade^3)x^2 - (bc^2d^3 + (bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e}{}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e + 2*a*c
^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b
*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*log((c^2*e*
x^2 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^2*x^2 - 1) - 2*e)/(e*x^2 + d))
 + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2
*e^2 + b*d*e^3)*x^2)*arccsc(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^
2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2
*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c^2*d^3*e
 + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^5*
e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4
+ 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*
c^2*d^3*e + 2*a*d^2*e^2 + 4*(a*c^4*d^3*e + 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2
- (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e +
2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*sqrt(c^2*x^
2 - 1)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*
d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arccsc(c*x) + 4*(b*c^4*d^4 + 2*b*c
^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*
c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1
)) + (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^
2 - 1))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d...
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.112.7 Maxima [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 1/4*(2*e*x^2*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x
- 1)) + 4*(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)*integrate(1/4*(2*c^2*e*x^3 + c
^2*d*x)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^4*x^6 + (2*c^2*d*e^
3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2 + (c^2*e^4*x^6 + (2*c
^2*d*e^3 - e^4)*x^4 - d^2*e^2 + (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^(log(c*x +
1) + log(c*x - 1))), x))*b/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)
```

3.112.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.113 $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

3.113.1 Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a+b\csc^{-1}(cx)}{4e(d+ex^2)^2} - \frac{bcx\arctan\left(\sqrt{-1+c^2x^2}\right)}{4d^2e\sqrt{c^2x^2}} + \frac{bc(3c^2d+2e)x\arctan\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}(c^2d+e)^{3/2}\sqrt{c^2x^2}}$$

output
```
1/4*(-a-b*arccsc(c*x))/e/(e*x^2+d)^2-1/4*b*c*x*arctan((c^2*x^2-1)^(1/2))/d
^2/e/(c^2*x^2)^(1/2)+1/8*b*c*(3*c^2*d+2*e)*x*arctan(e^(1/2)*(c^2*x^2-1)^(1
/2)/(c^2*d+e)^(1/2))/d^2/(c^2*d+e)^(3/2)/e^(1/2)/(c^2*x^2)^(1/2)+1/8*b*c*x
*(c^2*x^2-1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)/(c^2*x^2)^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.99

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$$
$$= \frac{1}{16}\left(-\frac{4a}{e(d+ex^2)^2}+\frac{2bc\sqrt{1-\frac{1}{c^2x^2}}x}{d(c^2d+e)(d+ex^2)}-\frac{4b\csc^{-1}(cx)}{e(d+ex^2)^2}+\frac{4b\arcsin\left(\frac{1}{cx}\right)}{d^2e}\right.$$
$$+\frac{b(3c^2d+2e)\log\left(\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}-i\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}$$
$$\left.+\frac{b(3c^2d+2e)\log\left(-\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(-\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2(-c^2d-e)^{3/2}\sqrt{e}}\right)$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

output
```
((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x)/(d*(c^2*d + e)
*(d + e*x^2)) - (4*b*ArcCsc[c*x])/(e*(d + e*x^2)^2) + (4*b*ArcSin[1/(c*x)]
)/(d^2*e) + (b*(3*c^2*d + 2*e)*Log[(16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(I*S
qrt[e] + c*(c*Sqrt[d] - I*Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b
*(3*c^2*d + 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt
[e]) + (b*(3*c^2*d + 2*e)*Log[(-16*d^2*Sqrt[-(c^2*d) - e]*Sqrt[e]*(-Sqrt[e
] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(b*(
3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*(-(c^2*d) - e)^(3/2)*Sqrt[e
]))/16
```

3.113.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5760, 354, 114, 27, 174, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{ 5760}$$

$$-\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}(ex^2+d)^2}dx}{4e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}(ex^2+d)^2}dx^2}{8e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 114}$$

$$-\frac{bcx\left(-\frac{\int -\frac{-ex^2c^2+2dc^2+2e}{2x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bcx\left(\frac{\int \frac{2\left(dc^2+e\right)-c^2ex^2}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 174}$$

$$-\frac{bcx\left(\frac{\frac{2\left(c^2d+e\right)\int \frac{1}{x^2\sqrt{c^2x^2-1}}dx^2}{d}-\frac{e\left(3c^2d+2e\right)\int \frac{1}{\sqrt{c^2x^2-1}\left(ex^2+d\right)}dx^2}{d}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{bcx\left(\frac{\frac{4\left(c^2d+e\right)\int \frac{1}{\frac{x^4}{c^2}+\frac{1}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}-\frac{2e\left(3c^2d+2e\right)\int \frac{1}{\frac{ex^4}{c^2}+d+\frac{e}{c^2}}d\sqrt{c^2x^2-1}}{c^2d}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{a+b\csc^{-1}(cx)}{4e\left(d+ex^2\right)^2}-\frac{bcx\left(\frac{\frac{4\arctan\left(\sqrt{c^2x^2-1}\right)\left(c^2d+e\right)}{d}-\frac{2\sqrt{e}\left(3c^2d+2e\right)\arctan\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{\sqrt{c^2d+e}}\right)}{d\sqrt{c^2d+e}}}{2d(c^2d+e)}-\frac{e\sqrt{c^2x^2-1}}{d(c^2d+e)(d+ex^2)}\right)}{8e\sqrt{c^2x^2}}$$

3.113. $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcCsc[c*x])/(e*(d + e*x^2)^2) - (b*c*x*(-((e*Sqrt[-1 + c^2*x^
2])/(d*(c^2*d + e)*(d + e*x^2))) + ((4*(c^2*d + e)*ArcTan[Sqrt[-1 + c^2*x^
2]])/d - (2*Sqrt[e]*(3*c^2*d + 2*e)*ArcTan[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/Sq
rt[c^2*d + e]])/(d*Sqrt[c^2*d + e]))/(2*d*(c^2*d + e))))/(8*e*Sqrt[c^2*x^2
])
```

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(168) = 336$.

Time = 7.18 (sec) , antiderivative size = 894, normalized size of antiderivative = 4.63

method	result
parts	$-\frac{a}{4e(e x^2+d)^2} + \frac{b\left(-\frac{c^6 \operatorname{arccsc}(cx)}{4e\left(c^2 e x^2+c^2 d\right)^2} - \frac{c\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\, c^4 d e x^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2 d}{e}}\cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a c^6}{4e\left(c^2 e x^2+c^2 d\right)^2}+b c^6\left(-\frac{\operatorname{arccsc}(cx)}{4e\left(c^2 e x^2+c^2 d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\, c^4 d e x^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{a c^6}{4e\left(c^2 e x^2+c^2 d\right)^2}+b c^6\left(-\frac{\operatorname{arccsc}(cx)}{4e\left(c^2 e x^2+c^2 d\right)^2}-\frac{\sqrt{c^2x^2-1}\left(4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\frac{c^2d+e}{e}}\, c^4 d e x^2+4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arccsc(c*x)-1/1
6*c*(c^2*x^2-1)^(1/2)*(4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*
c^4*d*e*x^2+4*arctan(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^4*d^2-3*l
n(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c
*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)
/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-3*l
n(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e
*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)
^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+4*arcta
n(1/(c^2*x^2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*e^2*c^2*x^2+4*arctan(1/(c^2*x^
2-1)^(1/2))*(-(c^2*d+e)/e)^(1/2)*c^2*d*e+2*(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e
)^(1/2)*c^2*d*e-2*ln(-2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*
e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(-2*((c^2*x^2-1
)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*e*x+(-c^2*d*e)^
(1/2)))*c^2*d*e-2*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e
)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2-2*ln(2*((c^2*x^2-1)^(
1/2)*(-(c^2*d+e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2
)))*c^2*d*e)/((c^2*x^2-1)/c^2/x^2)^(1/2)/x/d^2/(-(c^2*d+e)/e)^(1/2)/(c^2*d
+e)/(c*e*x+(-c^2*d*e)^(1/2))/(-c*e*x+(-c^2*d*e)^(1/2)))
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(165) = 330$.

Time = 0.50 (sec) , antiderivative size = 890, normalized size of antiderivative = 4.61

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^3}\,dx$$

$$= \left[-\frac{4\,ac^4d^4 + 8\,ac^2d^3e + 4\,ad^2e^2 + (3\,bc^2d^3 + (3\,bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + 2\,(3\,bc^2d^2e + 2\,bde^2)x^2)\sqrt{-c^2}}{}\right.$$

$$-\frac{2\,ac^4d^4 + 4\,ac^2d^3e + 2\,ad^2e^2 - (3\,bc^2d^3 + (3\,bc^2de^2 + 2\,be^3)x^4 + 2\,bd^2e + 2\,(3\,bc^2d^2e + 2\,bde^2)x^2)\sqrt{c^2de}}{}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 + 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^
2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sq
rt(-c^2*d*e - e^2)*log((c^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e - e^2)*sqrt(c^
2*x^2 - 1) - 2*e)/(e*x^2 + d)) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)
*arccsc(c*x) + 8*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 +
 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*
x^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^2*d^3*e + b*d^2*e^2 + (b*c^
2*d^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 +
d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2
*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + 4*a*c^2*d^3*e + 2*a*d^2*
e^2 - (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^
2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*arctan(sqrt(c^2*d*e + e^2)*s
qrt(c^2*x^2 - 1)/(c^2*d + e)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*
arccsc(c*x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 +
2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x
^2)*arctan(-c*x + sqrt(c^2*x^2 - 1)) - (b*c^2*d^3*e + b*d^2*e^2 + (b*c^2*d
^2*e^2 + b*d*e^3)*x^2)*sqrt(c^2*x^2 - 1))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4
*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^
2*d^4*e^3 + d^3*e^4)*x^2)]
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.113.7 Maxima [F]

$$\int \frac{x(a + b\csc^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x}{(ex^2 + d)^3} \, dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(4*(c^2*e^3*x^4 + 2*c^2*d*e^2*x^2 + c^2*d^2*e)*integrate(1/4*x*e^(1/2
*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 -
 d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^
4 - d^2*e + (c^2*d^2*e - 2*d*e^2)*x^2)*e^(log(c*x + 1) + log(c*x - 1))), x
) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/(e^3*x^4 + 2*d*e^2*x^2 + d^
2*e) - 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

3.113.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.114 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^3}\,dx$

3.114.1 Optimal result

Integrand size = 21, antiderivative size = 704

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^3}\,dx = &-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2(c^2d+e)\left(e+\frac{d}{x^2}\right)x}+\frac{e^2(a+b\csc^{-1}(cx))}{4d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e(a+b\csc^{-1}(cx))}{d^3\left(e+\frac{d}{x^2}\right)}\\
&+\frac{i(a+b\csc^{-1}(cx))^2}{2bd^3}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d+e}}\\
&-\frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}x}\right)}{8d^3(c^2d+e)^{3/2}}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}\\
&-\frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{2d^3}\end{aligned}$$

output
```
1/4*e^2*(a+b*arccsc(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arccsc(c*x))/d^3/(e+d/x^2
)+1/2*I*(a+b*arccsc(c*x))^2/b/d^3-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1
-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arcc
sc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e
)^(1/2)))/d^3-1/2*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsc(c*x))*ln(1+I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*
b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)
^(1/2)))/d^3+1/2*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3+1/2*I*b*polylog(2,I*c*(I
/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/d^3-1/8*b*
(c^2*d+2*e)*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(1/2))*e^(1/2
)/d^3/(c^2*d+e)^(3/2)+b*arctan((c^2*d+e)^(1/2)/c/x/e^(1/2)/(1-1/c^2/x^2)^(
1/2))*e^(1/2)/d^3/(c^2*d+e)^(1/2)-1/8*b*c*e*(1-1/c^2/x^2)^(1/2)/d^2/(c^2*d
+e)/(e+d/x^2)/x
```

3.114.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2114 vs. $2(704) = 1408$.

Time = 6.06 (sec) , antiderivative size = 2114, normalized size of antiderivative = 3.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d
+ e*x^2])/(2*d^3) + b*((((5*I)/16)*Sqrt[e]*(-(ArcCsc[c*x]/((-I)*Sqrt[d]*Sq
rt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(Sqrt
[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(S
qrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))
/d^(5/2) - (((5*I)/16)*Sqrt[e]*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) -
 (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*
c*Sqrt[d] + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) -
 e]*(Sqrt[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/d^(5/2) + (Sq
rt[e]*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sq
rt[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) -
 ArcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*
d + e]*(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*
x))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(
16*d^2) + (Sqrt[e]*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2
*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[
e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e
]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 -
1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)
^(3/2))))/(16*d^2) - ((I/16)*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2...
```

3.114.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$$

$$\Big\downarrow \text{ 5764}$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^5}d\frac{1}{x}$$

$$\Big\downarrow \text{ 5232}$$

$$-\int \left(\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^3x}-\frac{2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)^2x}+\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1+\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}+\frac{e^2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{4d^3\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{d^3\left(\frac{d}{x^2}+e\right)}+\frac{i\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)^2}{2bd^3}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}+\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{2d^3}-\frac{b\sqrt{e}(c^2d+2e)\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{8d^3\left(c^2d+e\right)^{3/2}}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}x\sqrt{1-\frac{1}{c^2x^2}}}\right)}{d^3\sqrt{c^2d+e}}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}}{8d^2x\left(c^2d+e\right)\left(\frac{d}{x^2}+e\right)}$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
-1/8*(b*c*e*Sqrt[1 - 1/(c^2*x^2)])/(d^2*(c^2*d + e)*(e + d/x^2)*x) + (e^2*
(a + b*ArcSin[1/(c*x)]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcSin[1/(c*x)]
))/(d^3*(e + d/x^2)) + ((I/2)*(a + b*ArcSin[1/(c*x)])^2)/(b*d^3) + (b*Sqrt
[e]*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)])/(d^3*Sqrt
[c^2*d + e]) - (b*Sqrt[e]*(c^2*d + 2*e)*ArcTan[Sqrt[c^2*d + e]/(c*Sqrt[e]*
Sqrt[1 - 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d + e)^(3/2)) - ((a + b*ArcSin[1/(c
*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d +
 e])])/(2*d^3) - ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSi
n[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSin[1/(c*x
)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(2*d^3) - ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[
1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) + ((I/2)*b*PolyLog[2, ((-
I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/d^3 + (
(I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^
2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, ((-I)*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)
]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3 + ((I/2)*b*PolyLog[2, (I*c*Sqrt[-d]*
E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/d^3
```

3.114. $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^3}\,dx$

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.114.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.31 (sec) , antiderivative size = 3479, normalized size of antiderivative = 4.94

method	result	size
parts	Expression too large to display	3479
derivativedivides	Expression too large to display	3553
default	Expression too large to display	3553

input
```
int((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a/d^3*ln(x)+1/2*a/d^2/(e*x^2+d)-1/2*a/d^3*ln(e*x^2+d)+1/4*a/d/(e*x^2+d)^2+
b*(-5/4*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e
^2)*arccsc(c*x)^2/d^3/(c^4*d^2+2*c^2*d*e+e^2)+I/(c^2*d+e)/d^3*e*arccsc(c*x
)^2+1/2*I/(c^2*d+e)/d^2*c^2*sum((_R1^2*c^2*d-2*c^2*d-4*e)/(_R1^2*c^2*d-c^2
*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-
I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2
+c^2*d))+5/4*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+
2*e^2)*ln(1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/
2)+2*e))*arccsc(c*x)/d^3/(c^4*d^2+2*c^2*d*e+e^2)+1/2*I*arccsc(c*x)^2*(c^2*
d+2*(e*(c^2*d+e))^(1/2)+2*e)/(c^2*d+e)/d^3-1/8*e*(8*c^6*d^2*arccsc(c*x)*x^
2+6*c^6*d*e*arccsc(c*x)*x^4+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^5*d^2*x+((c^2*x^
2-1)/c^2/x^2)^(1/2)*c^5*d*e*x^3-I*c^4*d^2-2*I*c^4*d*e*x^2-I*e^2*c^4*x^4+8*
c^4*d*e*arccsc(c*x)*x^2+6*arccsc(c*x)*e^2*c^4*x^4)/d^3/(c^2*d+e)/(c^2*e*x^
2+c^2*d)^2-5/8*I*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2
)*e+2*e^2)*polylog(2,d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*
d+e))^(1/2)+2*e))/d^3/(c^4*d^2+2*c^2*d*e+e^2)+1/4*I*polylog(2,d*c^2*(I/c/x
+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*(c^2*d+2*(e*(c^
2*d+e))^(1/2)+2*e)/(c^2*d+e)/d^3-1/2*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*ln(
1-d*c^2*(I/c/x+(1-1/c^2/x^2)^(1/2))^2/(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e))*a
rccsc(c*x)/(c^2*d+e)/d^3+1/2*I/(c^2*d+e)/d^3*e*sum((_R1^2*c^2*d-2*c^2*d...
```

3.114.5 Fricas [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.114.7 Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \int \frac{b\,\text{arccsc}\,(cx)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

3.114.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.115 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 1144

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}}{16e^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\csc^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\csc^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e(c^2d+e)^{3/2}} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e(c^2d+e)^{3/2}} - \frac{3b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{c^2d+e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3ib\operatorname{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3ib\operatorname{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

output
```
-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccsc(c*x))*ln(1+I*
c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2
)/(-d)^(1/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccsc(c
*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1
/2)))/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*I*b*polylo
g(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/e^(5/2)/(-d)^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*I*b*polylog(2,-
I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/e^(5
/2)/(-d)^(1/2)-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*
d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e/(c^2*d+e)^(3/2)/d^(1/2)-1/16*b*arctanh((
c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))
/e/(c^2*d+e)^(3/2)/d^(1/2)+1/16*(a+b*arccsc(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x
+(-d)^(1/2)*e^(1/2))^2+3/16*(a+b*arccsc(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2)
)-1/16*(a+b*arccsc(c*x))*(-d)^(1/2)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/1
6*(a+b*arccsc(c*x))/e^2/(d/x+(-d)^(1/2)*e^(1/2))-3/16*b*arctanh((c^2*d-(-d
)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/e^2/d...
```

3.115.2 Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 2067, normalized size of antiderivative = 1.81

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

output
```
(a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((5*(-(ArcCsc[c*x]/((-I)*Sqr
t[d]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e
]*(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])
*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sq
rt[d]))/(16*e^2) + (5*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcS
in[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d]
 + Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt
[d] - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*e^2) + ((I/16)*Sqr
t[d]*((I*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqr
t[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) -
ArcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d
 + e]*(I*Sqrt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x
))/((2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/e^
2 - ((I/16)*Sqrt[d]*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^
2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt
[e]*x)^2) - ArcSin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[
e]*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 -
 1/(c^2*x^2)])*x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e
)^(3/2))))/e^2 - (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*Arc...
```

3.115.3 Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5172, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+b\csc^{-1}(cx))}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{5764}$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{5172}$$

$$-\int \left(-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^3}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^3}-\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)d}{8e^2\left(-\frac{d^2}{x^2}-ed\right)}-\frac{3\left(a+b\arcsin\left(\right.\right.}{16e^2\left(\sqrt{-d}\sqrt{e}\right.}\right.$$

$\downarrow$ 2009

$$-\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{-d}\sqrt{1-\frac{1}{c^2x^2}}c}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{-d}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{3b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}}-\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(dc^2+e\right)^{3/2}}-\frac{3b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e^2\sqrt{dc^2+e}}-\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}e\left(dc^2+e\right)^{3/2}}-\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16\sqrt{-d}e^{5/2}}-\frac{3ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

3.115. $\int \frac{x^4\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

output
```
-1/16*(b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(e^(3/2)*(c^2*d + e)*(Sqrt[-d]*
Sqrt[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 - 1/(c^2*x^2)])/(16*e^(3/2)*(c^2*d
+ e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcSin[1/(c*x)]))/(16*e^
(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcSin[1/(c*x)]))/(16*e^2*(S
qrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcSin[1/(c*x)]))/(16*e^(3/2)*(
Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcSin[1/(c*x)]))/(16*e^2*(Sqrt[-d]
*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sq
rt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) -
(3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqr
t[1 - 1/(c^2*x^2)])]])/(16*Sqrt[d]*e^2*Sqrt[c^2*d + e]) - (b*ArcTanh[(c^2*d
 + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])
])/(16*Sqrt[d]*e*(c^2*d + e)^(3/2)) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt
[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])]])/(16*Sqrt[d]*e^
2*Sqrt[c^2*d + e]) - (3*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I
*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) + (
3*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sq
rt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSin[1/(c*x
)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e
])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[
-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e...
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5172
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (d + e*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (G
tQ[p, 0] || IGtQ[n, 0])
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.115. $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

3.115.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 47.85 (sec) , antiderivative size = 1804, normalized size of antiderivative = 1.58

method	result	size
parts	Expression too large to display	1804
derivativedivides	Expression too large to display	1827
default	Expression too large to display	1827

input
```
int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((-5/8/e*x^3-3/8*d/e^2*x)/(e*x^2+d)^2+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2)))+b/c^5*(-1/8*x*c^7*(3*d^2*c^4*arccsc(c*x)+5*c^4*d*e*arccsc(c*x)*
x^2+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+((c^2*x^2-1)/c^2/x^2)^(1/2)*e^2*
c^3*x^3+3*c^2*d*e*arccsc(c*x)+5*e^2*arccsc(c*x)*c^2*x^2)/e^2/(c^2*d+e)/(c^
2*e*x^2+c^2*d)^2+3/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2
*d+e))^(1/2)*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c^3*arctan(c*d
*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))
/(c^2*d+e)^2/e^2/d^2-3/16/(c^2*d+e)/e^2*c^8*d*sum(_R1/(_R1^2*c^2*d-c^2*d-2
*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/
x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2
*d))+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)
*c^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(I/c/x+(1-1/c
^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e
/d^3-3/16/(c^2*d+e)/e^2*c^8*d*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(
c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2
)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/2*((c^2
*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*c
*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e
)*d)^(1/2))/(c^2*d+e)/e/d^3+1/2*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2
)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*...
```

3.115.5 Fricas [F]

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccsc(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.115.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.116 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.116.1 Optimal result

Integrand size = 21, antiderivative size = 1144

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}}{16\sqrt{-d}\sqrt{e}\,(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\csc^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\csc^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d+e\right)^{3/2}} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} + \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d+e\right)^{3/2}} - \frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{c^2d+e}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{ib\,\mathrm{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

output
```
1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1
/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e)^(3/2)+1/16*b*arctanh((c^2*d+(-d)^(1/2)*
e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d+e
)^(3/2)+1/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccsc(c*x))*
ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))
/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*a
rccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*
d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b
*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*I*b*pol
ylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)
))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccsc(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)
^(1/2)*e^(1/2))^2+1/16*(a+b*arccsc(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/1
6*(-a-b*arccsc(c*x))/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-
a-b*arccsc(c*x))/d/e/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*arctanh((c^2*d-(-d)^(
1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(3/2)/...
```

3.116.2 Mathematica [A] (warning: unable to verify)

Time = 6.06 (sec) , antiderivative size = 2075, normalized size of antiderivative = 1.81

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(-1/16*(-(ArcCsc[c*x]/((-I)*Sqrt[
d]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*
(Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x
))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt
[d])/(d*e) - (-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[1/(c*x
)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-
(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d] - I*S
qrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d])/(16*d*e) - ((I/16)*((I*c*Sqrt[e]
*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))
 - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c*x)]/(d
*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sqrt[e] +
 c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*d + e)*
((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/16
)*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqrt[d
] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin
[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d + e]
*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))
/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/(Sqrt[d
]*e) - (Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 - ...
```

3.116.3 Rubi [A] (verified)

Time = 3.47 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5764, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+b \csc^{-1}(cx))}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{ 5764}$$

$$-\int \frac{a+b \arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3 x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{ 5232}$$

$$-\int \left(\frac{a+b \arcsin\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b \arcsin\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^3}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\,(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{1-\frac{1}{c^2x^2}}c}{16\sqrt{-d}\sqrt{e}\,(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}}+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(dc^2+e\right)^{3/2}}-\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}e\sqrt{dc^2+e}}+\frac{b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(dc^2+e\right)^{3/2}}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{3/2}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^3,x]
```

3.116. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

output
```
-1/16*(b*c*Sqrt[1 - 1/(c^2*x^2)])/(Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*
Sqrt[e] - d/x)) - (b*c*Sqrt[1 - 1/(c^2*x^2)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d
+ e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcSin[1/(c*x)])/(16*Sqrt[-d]*Sqrt
[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcSin[1/(c*x)])/(16*d*e*(Sqrt[-d
]*Sqrt[e] - d/x)) - (a + b*ArcSin[1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]
*Sqrt[e] + d/x)^2) - (a + b*ArcSin[1/(c*x)])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d
/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e
]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d + e)^(3/2)) - (b*ArcTanh[(c^
2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2
)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[
e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*(c^
2*d + e)^(3/2)) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqr
t[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(3/2)*e*Sqrt[c^2*d + e]) + ((a
 + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e
] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcSin[1/(c*x)])*
Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])
/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*
E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2
)) - ((a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))
/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((I/16)*b*Poly...
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5764
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(
m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
&& IntegerQ[m] && IntegerQ[p]
```

3.116. $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx$

3.116.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 64.23 (sec) , antiderivative size = 1278, normalized size of antiderivative = 1.12

method	result	size
parts	Expression too large to display	1278
derivativedivides	Expression too large to display	1301
default	Expression too large to display	1301

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*((1/8/d*x^3-1/8/e*x)/(e*x^2+d)^2+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1
/2)))+b/c^3*(1/8*x*c^5*(c^4*d*e*arccsc(c*x)*x^2-d^2*c^4*arccsc(c*x)+((c^2*
x^2-1)/c^2/x^2)^(1/2)*e^2*c^3*x^3+((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x+e^
2*arccsc(c*x)*c^2*x^2-c^2*d*e*arccsc(c*x))/d/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)
^2-1/16/d/(c^2*d+e)*c^4*sum(1/_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*l
n((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2
))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/16/d/(c^2*d+e
)*c^4*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^
2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(
c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-1/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2)*(c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*c*arctan(c*d*(I/c/x+(1-1/c
^2/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)/e/d
^3+1/8*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c
^2*d+2*c^2*d*e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*c*arctan(c*d*(I/c/x+(1-1/c^2
/x^2)^(1/2))/((-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/(c^2*d+e)^2/e/d
^3-1/8*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^
(1/2)+2*e)*c*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e
))^(1/2)+2*e)*d)^(1/2))/(c^2*d+e)/e/d^3+1/8*((c^2*d+2*(e*(c^2*d+e))^(1/2)+
2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-2*(e*(c^2*d+e))^(1/2)*
e+2*e^2)*c*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+...
```

3.116.5 Fricas [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsc(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.116.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.116.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^3, x)
```

3.117 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx$

3.117.1 Optimal result

Integrand size = 18, antiderivative size = 1134

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx = -\frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d+e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}(a+b\csc^{-1}(cx))}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5(a+b\csc^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{be\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(c^2d+e)^{3/2}} + \frac{5b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d+e}} - \frac{be\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(c^2d+e)^{3/2}} + \frac{5b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d+e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3(a+b\csc^{-1}(cx))\log\left(1-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3(a+b\csc^{-1}(cx))\log\left(1+\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\csc^{-1}(cx)}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

output
```
-1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(
1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d+e)^(3/2)-1/16*b*e*arctanh((c^2*d+(-d)^(
1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/2)/(c^
2*d+e)^(3/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccsc(c
*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1
/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccsc(c*x))*ln(1-I*c*(I/c/x+(1-1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(
a+b*arccsc(c*x))*ln(1+I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*I*b*polylog(2,I*c*(I/c/x+(1-1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/1
6*I*b*polylog(2,I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d
+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*I*b
*polylog(2,-I*c*(I/c/x+(1-1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(c^2*d+e)^
(1/2)))/(-d)^(5/2)/e^(1/2)+1/16*(a+b*arccsc(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x
+(-d)^(1/2)*e^(1/2))^2-5/16*(a+b*arccsc(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2)
)-1/16*(a+b*arccsc(c*x))*e^(1/2)/(-d)^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/1
6*(a+b*arccsc(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))+5/16*b*arctanh((c^2*d-(-d
)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d+e)^(1/2)/(1-1/c^2/x^2)^(1/2))/d^(5/...
```

3.117.2 Mathematica [A] (warning: unable to verify)

Time = 6.05 (sec) , antiderivative size = 2060, normalized size of antiderivative = 1.82

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^3,x]
```

output
```
(a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((-3*(-(ArcCsc[c*x]/((-I)*Sqrt[d
]*Sqrt[e] + e*x)) + (I*(ArcSin[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
Sqrt[e] + c*((-I)*c*Sqrt[d] - Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x)
)/(Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[
d]))/(16*d^2) - (3*(-(ArcCsc[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSin[
1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(-Sqrt[e] + c*((-I)*c*Sqrt[d] +
Sqrt[-(c^2*d) - e]*Sqrt[1 - 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) - e]*(Sqrt[d]
 - I*Sqrt[e]*x))]/Sqrt[-(c^2*d) - e]))/Sqrt[d]))/(16*d^2) + ((I/16)*((I*c*
Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*((-I)*Sqrt[d] + Sqrt
[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSin[1/(c
*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(4*d*Sqrt[e]*Sqrt[c^2*d + e]*(I*Sq
rt[e] + c*(c*Sqrt[d] - Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*x))/((2*c^2*
d + e)*((-I)*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3/2) - ((I
/16)*(((-I)*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d + e)*(I*Sqr
t[d] + Sqrt[e]*x)) - ArcCsc[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - Arc
Sin[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c^2*d +
 e]*((-I)*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])*
x))/((2*c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d*(c^2*d + e)^(3/2))))/d^(3
/2) - (3*(Pi^2 - 4*Pi*ArcCsc[c*x] + 8*ArcCsc[c*x]^2 - 32*ArcSin[Sqrt[1 ...
```

3.117.3 Rubi [A] (verified)

Time = 4.25 (sec) , antiderivative size = 1198, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5754, 5232, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5754}$$

$$-\int \frac{a+b\arcsin\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{5232}$$

$$-\int \left(\frac{\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^3}-\frac{2\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)^2}+\frac{a+b\arcsin\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{e}\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+\frac{5b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}}-\frac{be\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(dc^2+e)^{3/2}}+\frac{5b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{dc^2+e}}-\frac{be\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{dc^2+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(dc^2+e)^{3/2}}-\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3\left(a+b\arcsin\left(\frac{1}{cx}\right)\right)\log\left(\frac{i\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{dc^2+e}}+1\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3ib\,\text{PolyLog}\left(2,-\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3ib\,\text{PolyLog}\left(2,\frac{ic\sqrt{-d}e^{i\arcsin\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{dc^2+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^3,x]
```

3.117. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx$

output
```
-1/16*(b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/((-d)^(3/2)*(c^2*d + e)*(Sqrt[-d
]*Sqrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 - 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^
2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcSin[1/(c*x)]))/(16
*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcSin[1/(c*x)]))/(16*
d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcSin[1/(c*x)]))/(16*(-d)
^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcSin[1/(c*x)]))/(16*d^2*(
Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*
Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d + e)^(
3/2)) + (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d
+ e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d + e]) - (b*e*ArcTanh[
(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*
x^2)])])/(16*d^(5/2)*(c^2*d + e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d + e]*Sqrt[1 - 1/(c^2*x^2)])])/(16*d^(5/2
)*Sqrt[c^2*d + e]) - (3*(a + b*ArcSin[1/(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I
*ArcSin[1/(c*x)]))/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) +
 (3*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(
Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSin[1/
(c*x)])*Log[1 - (I*c*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d
 + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcSin[1/(c*x)])*Log[1 + (I*c
*Sqrt[-d]*E^(I*ArcSin[1/(c*x)]))/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)...
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5232
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSin[c*x])^n, (
f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d +
 e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 5754
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSin[x/c])^n/x^(2*(p + 1)))
, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

3.117. $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^3}\,dx$

3.117.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 106.52 (sec) , antiderivative size = 1798, normalized size of antiderivative = 1.59

method	result	size
parts	Expression too large to display	1798
derivativedivides	Expression too large to display	1823
default	Expression too large to display	1823

input
```
int((a+b*arccsc(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x/d/(e*x^2+d)^2+3/8*a/d^2*x/(e*x^2+d)+3/8*a/d^2/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))+b/c*(1/8*x*c^3*(5*d^2*c^4*arccsc(c*x)+3*c^4*d*e*arccsc(c*x
)*x^2-((c^2*x^2-1)/c^2/x^2)^(1/2)*c^3*d*e*x-((c^2*x^2-1)/c^2/x^2)^(1/2)*e^
2*c^3*x^3+5*c^2*d*e*arccsc(c*x)+3*e^2*arccsc(c*x)*c^2*x^2)/d^2/(c^2*d+e)/(
c^2*e*x^2+c^2*d)^2+1/2*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d
+2*(e*(c^2*d+e))^(1/2)+2*e)*e*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((-c^
2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/c^3/d^5/(c^2*d+e)+1/2*((c^2*d+2*(
e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*e*arcta
nh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(
1/2))/c^3/d^5/(c^2*d+e)-3/16/d/(c^2*d+e)*c^4*sum(1/_R1/(_R1^2*c^2*d-c^2*d-
2*e)*(I*arccsc(c*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c
/x-(1-1/c^2/x^2)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^
2*d))-3/16/d^2/(c^2*d+e)*c^2*e*sum(_R1/(_R1^2*c^2*d-c^2*d-2*e)*(I*arccsc(c
*x)*ln((_R1-I/c/x-(1-1/c^2/x^2)^(1/2))/_R1)+dilog((_R1-I/c/x-(1-1/c^2/x^2)
^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(-2*c^2*d-4*e)*_Z^2+c^2*d))-5/8*(-(c^2
*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*((e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*
e+2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*arctan(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2))/((
-c^2*d+2*(e*(c^2*d+e))^(1/2)-2*e)*d)^(1/2))/d^4/(c^2*d+e)^2/c-1/2*((c^2*d+
2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*(-(e*(c^2*d+e))^(1/2)*c^2*d+2*c^2*d*e-
2*(e*(c^2*d+e))^(1/2)*e+2*e^2)*e*arctanh(c*d*(I/c/x+(1-1/c^2/x^2)^(1/2)...
```

3.117.5 Fricas [F]

$$\int \frac{a + b \csc^{-1}(cx)}{(d + ex^2)^3} \, dx = \int \frac{b \operatorname{arccsc}(cx) + a}{(ex^2 + d)^3} \, dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \csc^{-1}(cx)}{(d + ex^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \csc^{-1}(cx)}{(d + ex^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.117.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^3} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{(d + ex^2)^3} dx = \int \frac{a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^3} dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^3, x)
```

3.118 $\int x^5\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

3.118.1 Optimal result

Integrand size = 23, antiderivative size = 403

$$\begin{aligned}\int x^5\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx &= -\frac{b(23c^4d^2+12c^2de-75e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{c^2x^2}} \\ &\quad -\frac{b(29c^2d-25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{c^2x^2}} \\ &\quad +\frac{d^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}-\frac{2d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3} \\ &\quad +\frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^3}-\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{105e^3\sqrt{c^2x^2}} \\ &\quad +\frac{b(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3)\,x\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

output
```
1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*a
rccsc(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arccsc(c*x))/e^3-8/105*b*c*d^(7/2
)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)+
1/1680*b*(105*c^6*d^3-35*c^4*d^2*e+63*c^2*d*e^2+75*e^3)*x*arctanh(e^(1/2)*
(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(c^2*x^2)^(1/2)-1/840*b*(
29*c^2*d-25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)
+1/42*b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)-1/1680*b
*(23*c^4*d^2+12*c^2*d*e-75*e^2)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^5/e^
2/(c^2*x^2)^(1/2)
```

3.118.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.56 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.81

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$=\frac{32a(d+ex^2)^2\left(8d^2-12dex^2+15e^2x^4\right)+\frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)\left(75e^2+2c^2e(19d+25ex^2)+c^4(-41d^2+22dex^2+40e^2x^4)\right)}{c^5}+\cdots}{}$$

input
```
Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
(32*a*(d + e*x^2)^2*(8*d^2 - 12*d*e*x^2 + 15*e^2*x^4) + (2*b*e*Sqrt[1 - 1/
(c^2*x^2)]*x*(d + e*x^2)*(75*e^2 + 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-41*d^
2 + 22*d*e*x^2 + 40*e^2*x^4)))/c^5 + (b*(-128*c^4*d^4*Sqrt[1 + d/(e*x^2)]*
AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] - (e*(105*c^6*d^3 - 35
*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*
x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]
))/(c^5*x) + 32*b*(d + e*x^2)^2*(8*d^2 - 12*d*e*x^2 + 15*e^2*x^4)*ArcCsc[c
*x])/(3360*e^3*Sqrt[d + e*x^2])
```

3.118.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {5762, 27, 7282, 2118, 27, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int\frac{(ex^2+d)^{3/2}(15e^2x^4-12dex^2+8d^2)}{105e^3x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^2(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{(d+ex^2)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{x\sqrt{c^2x^2-1}}dx}{105e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 7282$$

$$\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}\left(15e^2x^4-12dex^2+8d^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 2118$$

$$\frac{bcx\left(\frac{\int \frac{3e\left(ex^2+d\right)^{3/2}\left(16c^2d^2-\left(29c^2d-25e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{3c^2e}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{bcx\left(\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(16c^2d^2-\left(29c^2d-25e\right)ex^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 171$$

$$\frac{bcx\left(\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4+12dec^2-75e^2\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

3.118. $\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\Big\downarrow \; 27$$

$$\frac{bcx\left(\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4+12dec^2-75e^2\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}+\frac{5e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 171$$

$$\frac{bcx\left(\frac{\frac{\frac{\int \frac{128d^4c^6+e\left(105d^3c^6-35d^2ec^4+63de^2c^2+75e^3\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}+5e\sqrt{\;}\cdots}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{bcx\left(\frac{\frac{\frac{\int \frac{128d^4c^6+e\left(105d^3c^6-35d^2ec^4+63de^2c^2+75e^3\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(29c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}+5e\sqrt{\;}\cdots}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 175$$

3.118. $\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{bcx\left(\frac{\frac{\frac{128c^6d^4\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{\cdots}}{\cdots}}{2c^2}\cdots\right.}{210e^3\sqrt{c^2x^2}}\cdots$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow\ 66$$

$$\frac{bcx\left(\frac{\frac{\frac{128c^6d^4\int\frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c^2\cdots}}{\cdots}}{2c^2}\cdots\right.}{210e^3\sqrt{c^2x^2}}\cdots$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow\ 104$$

$$\frac{bcx\left(\frac{\frac{\frac{256c^6d^4\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c^2x^2-\cdots}}{\cdots}}{2c^2}\cdots\right.}{210e^3\sqrt{c^2x^2}}\cdots$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow\ 217$$

3.118. $\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{bcx\left(\frac{\frac{\frac{2e\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\int\frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-256c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{c^2x \cdots}}{\cdots}}{2c^2} \cdots\right)}{210e^3\sqrt{c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 221$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}+$$

$$\frac{bcx\left(\frac{\frac{\frac{\frac{2\sqrt{e}\left(105c^6d^3-35c^4d^2e+63c^2de^2+75e^3\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-256c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(23c^4d^2+12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2} \cdots}{2c^2} \cdots\right)}{210e^3\sqrt{c^2x^2}}$$

input
```
Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
(d^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/
2)*(a + b*ArcCsc[c*x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcCsc[c*x]))/
(7*e^3) + (b*c*x*((5*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(5/2))/c^2 + (-1/2*(
(29*c^2*d - 25*e)*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/c^2 + (-((e*(23*
c^4*d^2 + 12*c^2*d*e - 75*e^2)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) +
(-256*c^6*d^(7/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (
2*Sqrt[e]*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*ArcTanh[(Sq
rt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2))/(2*c^
2)))/(210*e^3*Sqrt[c^2*x^2])
```

3.118. $\int x^5\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.118.4 Maple [F]

$$\int x^5(a+b\,\operatorname{arccsc}(cx))\sqrt{e\,x^2+d}dx$$

input
```
int(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.118.5 Fricas [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 1699, normalized size of antiderivative = 4.22

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/6720*(128*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(-d) + 8*d^2)/x^4) + (105*b*c^6*d^3 - 35*b*c^4*d^2*e + 63*b*c^2
*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^
4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqr
t(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 -
64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*
x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x^4 - 4
1*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(11*b*c^5*d*e^2 + 25*b*c^3
*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^3), -1/6720*(256*b*c
^7*d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (105*b*c^6*d^3
 - 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 + 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 +
 c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d
- c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(240*a*c^7*e^3
*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + 16*(15*b*
c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*arccsc(
c*x) + (40*b*c^5*e^3*x^4 - 41*b*c^5*d^2*e + 38*b*c^3*d*e^2 + 75*b*c*e^3 +
2*(11*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))
/(c^7*e^3), 1/3360*(64*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e...
```

3.118.6 Sympy [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int x^5(a+b\operatorname{acsc}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**5*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.118.7 Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.118.8 Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^5, x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.119 $\int x^3\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

3.119.1 Optimal result

Integrand size = 23, antiderivative size = 294

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{b(c^2d+9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{c^2x^2}} + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2} + \frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} + \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^2\sqrt{c^2x^2}} - \frac{b(15c^4d^2-10c^2de-9e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{c^2x^2}}$$

output
```
-1/3*d*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arcc
sc(c*x))/e^2+2/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)
^(1/2))/e^2/(c^2*x^2)^(1/2)-1/120*b*(15*c^4*d^2-10*c^2*d*e-9*e^2)*x*arctan
h(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(c^2*x^2)^(1/2)
+1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e/(c^2*x^2)^(1/2)+1/120*b*(c
^2*d+9*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(c^2*x^2)^(1/2)
```

3.119.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.44 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$= \frac{16a(d+ex^2)^2(-2d+3ex^2) + \frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(9e+c^2(7d+6ex^2))}{c^3} + \frac{b\left(16c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+\right.}{}}{240e^2\sqrt{d+e}}$$

input
```
Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
(16*a*(d + e*x^2)^2*(-2*d + 3*e*x^2) + (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x^2)*(9*e + c^2*(7*d + 6*e*x^2)))/c^3 + (b*(16*c^2*d^3*Sqrt[1 + d/(e*x^
2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (e*(15*c^4*d^2 -
 10*c^2*d*e - 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF
1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) + 16
*b*(d + e*x^2)^2*(-2*d + 3*e*x^2)*ArcCsc[c*x])/(240*e^2*Sqrt[d + e*x^2])
```

3.119.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5762, 27, 435, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int -\frac{(2d-3ex^2)(ex^2+d)^{3/2}}{15e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} - \frac{d(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x\sqrt{c^2x^2-1}}dx}{15e^2\sqrt{c^2x^2}} + \frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2} - \frac{d(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{435}$$

$$-\frac{bcx\int \frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{171}$$

$$-\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(8c^2d^2-e\left(dc^2+9e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{3e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(8c^2d^2-e\left(dc^2+9e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{3e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{171}$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{16d^3c^4+e\left(15d^2c^4-10dec^2-9e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{16d^3c^4+e\left(15d^2c^4-10dec^2-9e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{175}$$

3.119. $\int x^3\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\ dx$

$$-\frac{bcx\left(\frac{\frac{16c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 66$$

$$-\frac{bcx\left(\frac{\frac{16c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 104$$

$$-\frac{bcx\left(\frac{\frac{32c^4d^3\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}+$$
$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 217$$

$$-\frac{bcx\left(\frac{\frac{2e\left(15c^4d^2-10c^2de-9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-32c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}$$

$$\downarrow 221$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-$$
$$\frac{bcx\left(\frac{\frac{\frac{2\sqrt{e}\left(15c^4d^2-10c^2de-9e^2\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-32c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^2\sqrt{c^2x^2}}$$

3.119. $\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$

input
```
Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
-1/3*(d*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/e^2 + ((d + e*x^2)^(5/2)*(a
 + b*ArcCsc[c*x]))/(5*e^2) - (b*c*x*((-3*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^
(3/2))/(2*c^2) + (-((e*(c^2*d + 9*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c
^2) + (-32*c^4*d^(5/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])
] + (2*Sqrt[e]*(15*c^4*d^2 - 10*c^2*d*e - 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2)))/(30*e^2*Sqrt[c^2*x
^2])
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.119.4 Maple [F]

$$\int x^{3}(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^{2}+d}dx$$

input
```
int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 1379, normalized size of antiderivative = 4.69

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(16*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt
(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 +
4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) +
e^2) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e
^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b
*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/480
*(32*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*s
qrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (15*b*
c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*
c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c
^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5
*d*e*x^2 - 16*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2
)*arccsc(c*x) + (6*b*c^3*e^2*x^2 + 7*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 -
 1))*sqrt(e*x^2 + d))/(c^5*e^2), 1/240*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2
 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^
2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2
 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*s
qrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d...
```

3.119.6 Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^3(a+b\operatorname{acsc}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.119.7 Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.119.8 Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arccsc}\,(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^3, x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx = \int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\text{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.120 $\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$

3.120.1 Optimal result

Integrand size = 21, antiderivative size = 195

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}-\frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e\sqrt{c^2x^2}}+\frac{b(3c^2d+e)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e-1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)
^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/6*b*(3*c^2*d+e)*x*ar
ctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(c^2*x^2)^(
1/2)+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(c^2*x^2)^(1/2)
```

3.120.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.57 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$=\frac{-\frac{2bd^2\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cex}+\frac{-\frac{b\left(3c^2d+e\right)\sqrt{1-\frac{1}{c^2x^2}}\,x^3\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right)}{\sqrt{1-c^2x^2}}+\frac{2\left(d+ex^2\right)\left(be\sqrt{1-\frac{1}{c^2x^2}}\,x+2\cdots\right.}{\cdots}}{c}}{12\sqrt{d+ex^2}}$$

input
```
Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
((-2*b*d^2*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(
e*x^2))])/(c*e*x) + (-((b*(3*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 +
 (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*
x^2]) + (2*(d + e*x^2)*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x + 2*a*c*(d + e*x^2) +
2*b*c*(d + e*x^2)*ArcCsc[c*x]))/e)/c)/(12*Sqrt[d + e*x^2])
```

3.120.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5760, 354, 113, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5760}$$

$$\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}}{x\sqrt{c^2x^2-1}}dx}{3e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{6e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{113}$$

$$\frac{bcx\left(\frac{\int \frac{2c^2d^2+e\left(3dc^2+e\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{2c^2d^2+e\left(3dc^2+e\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{175}$$

$$\frac{bcx\left(\frac{2c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e(3c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 66$$

$$\frac{bcx\left(\frac{2c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e(3c^2d+e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 104$$

$$\frac{bcx\left(\frac{4c^2d^2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e(3c^2d+e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 217$$

$$\frac{bcx\left(\frac{2e(3c^2d+e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-4c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e}+\frac{bcx\left(\frac{\frac{2\sqrt{e}(3c^2d+e)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-4c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{c^2x^2}}$$

input
```
Int[x*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

3.120. $\int x\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

output
```
((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e) + (b*c*x*((e*Sqrt[-1 + c^2*x
^2]*Sqrt[d + e*x^2])/c^2 + (-4*c^2*d^(3/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*(3*c^2*d + e)*ArcTanh[(Sqrt[e]*Sqrt[-1
+ c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2)))/(6*e*Sqrt[c^2*x^2])
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.120.4 Maple [F]

$$\int x(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^{2}+d}dx$$

input
```
int(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 1098, normalized size of antiderivative = 5.63

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 +
2*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*x
))*sqrt(e*x^2 + d))/(c^3*e), -1/24*(4*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^2*x
^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^
2*d^2 - d*e)*x^2 - d^2)) - (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c
^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d -
c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(2*a*c^3*e*x^2 +
 2*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arccsc(c*
x))*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2
*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)
*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d + b*e)*sqr
t(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 +
d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2*a*c^3*e*
x^2 + 2*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 + b*c^3*d)*arcc
sc(c*x))*sqrt(e*x^2 + d))/(c^3*e), -1/12*(2*b*c^3*d^(3/2)*arctan(-1/2*sqrt
(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*...
```

3.120.6 Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x(a+b\operatorname{acsc}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.120.7 Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arccsc}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(e*x^2 + d)^(3/2)*a/e + 1/3*(3*e*integrate(1/3*(c^2*e*x^3 + c^2*d*x)*e
^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (
c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x) + (e*x^2*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)) + d*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
)*sqrt(e*x^2 + d))*b/e
```

3.120.8 Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arccsc}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.121 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx$

3.121.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

3.121.2 Mathematica [N/A]

Not integrable

Time = 6.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x, x]
```

3.121.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
$Aborted
```

3.121.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.121.4 Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2+d}}{x}dx$$

input
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

output
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x)
```

3.121.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x, x)
```

3.121.6 Sympy [N/A]

Not integrable

Time = 10.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\sqrt{d+ex^2}}{x}\,dx$$

input
```
integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x,x)
```

output
```
Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x, x)
```

3.121.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.121.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x, x)
```

3.121.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{asin}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x, x)
```

3.122 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx$

3.122.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

3.122.2 Mathematica [N/A]

Not integrable

Time = 10.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx=\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3, x]
```

3.122.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{ 5772}$$

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.122.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.122.4 Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\,\operatorname{arccsc}\left(cx\right)\right)\sqrt{e\,x^{2}+d}}{x^{3}}dx$$

input
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

output
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

3.122.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

3.122.6 Sympy [N/A]

Not integrable

Time = 14.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

input
```
integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**3,x)
```

output
```
Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**3, x)
```

3.122.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.122.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\operatorname{arccsc}(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^3, x)
```

3.122.9 Mupad [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^3, x)
```

3.123 $\int x^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right),x\right)$$

output
```
Unintegrable(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.123.2 Mathematica [N/A]

Not integrable

Time = 10.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

input
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

3.123.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5772}$$

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)dx$$

input
```
Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
$Aborted
```

3.123.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.123.4 Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+b\,\operatorname{arccsc}(cx))\sqrt{e\,x^2+d}dx$$

input
```
int(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.123.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

3.123.6 Sympy [N/A]

Not integrable

Time = 118.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2(a+b\,\mathrm{acsc}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.123.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.123.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*x^2, x)
```

3.123.9 Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.124 $\int \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

3.124.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\csc^{-1}(cx)),x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.124.2 Mathematica [N/A]

Not integrable

Time = 17.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx = \int \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

3.124.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$$
$$\Big\downarrow \; 5772$$
$$\int \sqrt{d+ex^2}(a+b\csc^{-1}(cx))dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
$Aborted
```

3.124.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.124.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a+b\,\operatorname{arccsc}(cx))\sqrt{e\,x^2+d}dx$$

input
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.124.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a), x)
```

3.124.6 Sympy [N/A]

Not integrable

Time = 47.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int (a+b\,\mathrm{acsc}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.124.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\operatorname{arccsc}(cx)+a)\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a), x)
```

3.124.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.125 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2} dx$

3.125.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2} dx = \text{Int}\left(\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}, x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

3.125.2 Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2} dx = \int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2} dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2, x]
```

3.125.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.125.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.125.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\sqrt{e\,x^2+d}}{x^2}dx$$

input
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

output
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

3.125.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^2, x)
```

3.125.6 Sympy [N/A]

Not integrable

Time = 7.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}}{x^2}\,dx$$

input
```
integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

output
```
Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**2, x)
```

3.125.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.125.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^2, x)
```

3.125.9 Mupad [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^2, x)
```

3.126 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx$

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 328

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx &= -\frac{2bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{c^2x^2}} \\ &\quad -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{c^2x^2}}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3} \\ &\quad +\frac{2bc^2(c^2d+2e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\ &\quad -\frac{b(c^2d+e)(2c^2d+3e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

output
```
-1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/d/x^3-2/9*b*c*(c^2*d+2*e)*(c^2*x^2-
1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^
2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)+2/9*b*c^2*(c^2*d+2*e)*x*EllipticE(c*x,(-e/c
^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2
-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/9*b*(c^2*d+e)*(2*c^2*d+3*e)*x*EllipticF(c*x,
(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(
c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.126.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$= -\frac{\sqrt{d+ex^2}\left(3a(d+ex^2)+bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2+4ex^2)+3b(d+ex^2)\csc^{-1}(cx)\right)}{9dx^3}$$
$$+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d+2e)\,E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)\middle|-\frac{e}{c^2d}\right)-(2c^4d^2+5c^2de+3e^2)\operatorname{EllipticF}\left(\right.\right.}{9\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

output
```
-1/9*(Sqrt[d + e*x^2]*(3*a*(d + e*x^2) + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
2*c^2*d*x^2 + 4*e*x^2) + 3*b*(d + e*x^2)*ArcCsc[c*x]))/(d*x^3) + ((I/9)*b*
c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d + 2*e)*Ellip
ticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (2*c^4*d^2 + 5*c^2*d*e + 3*e
^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d*Sqrt[
1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.126.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5762, 27, 376, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{5762}$$
$$\frac{bcx\int -\frac{(ex^2+d)^{3/2}}{3dx^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3dx^3}$$
$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}}{x^4\sqrt{c^2x^2-1}}dx}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 376

$$-\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{e\left(dc^2+3e\right)x^2+2d\left(dc^2+2e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 25

$$-\frac{bcx\left(\frac{1}{3}\int \frac{e\left(dc^2+3e\right)x^2+2d\left(dc^2+2e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 445

$$-\frac{bcx\left(\frac{1}{3}\left(\frac{\int \frac{de\left(-2\left(dc^2+2e\right)x^2c^2+dc^2+3e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 27

$$-\frac{bcx\left(\frac{1}{3}\left(e\int \frac{-2\left(dc^2+2e\right)x^2c^2+dc^2+3e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 399

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(2c^2d+3e\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{2c^2\left(c^2d+2e\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{\cdots}}{3x^3}\cdots}{3d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 323

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\left(c^2d+e\right)\left(2c^2d+3e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{2c^2\left(c^2d+2e\right)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{x}\right)+d\cdots}{3d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

3.126. $\displaystyle\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^4}\,dx$

$$\Big\downarrow \; 323$$

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\,(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\,(c^2d+2e)\sqrt{d+ex}}{x}\right.\right.}{3d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \; 321$$

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\,(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{2\sqrt{c^2x^2-1}\,(c^2d+2e)\sqrt{c}}{x}\right.\right.}{3d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \; 331$$

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\sqrt{1-c^2x^2}\,(c^2d+2e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{2\sqrt{c^2x^2-1}\,(c}{}\right.\right.}{3d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \; 330$$

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c^2\sqrt{1-c^2x^2}\,(c^2d+2e)\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{2\sqrt{c}}{}\right.\right.}{3d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \; 327$$

$$-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3dx^3}-$$

$$\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}\,(c^2d+e)\,(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{2c\sqrt{1-c^2x^2}\,(c^2d+2e)\sqrt{d+ex^2}\,E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+2\right.\right.}{3d\sqrt{c^2x^2}}$$

3.126. $\displaystyle\int\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^4}\,dx$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^4,x]
```

output
```
-1/3*((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(d*x^3) - (b*c*x*((d*Sqrt[-1
+ c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + ((2*(c^2*d + 2*e)*Sqrt[-1 + c^2*x^2]
*Sqrt[d + e*x^2])/x + e*((-2*c*(c^2*d + 2*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 +
(e*x^2)/d]) + ((c^2*d + e)*(2*c^2*d + 3*e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x
^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[
d + e*x^2])))/3))/(3*d*Sqrt[c^2*x^2])
```

3.126.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.126. $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx$

3.126.4 Maple [F]

$$\int \frac{(a + b \operatorname{arccsc}(cx)) \sqrt{e x^2 + d}}{x^4} dx$$

input
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

output
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{x^4} dx = -\frac{\left(3\, acdex^2 + 3\, acd^2 + 3\left(bcdex^2 + bcd^2\right) \operatorname{arccsc}(cx) + \left(bcd^2 + 2\left(bc^3 d^2 + 2\, bcde\right)x^2\right)\sqrt{c^2x^2 - 1}\right)\sqrt{ex^2 + d}}{}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/9*((3*a*c*d*e*x^2 + 3*a*c*d^2 + 3*(b*c*d*e*x^2 + b*c*d^2)*arccsc(c*x) +
 (b*c*d^2 + 2*(b*c^3*d^2 + 2*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 +
 d) + (2*(b*c^6*d^2 + 2*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d))
 - (2*b*c^6*d^2 + (4*b*c^4 + b*c^2)*d*e + 3*b*e^2)*x^3*elliptic_f(arcsin(c
*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^3)
```

3.126.6 Sympy [F]

$$\int \frac{\sqrt{d + ex^2}(a + b \csc^{-1}(cx))}{x^4} dx = \int \frac{(a + b \operatorname{acsc}(cx)) \sqrt{d + ex^2}}{x^4} dx$$

input
```
integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

output
```
Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**4, x)
```

3.126.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.126.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^4, x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^4, x)
```

3.127 $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

3.127.1 Optimal result

Integrand size = 23, antiderivative size = 453

$$\begin{aligned}
&\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx\\
&= -\frac{bc(24c^4d^2+19c^2de-31e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{c^2x^2}}\\
&\quad-\frac{bc(12c^2d-e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{c^2x^2}}\\
&\quad-\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{5dx^5}+\frac{2e(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{15d^2x^3}\\
&\quad+\frac{bc^2(24c^4d^2+19c^2de-31e^2)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\\
&\quad-\frac{b(c^2d+e)\,(24c^4d^2+7c^2de-30e^2)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{225d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

output
```
-1/5*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*a
rccsc(c*x))/d^2/x^3+2/15*b*c*e^2*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^
2*x^2)^(1/2)-1/45*b*c*e*(2*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/
(c^2*x^2)^(1/2)-1/75*b*c*(8*c^4*d^2+3*c^2*d*e-2*e^2)*(c^2*x^2-1)^(1/2)*(e*
x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)-1/25*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2
)/x^4/(c^2*x^2)^(1/2)-1/45*b*c*e*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(
c^2*x^2)^(1/2)-1/75*b*c*(4*c^2*d+e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^
2/(c^2*x^2)^(1/2)-2/15*b*c^2*e^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x
^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2
/d)^(1/2)+1/45*b*c^2*e*(2*c^2*d+e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2
*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x
^2/d)^(1/2)+1/75*b*c^2*(8*c^4*d^2+3*c^2*d*e-2*e^2)*x*EllipticE(c*x,(-e/c^2
/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2
-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/75*b*c^2*(8*c^2*d-e)*(c^2*d+e)*x*EllipticF(c
*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2
)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)-2/45*b*c^2*e*(c^2*d+e)*x*EllipticF(c*x
,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/
(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)+2/15*b*e^2*(c^2*d+e)*x*EllipticF(c*x,(-e
/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c
^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.127.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.69 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx =$$

$$-\frac{\sqrt{d+ex^2}\Big(15a(3d^2+dex^2-2e^2x^4)+bc\sqrt{1-\frac{1}{c^2x^2}}x(-31e^2x^4+dex^2(8+19c^2x^2)+3d^2(3+4c^2x^2+8}{225d^2x^5}$$

$$+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2+19c^2de-31e^2)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\middle|-\frac{e}{c^2d}\right)+(-24c^6d^3-31c^4d^2}{225\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6,x]
```

output
```
-1/225*(Sqrt[d + e*x^2]*(15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 -
 1/(c^2*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4*c^2
*x^2 + 8*c^4*x^4)) + 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcCsc[c*x]))/(d^2
*x^5) + ((I/225)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2
4*c^4*d^2 + 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c
^2*d))] + (-24*c^6*d^3 - 31*c^4*d^2*e + 23*c^2*d*e^2 + 30*e^3)*EllipticF[I
*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*
Sqrt[d + e*x^2])
```

3.127.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5762, 27, 442, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{x^6}\,dx$$

$$\Big\downarrow \; 5762$$

$$\frac{bcx\int -\frac{(3d-2ex^2)(ex^2+d)^{3/2}}{15d^2x^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\int \frac{(3d-2ex^2)(ex^2+d)^{3/2}}{x^6\sqrt{c^2x^2-1}}dx}{15d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 442$$

$$-\frac{bcx\left(\frac{3d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}-\frac{1}{5}\int -\frac{\sqrt{ex^2+d}\left((3c^2d-10e)ex^2+d(12c^2d-e)\right)}{x^4\sqrt{c^2x^2-1}}dx\right)}{15d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{bcx\left(\frac{1}{5}\int \frac{\sqrt{ex^2+d}\left((3c^2d-10e)ex^2+d(12c^2d-e)\right)}{x^4\sqrt{c^2x^2-1}}dx+\frac{3d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 442

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{d\sqrt{c^2x^2-1}(12c^2d-e)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{2e(6d^2c^4+4dec^2-15e^2)x^2+d(24d^2c^4+19dec^2-31e^2)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\frac{3d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 25

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{2e(6d^2c^4+4dec^2-15e^2)x^2+d(24d^2c^4+19dec^2-31e^2)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}(12c^2d-e)\sqrt{d+ex^2}}{3x^3}\right)+\frac{3d\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{15d^2\sqrt{c^2x^2}}+\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 445

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(2\left(6d^2c^4+4dec^2-15e^2\right)-c^2\left(24d^2c^4+19dec^2-31e^2\right)x^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}(12c}{3x}\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 27

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int \frac{2(6d^2c^4+4dec^2-15e^2)-c^2(24d^2c^4+19dec^2-31e^2)x^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}(12c}{3x}\right.\right.}{15d^2\sqrt{c^2x^2}}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 399

3.127. $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}dx$

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\sqrt{c^2x^2-1}(24c^4d^2+\cdots\right.\right.\right.}{15d^2\sqrt{c^2x^2}\cdots}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\downarrow$ 323

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\sqrt{c^2x^2-\cdots}\right.\right.\right.}{15d^2\sqrt{c^2x^2}\cdots}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\downarrow$ 323

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\cdots\right.\right.}{15d^2\sqrt{c^2x^2}\cdots}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\downarrow$ 321

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(24c^4d^2+19c^2de-31e^2)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\cdots\right.\right.\right.\right.}{15d^2\sqrt{c^2x^2}\cdots}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\downarrow$ 331

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\int \cdots}{e\sqrt{c^2x^2-1}\cdots}\right.\right.\right.\right.}{15d^2\sqrt{c^2x^2}\cdots}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\downarrow$ 330

3.127. $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\sqrt{d-}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{15d^2\sqrt{c}}$$

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\downarrow$ 327

$$\frac{2e(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{5dx^5}-$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\sqrt{d+ex^2}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{15d^2\sqrt{c}}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(d*x^5) + (2*e*(d + e*x^2)^(3
/2)*(a + b*ArcCsc[c*x]))/(15*d^2*x^3) - (b*c*x*((3*d*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(3/2))/(5*x^5) + ((d*(12*c^2*d - e)*Sqrt[-1 + c^2*x^2]*Sqrt[d +
e*x^2])/(3*x^3) + (((24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[-1 + c^2*x^2]*
Sqrt[d + e*x^2])/x + e*(-((c*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[1 - c
^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 +
 c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + e)*(24*c^4*d^2 + 7*c^2*d*e - 3
0*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c
^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3)/5))/(15*d^2*Sqrt[c^
2*x^2])
```

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] && !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&& !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.127. $\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.127.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arccsc}\,(cx))\,\sqrt{e\,x^2+d}}{x^6}dx$$

input
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

output
```
int((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \frac{(30\,acde^2x^4 - 15\,acd^2ex^2 - 45\,acd^3 + 15\,(2\,bcde^2x^4 - bcd^2ex^2 - 3\,bcd^3)\,\mathrm{arccsc}\,(cx) - (9\,bcd^3 + (24\,bc^5d^3 -}{}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/225*((30*a*c*d*e^2*x^4 - 15*a*c*d^2*e*x^2 - 45*a*c*d^3 + 15*(2*b*c*d*e^2
*x^4 - b*c*d^2*e*x^2 - 3*b*c*d^3)*arccsc(c*x) - (9*b*c*d^3 + (24*b*c^5*d^3
 + 19*b*c^3*d^2*e - 31*b*c*d*e^2)*x^4 + 4*(3*b*c^3*d^3 + 2*b*c*d^2*e)*x^2)
*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((24*b*c^8*d^3 + 19*b*c^6*d^2*e - 31
*b*c^4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (24*b*c^8*d^3 + (1
9*b*c^6 + 12*b*c^4)*d^2*e - (31*b*c^4 - 8*b*c^2)*d*e^2 - 30*b*e^3)*x^5*ell
iptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^5)
```

3.127.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{(a+b\operatorname{acsc}(cx))\sqrt{d+ex^2}}{x^6}\,dx$$

input
```
integrate((a+b*acsc(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

output
```
Integral((a + b*acsc(c*x))*sqrt(d + e*x**2)/x**6, x)
```

3.127.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.127.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/x^6, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))))/x^6, x)
```

3.128 $\int x^3 (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\,dx$

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 374

$$\begin{aligned}\int x^3 (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\,dx = & -\frac{b(3c^4d^2-38c^2de-25e^2)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{c^2x^2}} \\ & + \frac{b(13c^2d+25e)\,x\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{c^2x^2}} \\ & + \frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{c^2x^2}} - \frac{d(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^2} \\ & + \frac{(d+ex^2)^{7/2}\,(a+b\csc^{-1}(cx))}{7e^2} + \frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{35e^2\sqrt{c^2x^2}} \\ & - \frac{b(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{c^2x^2}}\end{aligned}$$

output
```
-1/5*d*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arcc
sc(c*x))/e^2+2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)
^(1/2))/e^2/(c^2*x^2)^(1/2)-1/560*b*(35*c^6*d^3-35*c^4*d^2*e-63*c^2*d*e^2-
25*e^3)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)
/(c^2*x^2)^(1/2)+1/840*b*(13*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/
2)/c^3/e/(c^2*x^2)^(1/2)+1/42*b*x*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/c/e/(c
^2*x^2)^(1/2)-1/560*b*(3*c^4*d^2-38*c^2*d*e-25*e^2)*x*(c^2*x^2-1)^(1/2)*(e
*x^2+d)^(1/2)/c^5/e/(c^2*x^2)^(1/2)
```

3.128.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.59 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.81

$$\int x^3 (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\ dx = \frac{96a(d+ex^2)^3(-2d+5ex^2)+\frac{2be\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(75e^2+2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2+40e^2x^4}{c^5}}{}$$

input
```
Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(96*a*(d + e*x^2)^3*(-2*d + 5*e*x^2) + (2*b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d +
 e*x^2)*(75*e^2 + 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 +
40*e^2*x^4)))/c^5 + (3*b*(32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2,
1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))] + (e*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^
2*d*e^2 - 25*e^3)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1
, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^5*x) + 96*b*
(d + e*x^2)^3*(-2*d + 5*e*x^2)*ArcCsc[c*x])/(3360*e^2*Sqrt[d + e*x^2])
```

3.128.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {5762, 27, 435, 171, 27, 171, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\ dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int -\frac{(2d-5ex^2)(ex^2+d)^{5/2}}{35e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{(2d-5ex^2)(ex^2+d)^{5/2}}{x\sqrt{c^2x^2-1}}dx}{35e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{435}$$

$$-\frac{bcx\int \frac{(2d-5ex^2)(ex^2+d)^{5/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{171}$$

$$-\frac{bcx\left(\frac{\int \frac{(ex^2+d)^{3/2}\left(12c^2d^2-e\left(13dc^2+25e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{3c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\left(\frac{\int \frac{(ex^2+d)^{3/2}\left(12c^2d^2-e\left(13dc^2+25e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{6c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{171}$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4-38dec^2-25e^2\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)(d+ex^2)^{3/2}}{2c^2}}{6c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4-38dec^2-25e^2\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)(d+ex^2)^{3/2}}{2c^2}}{6c^2}-\frac{5e\sqrt{c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\csc^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^2}$$

3.128. $\int x^3(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\Big\downarrow \; 171$$

$$-\frac{bcx\left(\frac{\frac{3\left(\frac{\int \frac{32d^4c^6+e\left(35d^3c^6-35d^2ec^4-63de^2c^2-25e^3\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}-\cdots\right.}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\left(\frac{\frac{3\left(\frac{\int \frac{32d^4c^6+e\left(35d^3c^6-35d^2ec^4-63de^2c^2-25e^3\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2x^2-1}\left(13c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}-\cdots\right.}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \; 175$$

$$-\frac{bcx\left(\frac{\frac{3\left(\frac{32c^6d^4\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\cdots}{6c^2}\right.}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \; 66$$

3.128. $\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$-\frac{bcx\left(\frac{\frac{3\left(\frac{32c^6d^4\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\ldots}{6c^2}\ldots\right)}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \; 104$$

$$-\frac{bcx\left(\frac{\frac{3\left(\frac{64c^6d^4\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{c^2\ldots}}{\ldots}}{6c^2}\ldots\right)}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \; 217$$

$$-\frac{bcx\left(\frac{\frac{3\left(\frac{2e\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-64c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}-\frac{e\sqrt{\ldots}}{\ldots}}{6c^2}\ldots\right)}{70e^2\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \; 221$$

3.128. $\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\csc^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^2}-$$

$$\frac{bcx\left(\frac{\frac{3\left(\frac{\frac{2\sqrt{e}\left(35c^6d^3-35c^4d^2e-63c^2de^2-25e^3\right)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-64c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(3c^4d^2-38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}}{6c^2}\right)}{70e^2\sqrt{c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
-1/5*(d*(d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/e^2 + ((d + e*x^2)^(7/2)*(a
 + b*ArcCsc[c*x]))/(7*e^2) - (b*c*x*((-5*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^
(5/2))/(3*c^2) + (-1/2*(e*(13*c^2*d + 25*e)*Sqrt[-1 + c^2*x^2]*(d + e*x^2)
^(3/2))/c^2 + (3*((e*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*Sqrt[-1 + c^2*x^2]*
Sqrt[d + e*x^2])/c^2 + (-64*c^6*d^(7/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sq
rt[-1 + c^2*x^2])] + (2*Sqrt[e]*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2
- 25*e^3)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2
*c^2)))/(4*c^2))/(6*c^2)))/(70*e^2*Sqrt[c^2*x^2])
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.128. $\int x^3(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

3.128.4 Maple [F]

$$\int x^3 \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b \,\mathrm{arccsc}\left(c x\right)\right)dx$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

3.128.5 Fricas [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 1697, normalized size of antiderivative = 4.54

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
[1/6720*(96*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c
^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
 + d)*sqrt(-d) + 8*d^2)/x^4) - 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2
*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^
4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqr
t(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 +
 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x
^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*x) + (40*b*c^5*e^3*x^4 + 57*b
*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(53*b*c^5*d*e^2 + 25*b*c^3*e
^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^7*e^2), 1/6720*(192*b*c^7*
d^(7/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
 d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 3*(35*b*c^6*d^3 -
 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c
^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d -
c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(240*a*c^7*e^3*x
^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + 48*(5*b*c^7
*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arccsc(c*x)
+ (40*b*c^5*e^3*x^4 + 57*b*c^5*d^2*e + 164*b*c^3*d*e^2 + 75*b*c*e^3 + 2*(5
3*b*c^5*d*e^2 + 25*b*c^3*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^
7*e^2), 1/3360*(48*b*c^7*sqrt(-d)*d^3*log(((c^4*d^2 - 6*c^2*d*e + e^2)*...
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

output
```
Timed out
```

3.128.7 Maxima [F(-2)]

Exception generated.

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.128.8 Giac [F]

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^3, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int x^3 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.129 $\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

3.129.1 Optimal result

Integrand size = 21, antiderivative size = 262

$$\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\frac{b(7c^2d+3e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{c^2x^2}}+\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}-\frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{5e\sqrt{c^2x^2}}+\frac{b(15c^4d^2+10c^2de+3e^2)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{c^2x^2}}$$

output
```
1/5*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/e-1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)
^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/(c^2*x^2)^(1/2)+1/40*b*(15*c^4*d^2+10*
c^2*d*e+3*e^2)*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/
e^(1/2)/(c^2*x^2)^(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/(c^2*
x^2)^(1/2)+1/40*b*(7*c^2*d+3*e)*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/(c
^2*x^2)^(1/2)
```

3.129.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.32 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.95

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx = \frac{\frac{16a(d+ex^2)^3}{e}+\frac{2b\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)(3e+c^2(9d+2ex^2))}{c^3}+\frac{b\left(-\frac{8c^2d^3\sqrt{1+\frac{d}{ex^2}}\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{e}-\frac{(15c\cdots}{\cdots}\right)}{\cdots}}{80\sqrt{d+ex^2}}$$

input
```
Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
((16*a*(d + e*x^2)^3)/e + (2*b*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)*(3*e +
c^2*(9*d + 2*e*x^2)))/c^3 + (b*((-8*c^2*d^3*Sqrt[1 + d/(e*x^2)]*AppellF1[1
, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/e - ((15*c^4*d^2 + 10*c^2*d*e +
 3*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2
, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2]))/(c^3*x) + (16*b*(d + e*x^
2)^3*ArcCsc[c*x])/e)/(80*Sqrt[d + e*x^2])
```

3.129.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5760, 354, 113, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5760}$$

$$\frac{bcx\int \frac{(ex^2+d)^{5/2}}{x\sqrt{c^2x^2-1}}dx}{5e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{(ex^2+d)^{5/2}}{x^2\sqrt{c^2x^2-1}}dx^2}{10e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{113}$$

$$\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(4c^2d^2+e\left(7dc^2+3e\right)x^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(4c^2d^2+e\left(7dc^2+3e\right)x^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{171}$$

$$\frac{bcx\left(\frac{\frac{\int \frac{8d^3c^4+e\left(15d^2c^4+10dec^2+3e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\frac{\int \frac{8d^3c^4+e\left(15d^2c^4+10dec^2+3e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{175}$$

$$\frac{bcx\left(\frac{\frac{8c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2+10c^2de+3e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \text{66}$$

3.129. $\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{bcx\left(\frac{\frac{8c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(15c^4d^2+10c^2de+3e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 104$$

$$\frac{bcx\left(\frac{\frac{16c^4d^3\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(15c^4d^2+10c^2de+3e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 217$$

$$\frac{bcx\left(\frac{\frac{2e\left(15c^4d^2+10c^2de+3e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-16c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e}+\frac{bcx\left(\frac{\frac{\frac{2\sqrt{e}\left(15c^4d^2+10c^2de+3e^2\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-16c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{c^2x^2-1}\left(7c^2d+3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{10e\sqrt{c^2x^2}}$$

input
```
Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

3.129. $\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

output
```
((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e) + (b*c*x*((e*Sqrt[-1 + c^2*x
^2]*(d + e*x^2)^(3/2))/(2*c^2) + ((e*(7*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2]*Sq
rt[d + e*x^2])/c^2 + (-16*c^4*d^(5/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt
[-1 + c^2*x^2])] + (2*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*ArcTanh[(S
qrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c^2))/(4*c^2)))/(10
*e*Sqrt[c^2*x^2])
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.129. $\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx$

3.129.4 Maple [F]

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccsc}\left(cx\right)\right)dx$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 1375, normalized size of antiderivative = 5.25

$$\int x\left(d+ex^{2}\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
[1/160*(8*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2
*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 +
 d)*sqrt(-d) + 8*d^2)/x^4) + (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(
e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4
*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e
^2) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x
^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3
*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e), -1/160*(16*
b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e
*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (15*b*c^4*d
^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d
*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^
2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*
x^2 + 8*a*c^5*d^2 + 8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arccsc
(c*x) + (2*b*c^3*e^2*x^2 + 9*b*c^3*d*e + 3*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqr
t(e*x^2 + d))/(c^5*e), 1/80*(4*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*
e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^
2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2
*d*e + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2
 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)...
```

3.129.6 Sympy [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x(a+b\,\mathrm{acsc}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

output
```
Integral(x*(a + b*acsc(c*x))*(d + e*x**2)**(3/2), x)
```

3.129.7 Maxima [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*(5*e*integrate(1/5*(c^2*e^2*x^5 + 2*c^2*d*
e*x^3 + c^2*d^2*x)*e^(1/2*log(e*x^2 + d) + 1/2*log(c*x + 1) + 1/2*log(c*x
- 1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(c*x - 1)) - e), x
) + (e^2*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 2*d*e*x^2*arctan2(1
, sqrt(c*x + 1)*sqrt(c*x - 1)) + d^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)))*sqrt(e*x^2 + d))*b/e
```

3.129.8 Giac [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int x\,\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.130 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x} dx$

3.130.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x} dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x}, x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

3.130.2 Mathematica [N/A]

Not integrable

Time = 7.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x} dx = \int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x} dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x, x]
```

3.130.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}\,dx$$
$$\Big\downarrow \text{5772}$$
$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x,x]
```

output
```
$Aborted
```

3.130.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.130.4 Maple [N/A] (verified)

Not integrable

Time = 2.92 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsc}\,(cx))}{x}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x)
```

3.130.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x,
x)
```

3.130.6 Sympy [N/A]

Not integrable

Time = 72.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x,x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x, x)
```

3.130.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.130.8 Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x, x)
```

3.130.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x, x)
```

3.131 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx$

3.131.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}, x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

3.131.2 Mathematica [N/A]

Not integrable

Time = 11.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3, x]
```

3.131.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2} (a+b\csc^{-1}(cx))}{x^3} \, dx$$

$$\Big\downarrow \text{ 5772}$$

$$\int \frac{(d+ex^2)^{3/2} (a+b\csc^{-1}(cx))}{x^3} dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.131.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.131.4 Maple [N/A] (verified)

Not integrable

Time = 2.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}} (a+b\,\mathrm{arccsc}\,(cx))}{x^3} dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x)
```

3.131.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

3.131.6 Sympy [N/A]

Not integrable

Time = 63.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**3,x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

3.131.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.131.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^3, x)
```

3.131.9 Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^3, x)
```

3.132 $\int x^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\,dx$

3.132.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\,dx=\operatorname{Int}\left(x^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx)),x\right)$$

output
```
Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

3.132.2 Mathematica [N/A]

Not integrable

Time = 11.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\,dx=\int x^2(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

3.132.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+e x^2\right)^{3/2} \left(a+b \csc^{-1}(c x)\right) \, dx$$

$$\Big\downarrow \ 5772$$

$$\int x^2 \left(d+e x^2\right)^{3/2} \left(a+b \csc^{-1}(c x)\right)dx$$

input
```
Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
$Aborted
```

3.132.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.132.4 Maple [N/A] (verified)

Not integrable

Time = 1.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^{2} \left(e \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\operatorname{arccsc}\left(c x \right)\right)dx$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

3.132.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2 (d + ex^2)^{3/2} \left(a + b \csc^{-1}(cx)\right) \, dx = \int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arccsc}(cx) + a) x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arccsc(c*x))*sqrt(e*x^2
+ d), x)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (d + ex^2)^{3/2} \left(a + b \csc^{-1}(cx)\right) \, dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

output
```
Timed out
```

3.132.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + ex^2)^{3/2} \left(a + b \csc^{-1}(cx)\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.132.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d+ex^2)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int (ex^2+d)^{\frac{3}{2}} (b\,\mathrm{arccsc}\,(cx)+a)x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*x^2, x)
```

3.132.9 Mupad [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2 (d+ex^2)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int x^2 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.133 $\int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx$

3.133.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx = \mathrm{Int}\Big((d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\,, x\Big)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

3.133.2 Mathematica [N/A]

Not integrable

Time = 18.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx = \int (d+ex^2)^{3/2} (a+b\csc^{-1}(cx))\, dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

3.133.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)dx$$

input
```
Int[(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
$Aborted
```

3.133.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.133.4 Maple [N/A] (verified)

Not integrable

Time = 1.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsc}\left(cx\right)\right)dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

3.133.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arccsc}\,(cx)+a\right)dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d), x)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

output
```
Timed out
```

3.133.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.133.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\operatorname{arccsc}(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a), x)
```

3.133.9 Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.134 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

3.134.2 Mathematica [N/A]

Not integrable

Time = 36.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2, x]
```

3.134.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.134.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.134.4 Maple [N/A] (verified)

Not integrable

Time = 2.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}(a+b\,\operatorname{arccsc}(cx))}{x^2}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x)
```

3.134.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^2
, x)
```

3.134.6 Sympy [N/A]

Not integrable

Time = 98.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**2,x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

3.134.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.134.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arccsc}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^2, x)
```

3.134.9 Mupad [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\text{asin}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^2, x)
```

3.135 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx$

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

3.135.2 Mathematica [N/A]

Not integrable

Time = 5.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4, x]
```

3.135.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}\,dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^4}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^4,x]
```

output
```
$Aborted
```

3.135.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.135.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsc}\,(cx))}{x^4}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x)
```

3.135.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

3.135.6 Sympy [N/A]

Not integrable

Time = 65.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acsc}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**4,x)
```

output
```
Integral((a + b*acsc(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

3.135.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.135.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^4, x)
```

3.135.9 Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^4}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^4, x)
```

3.136 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

3.136.1 Optimal result

Integrand size = 23, antiderivative size = 416

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx = -\frac{bc(8c^4d^2+23c^2de+23e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{c^2x^2}} - \frac{4bc(c^2d+2e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{c^2x^2}} - \frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{c^2x^2}} - \frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5} + \frac{bc^2(8c^4d^2+23c^2de+23e^2)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)(8c^4d^2+19c^2de+15e^2)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{75d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
-1/5*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/d/x^5-1/25*b*c*(e*x^2+d)^(3/2)*(c^2
*x^2-1)^(1/2)/x^4/(c^2*x^2)^(1/2)-1/75*b*c*(8*c^4*d^2+23*c^2*d*e+23*e^2)*(
c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)-4/75*b*c*(c^2*d+2*e)*(c
^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(c^2*x^2)^(1/2)+1/75*b*c^2*(8*c^4*d^2+
23*c^2*d*e+23*e^2)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e
*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-1/75*b
*(c^2*d+e)*(8*c^4*d^2+19*c^2*d*e+15*e^2)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))
*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/
(e*x^2+d)^(1/2)
```

3.136.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.20 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx =$$

$$-\frac{\sqrt{d+ex^2}\left(15a(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(23e^2x^4+dex^2(11+23c^2x^2)+d^2(3+4c^2x^2+8c^4x^4))+15b(d\right.}{75dx^5}$$

$$+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2+23c^2de+23e^2)E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-(8c^6d^3+27c^4d^2e+34\right.}{75\sqrt{-c^2}d\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

output
```
-1/75*(Sqrt[d + e*x^2]*(15*a*(d + e*x^2)^2 + b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(
23*e^2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4))
+ 15*b*(d + e*x^2)^2*ArcCsc[c*x]))/(d*x^5) + ((I/75)*b*c*Sqrt[1 - 1/(c^2*x
^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Ellipt
icE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] - (8*c^6*d^3 + 27*c^4*d^2*e + 3
4*c^2*d*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(
Sqrt[-c^2]*d*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.136.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.652, Rules used = {5762, 27, 376, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx$$

$$\Big\downarrow \ 5762$$

$$\frac{bcx\int -\frac{(ex^2+d)^{5/2}}{5dx^6\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\int \frac{\left(ex^2+d\right)^{5/2}}{x^6\sqrt{c^2x^2-1}}dx}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 376$$

$$-\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}-\frac{1}{5}\int -\frac{\sqrt{ex^2+d}\left(e\left(dc^2+5e\right)x^2+4d\left(dc^2+2e\right)\right)}{x^4\sqrt{c^2x^2-1}}dx\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{bcx\left(\frac{1}{5}\int \frac{\sqrt{ex^2+d}\left(e\left(dc^2+5e\right)x^2+4d\left(dc^2+2e\right)\right)}{x^4\sqrt{c^2x^2-1}}dx+\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 442$$

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{e\left(4d^2c^4+11dec^2+15e^2\right)x^2+d\left(8d^2c^4+23dec^2+23e^2\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 25$$

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{e\left(4d^2c^4+11dec^2+15e^2\right)x^2+d\left(8d^2c^4+23dec^2+23e^2\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2e\right)\sqrt{d+ex^2}}{3x^3}\right)+\frac{d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}\right)}{5d\sqrt{c^2x^2}}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 445$$

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(4d^2c^4-\left(8d^2c^4+23dec^2+23e^2\right)x^2c^2+11dec^2+15e^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}\left(8c^4d^2+23c^2de+23e^2\right)\sqrt{d+ex^2}}{x}\right)+\frac{4d\sqrt{c^2x^2-1}\left(c^2d+2\right.}{3x^3}\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \; 27$$

3.136. $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^6}\,dx$

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int \frac{4d^2c^4-(8d^2c^4+23dec^2+23e^2)x^2c^2+11dec^2+15e^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{x}\right)+\frac{4d\sqrt{c^2x^2-1}(c^2d+2e}{3x^3}\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 399

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(8c^4d^2+19c^2de+15e^2)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}(8c^4d^2+23\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 323

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}}{}\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 323

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 321

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right.\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\big\downarrow$ 331

3.136. $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\int\frac{\sqrt{e}}{\sqrt{1-}}}{e\sqrt{c^2x^2-1}}\right.\right.\right.\right.}{5d\sqrt{c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\downarrow$ 330

$$-\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+}}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{5d\sqrt{c^2x}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}$$

$\downarrow$ 327

$$-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5dx^5}-$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}E}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right.\right.\right.\right.}{5d\sqrt{c^2x}}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(d*x^5) - (b*c*x*((d*Sqrt[-1
+ c^2*x^2]*(d + e*x^2)^(3/2))/(5*x^5) + ((4*d*(c^2*d + 2*e)*Sqrt[-1 + c^2*
x^2]*Sqrt[d + e*x^2])/(3*x^3) + (((8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[-
1 + c^2*x^2]*Sqrt[d + e*x^2])/x + e*(-((c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2
)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/
(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + e)*(8*c^4*d^2 + 19
*c^2*d*e + 15*e^2)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[
c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3)/5))/(5*
d*Sqrt[c^2*x^2])
```

3.136. $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.136. $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.136.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arccsc}\,(cx))}{x^6}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{(15\,acde^2x^4+30\,acd^2ex^2+15\,acd^3+15\,(bcde^2x^4+2\,bcd^2ex^2+bcd^3)\,\text{arccsc}\,(cx)+(3\,bcd^3+(8\,bc^5d^3+23$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*((15*a*c*d*e^2*x^4 + 30*a*c*d^2*e*x^2 + 15*a*c*d^3 + 15*(b*c*d*e^2*x
^4 + 2*b*c*d^2*e*x^2 + b*c*d^3)*arccsc(c*x) + (3*b*c*d^3 + (8*b*c^5*d^3 +
23*b*c^3*d^2*e + 23*b*c*d*e^2)*x^4 + (4*b*c^3*d^3 + 11*b*c*d^2*e)*x^2)*sqr
t(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((8*b*c^8*d^3 + 23*b*c^6*d^2*e + 23*b*c^
4*d*e^2)*x^5*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (8*b*c^8*d^3 + (23*b*c^
6 + 4*b*c^4)*d^2*e + (23*b*c^4 + 11*b*c^2)*d*e^2 + 15*b*e^3)*x^5*elliptic_
f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x^5)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**6,x)
```

output
```
Timed out
```

3.136.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.136.8 Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^6, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^6}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^6, x)
```

3.137 $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx$

3.137.1 Optimal result

Integrand size = 23, antiderivative size = 554

$$\begin{aligned}
&\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx = \\
&-\frac{bc(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{c^2x^2}} \\
&-\frac{bc(120c^4d^2+159c^2de-37e^2)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{c^2x^2}} \\
&-\frac{bc(30c^2d+11e)\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{c^2x^2}} \\
&-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7}+\frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5} \\
&+\frac{bc^2(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} \\
&-\frac{2b(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3675d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

output
```
-1/7*(e*x^2+d)^(5/2)*(a+b*arccsc(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*a
rccsc(c*x))/d^2/x^5-1/1225*b*c*(30*c^2*d+11*e)*(e*x^2+d)^(3/2)*(c^2*x^2-1)
^(1/2)/d/x^4/(c^2*x^2)^(1/2)-1/49*b*c*(e*x^2+d)^(5/2)*(c^2*x^2-1)^(1/2)/d/
x^6/(c^2*x^2)^(1/2)-1/3675*b*c*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-24
7*e^3)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)-1/3675*b*c*(1
20*c^4*d^2+159*c^2*d*e-37*e^2)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^
2*x^2)^(1/2)+1/3675*b*c^2*(240*c^6*d^3+528*c^4*d^2*e+193*c^2*d*e^2-247*e^3
)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2
/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-2/3675*b*(c^2*d+e)*(1
20*c^6*d^3+204*c^4*d^2*e+17*c^2*d*e^2-105*e^3)*x*EllipticF(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1
)^(1/2)/(e*x^2+d)^(1/2)
```

3.137.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.82 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.69

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^8}\,dx =$$

$$-\frac{\sqrt{d+ex^2}\Big(105a(5d-2ex^2)\,(d+ex^2)^2+bc\sqrt{1-\frac{1}{c^2x^2}}x(-247e^3x^6+de^2x^4(71+193c^2x^2)+3d^2ex^2(61+8}{3675d^2x^7}$$

$$+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3)\,E\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)-2(120}{3675\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d}}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

output
```
-1/3675*(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1
 - 1/(c^2*x^2)]*x*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*e*x
^2*(61 + 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 1
6*c^6*x^6)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsc[c*x]))/(d^2*x^7)
+ ((I/3675)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^
6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[
-c^2]*x], -(e/(c^2*d))] - 2*(120*c^8*d^4 + 324*c^6*d^3*e + 221*c^4*d^2*e^2
 - 88*c^2*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))
]))/(Sqrt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.137.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.91, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.739$, Rules used = {5762, 27, 442, 25, 442, 25, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right)}{x^8} \, dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int -\frac{\left(5d-2ex^2\right)\left(ex^2+d\right)^{5/2}}{35d^2x^8\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{\left(5d-2ex^2\right)\left(ex^2+d\right)^{5/2}}{x^8\sqrt{c^2x^2-1}}dx}{35d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{442}$$

$$-\frac{bcx\left(\frac{5d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{7x^7}-\frac{1}{7}\int -\frac{\left(ex^2+d\right)^{3/2}\left(\left(5c^2d-14e\right)ex^2+d\left(30dc^2+11e\right)\right)}{x^6\sqrt{c^2x^2-1}}dx\right)}{35d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{25}$$

$$-\frac{bcx\left(\frac{1}{7}\int \frac{\left(ex^2+d\right)^{3/2}\left(\left(5c^2d-14e\right)ex^2+d\left(30dc^2+11e\right)\right)}{x^6\sqrt{c^2x^2-1}}dx+\frac{5d\sqrt{c^2x^2-1}\left(d+ex^2\right)^{5/2}}{7x^7}\right)}{35d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{442}$$

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{d\sqrt{c^2x^2-1}\left(30c^2d+11e\right)\left(d+ex^2\right)^{3/2}}{5x^5}-\frac{1}{5}\int -\frac{\sqrt{ex^2+d}\left(2e\left(15d^2c^4+18dec^2-35e^2\right)x^2+d\left(120d^2c^4+159dec^2-37e^2\right)\right)}{x^4\sqrt{c^2x^2-1}}dx\right)+\frac{5d\sqrt{c^2}\ldots}{}\right)}{35d^2\sqrt{c^2x^2}}+\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$$\Big\downarrow \text{25}$$

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\int\frac{\sqrt{ex^2+d}\left(2e\left(15d^2c^4+18dec^2-35e^2\right)x^2+d\left(120d^2c^4+159dec^2-37e^2\right)\right)}{x^4\sqrt{c^2x^2-1}}dx+\frac{d\sqrt{c^2x^2-1}\left(30c^2d+11e\right)\left(d+ex^2\right)^{3/2}}{5x^5}\right)+\frac{5d\sqrt{c^2x^2}}{}\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\big\downarrow$ 442

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{d\sqrt{c^2x^2-1}\left(120c^4d^2+159c^2de-37e^2\right)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int-\frac{e\left(120d^3c^6+249d^2ec^4+71de^2c^2-210e^3\right)x^2+d\left(240d^3c^6+528d^2ec^4+193d\right.}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}\right.\right.\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\big\downarrow$ 25

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{e\left(120d^3c^6+249d^2ec^4+71de^2c^2-210e^3\right)x^2+d\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\left(120c^4d^2+159c^2de-\right.}{3x^3}\right.\right.\right.}{35d^2\sqrt{c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\big\downarrow$ 445

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int\frac{de\left(120d^3c^6+249d^2ec^4+71de^2c^2-\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)x^2c^2-210e^3\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}\left(240c^6d^3+528c^4d^2e+19\right.}{x}\right.\right.\right.\right.}{35}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\big\downarrow$ 27

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{120d^3c^6+249d^2ec^4+71de^2c^2-\left(240d^3c^6+528d^2ec^4+193de^2c^2-247e^3\right)x^2c^2-210e^3}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}\left(240c^6d^3+528c^4d^2e-\right.}{x}\right.\right.\right.\right.}{3}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\big\downarrow$ 399

3.137. $\displaystyle\int\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^8}\,dx$

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\left(240c^6d^3+528c^4d^2e+193c^2de^2-247e^3\right)\int \cdots}{e}\cdots\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 323

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\left(240c^6d^3+528c^4d^2e+193c^2de^2-\cdots\right.}{e}\cdots\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 323

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\left(240c^6d^3+528c^4d^2e+\cdots\right.}{\cdots}\cdots\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 321

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\left(240c^6d^3+528c^4d\cdots\right.}{\cdots}\cdots\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 331

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)\left(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\left(240c^6d\cdots\right.}{\cdots}\cdots\right.\right.\right.\right.\right.}{\cdots}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 330

3.137. $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^8}\,dx$

$$-\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(240c^6}{}\right.\right.\right.\right.\right.}{}$$

$$\frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7}$$

$\downarrow$ 327

$$\frac{2e(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{7dx^7}-$$
$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d+e)(120c^6d^3+204c^4d^2e+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}(240c^6d^3+5}{}\right.\right.\right.\right.\right.}{}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/x^8,x]
```

output
```
-1/7*((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(d*x^7) + (2*e*(d + e*x^2)^(5
/2)*(a + b*ArcCsc[c*x]))/(35*d^2*x^5) - (b*c*x*((5*d*Sqrt[-1 + c^2*x^2]*(d
 + e*x^2)^(5/2))/(7*x^7) + ((d*(30*c^2*d + 11*e)*Sqrt[-1 + c^2*x^2]*(d + e
*x^2)^(3/2))/(5*x^5) + ((d*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[-1 +
c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (((240*c^6*d^3 + 528*c^4*d^2*e + 193*c
^2*d*e^2 - 247*e^3)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x + e*(-((c*(240*c
^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[1 - c^2*x^2]*Sqrt[d
 + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt
[1 + (e*x^2)/d])) + (2*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d
*e^2 - 105*e^3)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x
], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3)/5)/7))/(35
*d^2*Sqrt[c^2*x^2])
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.137. $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx$

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.137. $\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.137.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arccsc}\,(cx))}{x^8}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.72

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{x^8}\,dx = \frac{(210\,acde^3x^6 - 105\,acd^2e^2x^4 - 840\,acd^3ex^2 - 525\,acd^4 + 105\,(2\,bcde^3$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="fricas")
```

output
```
1/3675*((210*a*c*d*e^3*x^6 - 105*a*c*d^2*e^2*x^4 - 840*a*c*d^3*e*x^2 - 525
*a*c*d^4 + 105*(2*b*c*d*e^3*x^6 - b*c*d^2*e^2*x^4 - 8*b*c*d^3*e*x^2 - 5*b*
c*d^4)*arccsc(c*x) - ((240*b*c^7*d^4 + 528*b*c^5*d^3*e + 193*b*c^3*d^2*e^2
 - 247*b*c*d*e^3)*x^6 + 75*b*c*d^4 + (120*b*c^5*d^4 + 249*b*c^3*d^3*e + 71
*b*c*d^2*e^2)*x^4 + 3*(30*b*c^3*d^4 + 61*b*c*d^3*e)*x^2)*sqrt(c^2*x^2 - 1)
)*sqrt(e*x^2 + d) - ((240*b*c^10*d^4 + 528*b*c^8*d^3*e + 193*b*c^6*d^2*e^2
 - 247*b*c^4*d*e^3)*x^7*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (240*b*c^10*
d^4 + 24*(22*b*c^8 + 5*b*c^6)*d^3*e + (193*b*c^6 + 249*b*c^4)*d^2*e^2 - (2
47*b*c^4 - 71*b*c^2)*d*e^3 - 210*b*e^4)*x^7*elliptic_f(arcsin(c*x), -e/(c^
2*d)))*sqrt(-d))/(c*d^3*x^7)
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsc(c*x))/x**8,x)
```

output
```
Timed out
```

3.137.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{x^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.137.8 Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\mathrm{arccsc}\,(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)/x^8, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{x^8}\,dx = \int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))))/x^8, x)
```

3.138 $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 321

$$\begin{aligned}\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx &= -\frac{b(19c^2d-9e)\,x\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{c^2x^2}} \\ &\quad+\frac{bx\sqrt{-1+c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} \\ &\quad-\frac{2d(d+ex^2)^{3/2}\,(a+b\csc^{-1}(cx))}{3e^3} \\ &\quad+\frac{(d+ex^2)^{5/2}\,(a+b\csc^{-1}(cx))}{5e^3}-\frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{15e^3\sqrt{c^2x^2}} \\ &\quad+\frac{b(45c^4d^2-10c^2de+9e^2)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

output
```
-2/3*d*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arcc
sc(c*x))/e^3-8/15*b*c*d^(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)
^(1/2))/e^3/(c^2*x^2)^(1/2)+1/120*b*(45*c^4*d^2-10*c^2*d*e+9*e^2)*x*arctan
h(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(c^2*x^2)^(1/2)
+1/20*b*x*(e*x^2+d)^(3/2)*(c^2*x^2-1)^(1/2)/c/e^2/(c^2*x^2)^(1/2)+d^2*(a+b
*arccsc(c*x))*(e*x^2+d)^(1/2)/e^3-1/120*b*(19*c^2*d-9*e)*x*(c^2*x^2-1)^(1/
2)*(e*x^2+d)^(1/2)/c^3/e^2/(c^2*x^2)^(1/2)
```

3.138.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.01 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.88

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$= \frac{16a(d+ex^2)(8d^2-4dex^2+3e^2x^4)+\frac{2be\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2)(9ex+c^2(-13dx+6ex^3))}{c^3}+\frac{b\left(-64c^2d^3\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},\right.\right.}{}}{24}$$

input
```
Integrate[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(16*a*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) + (2*b*e*Sqrt[1 - 1/(c^2
*x^2)]*(d + e*x^2)*(9*e*x + c^2*(-13*d*x + 6*e*x^3)))/c^3 + (b*(-64*c^2*d^
3*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))]
+ (e*(-45*c^4*d^2 + 10*c^2*d*e - 9*e^2)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[1 +
 (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*
x^2]))/(c^3*x) + 16*b*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*ArcCsc[c
*x])/(240*e^3*Sqrt[d + e*x^2])
```

3.138.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5762, 27, 7282, 2118, 27, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 5762$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}(3e^2x^4-4dex^2+8d^2)}{15e^3x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{5/2}(a+b\csc^{-1}(cx))}{5e^3}-\frac{2d(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}(3e^2x^4-4dex^2+8d^2)}{x\sqrt{c^2x^2-1}}dx}{15e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{7282}$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}(3e^2x^4-4dex^2+8d^2)}{x^2\sqrt{c^2x^2-1}}dx^2}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{2118}$$

$$\frac{bcx\left(\frac{\int \frac{e\sqrt{ex^2+d}\left(32c^2d^2-\left(19c^2d-9e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}}dx^2}{2c^2e}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(32c^2d^2-\left(19c^2d-9e\right)ex^2\right)}{x^2\sqrt{c^2x^2-1}}dx^2}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{171}$$

$$\frac{bcx\left(\frac{\frac{\int \frac{64d^3c^4+e\left(45d^2c^4-10dec^2+9e^2\right)x^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{27}$$

3.138. $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

$$\frac{bcx\left(\frac{\frac{\int \frac{64d^3c^4+e\left(45d^2c^4-10dec^2+9e^2\right)x^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 175$$

$$\frac{bcx\left(\frac{\frac{64c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 66$$

$$\frac{bcx\left(\frac{\frac{64c^4d^3\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 104$$

$$\frac{bcx\left(\frac{\frac{128c^4d^3\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 217$$

$$\frac{bcx\left(\frac{\frac{2e\left(45c^4d^2-10c^2de+9e^2\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-128c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

3.138. $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

$$\Big\downarrow \; 221$$

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}+\frac{bcx\left(\frac{\frac{\frac{2\sqrt{e}\left(45c^4d^2-10c^2de+9e^2\right)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-128c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\left(19c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{c^2x^2-1}(d+ex^2}{2c^2}\right.}{30e^3\sqrt{c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(d^2*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a
+ b*ArcCsc[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCsc[c*x]))/(5*e^3
) + (b*c*x*((3*e*Sqrt[-1 + c^2*x^2]*(d + e*x^2)^(3/2))/(2*c^2) + (-(((19*c
^2*d - 9*e)*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) + (-128*c^4*d^(5/2)
*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*(45*c^4
*d^2 - 10*c^2*d*e + 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d
+ e*x^2])])/c)/(2*c^2))/(4*c^2)))/(30*e^3*Sqrt[c^2*x^2])
```

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.138. $\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.138.4 Maple [F]

$$\int \frac{x^{5}\left(a+b\,\mathrm{arccsc}\left(c x\right)\right)}{\sqrt{e\,x^{2}+d}}d x$$

input
```
int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 1383, normalized size of antiderivative = 4.31

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\text{Too large to display}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(64*b*c^5*sqrt(-d)*d^2*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^
2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2
+ d)*sqrt(-d) + 8*d^2)/x^4) + (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt
(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 +
4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) +
e^2) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*
e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 -
13*b*c^3*d*e + 9*b*c*e^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), -
1/480*(128*b*c^5*d^(5/2)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 -
2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) -
(45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^
2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(24*a*c^5*e^2*x^4 - 3
2*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + 8*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*
b*c^5*d^2)*arccsc(c*x) + (6*b*c^3*e^2*x^2 - 13*b*c^3*d*e + 9*b*c*e^2)*sqrt
(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*sqrt(-d)*d^2*lo
g(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - (4
5*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c
^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d...
```

3.138.6 Sympy [F]

$$\int \frac{x^5(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{x^5(a + b\operatorname{acsc}(cx))}{\sqrt{d + ex^2}}\, dx$$

input
```
integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.138.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.138.8 Giac [F]

$$\int \frac{x^5(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^5}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^5\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

input
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.139 $\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 225

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{c^2x^2}} - \frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2} + \frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^2\sqrt{c^2x^2}} - \frac{b(3c^2d-e)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{c^2x^2}}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^2+2/3*b*c*d^(3/2)*x*arctan((e*x^2+
d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/(c^2*x^2)^(1/2)-1/6*b*(3*c^2*d-e)*
x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(c^2*x^
2)^(1/2)-d*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e^2+1/6*b*x*(c^2*x^2-1)^(1/2)
*(e*x^2+d)^(1/2)/c/e/(c^2*x^2)^(1/2)
```

3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.42 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{4bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+be(-3c^2d+e)\sqrt{1-\frac{1}{c^2x^2}}x^4\sqrt{1-c^2x^2}\sqrt{1+}}{12c}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(4*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^
2*x^2), -(d/(e*x^2))] + b*e*(-3*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt[
1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^
2)/d)] + 2*x*(-1 + c^2*x^2)*(d + e*x^2)*(-4*a*c*d + b*e*Sqrt[1 - 1/(c^2*x^
2)]*x + 2*a*c*e*x^2 + 2*b*c*(-2*d + e*x^2)*ArcCsc[c*x]))/(12*c*e^2*x*(-1 +
 c^2*x^2)*Sqrt[d + e*x^2])
```

3.139.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5762, 27, 435, 171, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 5762}$$

$$\frac{bcx\int -\frac{(2d-ex^2)\sqrt{ex^2+d}}{3e^2x\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bcx\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x\sqrt{c^2x^2-1}}dx}{3e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}$$

$$\Big\downarrow 435$$

$$-\frac{bcx\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx^2}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}$$

$$\Big\downarrow 171$$

$$-\frac{bcx\left(\frac{\int \frac{4c^2d^2+(3c^2d-e)ex^2}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}$$

$$\Big\downarrow 27$$

$$-\frac{bcx\left(\frac{\int \frac{4c^2d^2+(3c^2d-e)ex^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}$$

$$\Big\downarrow 175$$

$$-\frac{bcx\left(\frac{4c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e(3c^2d-e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}$$

$$\Big\downarrow 66$$

$$-\frac{bcx\left(\frac{4c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e(3c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}$$

$$\Big\downarrow 104$$

$$-\frac{bcx\left(\frac{8c^2d^2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e(3c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}$$

$$\Big\downarrow 217$$

3.139. $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

$$-\frac{bcx\left(\frac{2e(3c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-8c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}-\frac{bcx\left(\frac{\frac{2\sqrt{e}(3c^2d-e)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-8c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
-((d*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^2) + ((d + e*x^2)^(3/2)*(a + b
*ArcCsc[c*x]))/(3*e^2) - (b*c*x*(-((e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/
c^2) + (-8*c^2*d^(3/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])
] + (2*(3*c^2*d - e)*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[
d + e*x^2])])/c)/(2*c^2)))/(6*e^2*Sqrt[c^2*x^2])
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.139. $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

3.139.4 Maple [F]

$$\int \frac{x^3(a+b\operatorname{arccsc}(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 1107, normalized size of antiderivative = 4.92

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*b*c^3*sqrt(-d)*d*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^
2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)
*sqrt(-d) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^
4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c
*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^2 -
4*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arccsc(c
*x))*sqrt(e*x^2 + d))/(c^3*e^2), 1/24*(8*b*c^3*d^(3/2)*arctan(-1/2*sqrt(c^
2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 +
(c^2*d^2 - d*e)*x^2 - d^2)) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4
+ c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d
 - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e*x^
2 - 4*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arcc
sc(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*sqrt(-d)*d*log(((c^4*d^
2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c
^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + (3*b*c^2*d -
 b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt
(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*(2
*a*c^3*e*x^2 - 4*a*c^3*d + sqrt(c^2*x^2 - 1)*b*c*e + 2*(b*c^3*e*x^2 - 2*b*
c^3*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(4*b*c^3*d^(3/2)*arct
an(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(...
```

3.139.6 Sympy [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.139.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.139.8 Giac [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.140 $\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

3.140.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e} - \frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e\sqrt{c^2x^2}} + \frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{c^2x^2}}$$

output
```
-b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e/(c^2*x^
2)^(1/2)+b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/
(c^2*x^2)^(1/2)+(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e
```

3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.81

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e-c^2ex^2}{c^2d+e},1-c^2x^2\right)}{\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}}}+b\csc^{-1}(cx)\right)}{e}$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x*AppellF1[1/2, -1/2, 1,
3/2, (e - c^2*e*x^2)/(c^2*d + e), 1 - c^2*x^2])/Sqrt[(c^2*(d + e*x^2))/(c^
2*d + e)] + b*ArcCsc[c*x]))/e
```

3.140.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5760, 354, 140, 27, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{5760}$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}}{x\sqrt{c^2x^2-1}}dx}{e\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx^2}{2e\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{140}$$

$$\frac{bcx\left(e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\int \frac{d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{66}$$

$$\frac{bcx\left(d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{104}$$

$$\frac{bcx\left(2d\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \; 217$$

$$\frac{bcx\left(2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-2\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e}+\frac{bcx\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-2\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e\sqrt{c^2x^2}}$$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e + (b*c*x*(-2*Sqrt[d]*ArcTan[Sqrt[d
 + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt
[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c))/(2*e*Sqrt[c^2*x^2])
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.140.4 Maple [F]

$$\int \frac{x(a+b\,\mathrm{arccsc}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 870, normalized size of antiderivative = 6.59

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{bc\sqrt{-d}\log\left(\frac{(c^4d^2-6c^2de+e^2)x^4-8(c^2d^2-de)x^2+4\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{-d}+8d^2}{x^4}\right)+b\sqrt{e}\log\left(8c^4e^2x^4+c^4d\right.}{}\right.$$
$$-\frac{2bc\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{d}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)-b\sqrt{e}\log\left(8c^4e^2x^4+c^4d^2-6c^2de+8(c^4de-c^2e^2)x\right.}{4ce}$$
$$-\frac{bc\sqrt{d}\arctan\left(-\frac{\sqrt{c^2x^2-1}((c^2d-e)x^2-2d)\sqrt{ex^2+d}\sqrt{d}}{2(c^2dex^4+(c^2d^2-de)x^2-d^2)}\right)+b\sqrt{-e}\arctan\left(\frac{(2c^2ex^2+c^2d-e)\sqrt{c^2x^2-1}\sqrt{ex^2+d}\sqrt{-e}}{2(c^3e^2x^4-cde+(c^3de-ce^2)x^2)}\right)-2\sqrt{e}}{2ce}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(b*c*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e
)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-
d) + 8*d^2)/x^4) + b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(
c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*s
qrt(e*x^2 + d)*sqrt(e) + e^2) + 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))
/(c*e), -1/4*(2*b*c*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2))
 - b*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^
2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*s
qrt(e) + e^2) - 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e), 1/4*(b*c
*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4
*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^
2)/x^4) - 2*b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 -
 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)
) + 4*sqrt(e*x^2 + d)*(b*c*arccsc(c*x) + a*c))/(c*e), -1/2*(b*c*sqrt(d)*ar
ctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d
)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + b*sqrt(-e)*arctan(1/2*(2*c^
2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x
^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*sqrt(e*x^2 + d)*(b*c*arccsc(c*x)
+ a*c))/(c*e)]
```

3.140.6 Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x(a+b\operatorname{acsc}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.140.7 Maxima [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
(e*integrate((c^2*e*x^3 + c^2*d*x)*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x +
1) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log(
c*x - 1)) - e), x) + sqrt(e*x^2 + d)*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1
)))*b/e + sqrt(e*x^2 + d)*a/e
```

3.140.8 Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.141 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

3.141.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.141.2 Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]), x]
```

3.141.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.141.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.141.4 Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.141.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+d}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^3 + d*x), x)
```

3.141.6 Sympy [N/A]

Not integrable

Time = 7.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x*sqrt(d + e*x**2)), x)
```

3.141.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.141.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+d}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

3.141.9 Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.142 $\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

3.142.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.142.2 Mathematica [N/A]

Not integrable

Time = 5.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

3.142.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5772}$$

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.142.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.142.4 Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{x^{3}\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.142.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+dx^3}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e*x^5 + d*x^3), x)
```

3.142.6 Sympy [N/A]

Not integrable

Time = 27.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

3.142.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.142.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

3.142.9 Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.143 $\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

3.143.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.143.2 Mathematica [N/A]

Not integrable

Time = 36.98 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

3.143.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}dx$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.143.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.143.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\,\mathrm{arccsc}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.143.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsc(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

3.143.6 Sympy [N/A]

Not integrable

Time = 53.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\,\mathrm{acsc}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.143.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.143.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

3.143.9 Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.144 $\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

3.144.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\operatorname{Int}\left(\frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.144.2 Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2], x]
```

3.144.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \ 5772$$

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.144.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.144.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsc}\left(cx\right)}{\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.144.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.144.6 Sympy [N/A]

Not integrable

Time = 15.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.144.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.144.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.144.9 Mupad [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a+b\csc^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.145 $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

3.145.1 Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\,(a+b\csc^{-1}(cx))}{dx} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\,E\left(\arcsin(cx)\,\middle|\,-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
-(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/d/x-b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/
2)/d/(c^2*x^2)^(1/2)+b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^
(1/2)*(e*x^2+d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2
)-b*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^
2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.57

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+bc\sqrt{1-\frac{1}{c^2x^2}}x+b\csc^{-1}(cx)\right)}{dx} + \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|-\frac{c^2d}{e}\right)}{d\sqrt{-\frac{e}{d}}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((Sqrt[d + e*x^2]*(a + b*c*Sqrt[1 - 1/(c^2*x^2)]*x + b*ArcCsc[c*x]))/(d*x
)) + (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[S
qrt[-(e/d)]*x], -((c^2*d)/e)])/(d*Sqrt[-(e/d)]*Sqrt[1 - c^2*x^2]*Sqrt[d +
e*x^2])
```

3.145.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5762, 25, 27, 377, 27, 326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 5762}$$

$$\frac{bcx\int -\frac{\sqrt{ex^2+d}}{dx^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}}{dx^2\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{c^2x^2-1}}dx}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \ 377$$

$$-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-\int \frac{e\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\int \frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \ 326$$

$$-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{(c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \ 323$$

$$-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \ 323$$

$$-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \ 321$$

$$-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{dx}$$

3.145. $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

$$\Big\downarrow \; 331$$

$$-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx}$$

$$\Big\downarrow \; 330$$

$$-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx}$$

$$\Big\downarrow \; 327$$

$$-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{dx}-\frac{bcx\left(\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}\right)\right)}{d\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(d*x)) - (b*c*x*((Sqrt[-1 + c^2*x^
2]*Sqrt[d + e*x^2])/x - e*((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[
ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) - (
(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -
(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))))/(d*Sqrt[c^2*x^2]
)
```

3.145. $\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.145.4 Maple [F]

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x^2\sqrt{e\,x^2 + d}}dx$$

input
```
int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.43

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\left(bcd\,\mathrm{arccsc}\,(cx) + \sqrt{c^2x^2-1}bcd + acd\right)\sqrt{ex^2+d} + \left(bc^4dxE(\arcsin\,(cx)\mid -\frac{e}{c^2d}) - (bc^4d+be)xF(\mathrm{arcsi}\right.}{cd^2x}$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
-((b*c*d*arccsc(c*x) + sqrt(c^2*x^2 - 1)*b*c*d + a*c*d)*sqrt(e*x^2 + d) +
(b*c^4*d*x*elliptic_e(arcsin(c*x), -e/(c^2*d)) - (b*c^4*d + b*e)*x*ellipti
c_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^2*x)
```

3.145.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{a+b\operatorname{acsc}(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

3.145.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.145.8 Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.146 $\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 362

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx &= -\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}}-\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}}\\ &\quad-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}\\ &\quad+\frac{bc^2(2c^2d-5e)\,x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\,\middle|-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\\ &\quad-\frac{2b(c^2d-3e)(c^2d+e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

output
```
-1/3*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arccsc(c*x))*(e*x^
2+d)^(1/2)/d^2/x-1/9*b*c*(2*c^2*d-5*e)*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d
^2/(c^2*x^2)^(1/2)-1/9*b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(c^2*x^
2)^(1/2)+1/9*b*c^2*(2*c^2*d-5*e)*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x
^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2
/d)^(1/2)-2/9*b*(c^2*d-3*e)*(c^2*d+e)*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-
c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(
e*x^2+d)^(1/2)
```

3.146.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.53 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.69

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= -\frac{\sqrt{d+ex^2}\left(bc\sqrt{1-\frac{1}{c^2x^2}}x(d+2c^2dx^2-5ex^2)+3a(d-2ex^2)+3b(d-2ex^2)\csc^{-1}(cx)\right)}{9d^2x^3}$$
$$+\frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d-5e)E\left(i\operatorname{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+2(-c^4d^2+2c^2de+3e^2)\operatorname{EllipticF}\right.}{9\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/9*(Sqrt[d + e*x^2]*(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + 2*c^2*d*x^2 - 5*e*
x^2) + 3*a*(d - 2*e*x^2) + 3*b*(d - 2*e*x^2)*ArcCsc[c*x]))/(d^2*x^3) + ((I
/9)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d - 5*e)
*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(-(c^4*d^2) + 2*c^2*
d*e + 3*e^2)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2
]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.146.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5762, 27, 442, 25, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \ 5762$$
$$\frac{bcx\int -\frac{(d-2ex^2)\sqrt{ex^2+d}}{3d^2x^4\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$
$$\Big\downarrow \ 27$$

$$-\frac{bcx\int \frac{(d-2ex^2)\sqrt{ex^2+d}}{x^4\sqrt{c^2x^2-1}}dx}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\Big\downarrow$ 442

$$-\frac{bcx\left(\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int -\frac{(c^2d-6e)ex^2+d(2c^2d-5e)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\Big\downarrow$ 25

$$-\frac{bcx\left(\frac{1}{3}\int \frac{(c^2d-6e)ex^2+d(2c^2d-5e)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\Big\downarrow$ 445

$$-\frac{bcx\left(\frac{1}{3}\left(\frac{\int \frac{de\left(-\left(\left(2c^2d-5e\right)x^2c^2\right)+dc^2-6e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\Big\downarrow$ 27

$$-\frac{bcx\left(\frac{1}{3}\left(e\int \frac{-((2c^2d-5e)x^2c^2)+dc^2-6e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\Big\downarrow$ 399

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2(c^2d-3e)(c^2d+e)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2(2c^2d-5e)\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\Big\downarrow$ 323

3.146. $\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2(2c^2d-5e)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex^2}}{x}\right)+ d\cdots}{3d^2\sqrt{c^2x^2}}$$
$$\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\downarrow$ 323

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(2c^2d-5e)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{d+ex\cdots}}{x}\cdots}{3d^2\sqrt{c^2x^2}}$$
$$\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\downarrow$ 321

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2(2c^2d-5e)\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}(2c^2d-5e)\sqrt{c\cdots}}{x}\cdots}{3d^2\sqrt{c^2x^2}}$$
$$\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\downarrow$ 331

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(2c^2d-5e)\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{\sqrt{c^2x^2-1}(2c\cdots}{\cdots}\cdots}{3d^2\sqrt{c^2x^2}}$$
$$\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\downarrow$ 330

$$-\frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}(2c^2d-5e)\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\sqrt{c^2\cdots}\cdots}{3d^2\sqrt{c^2x^2}}$$
$$\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3}$$

$\downarrow$ 327

3.146. $\int\frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

$$\frac{2e\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{3dx^3} - \frac{bcx\left(\frac{1}{3}\left(e\left(\frac{2\sqrt{1-c^2x^2}(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}} - \frac{c\sqrt{1-c^2x^2}(2c^2d-5e)\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\right.\right.}{3d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/3*(Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/(d*x^3) + (2*e*Sqrt[d + e*x^2]*
(a + b*ArcCsc[c*x]))/(3*d^2*x) - (b*c*x*((d*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*
x^2])/(3*x^3) + (((2*c^2*d - 5*e)*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x +
e*(-((c*(2*c^2*d - 5*e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin
[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + (2*(c^
2*d - 3*e)*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[Arc
Sin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])))/3))/(3
*d^2*Sqrt[c^2*x^2])
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.146. $\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.146.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsc}\,(cx)}{x^4\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= \frac{\left(6\,acdex^2-3\,acd^2+3\,(2\,bcdex^2-bcd^2)\,\mathrm{arccsc}\,(cx)-(bcd^2+(2\,bc^3d^2-5\,bcde)x^2)\sqrt{c^2x^2-1}\right)\sqrt{ex^2+d}\ \cdots}{9\ \cdots}$$

input
```
integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
1/9*((6*a*c*d*e*x^2 - 3*a*c*d^2 + 3*(2*b*c*d*e*x^2 - b*c*d^2)*arccsc(c*x)
- (b*c*d^2 + (2*b*c^3*d^2 - 5*b*c*d*e)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2
+ d) - ((2*b*c^6*d^2 - 5*b*c^4*d*e)*x^3*elliptic_e(arcsin(c*x), -e/(c^2*d)
) - (2*b*c^6*d^2 - (5*b*c^4 - b*c^2)*d*e - 6*b*e^2)*x^3*elliptic_f(arcsin(
c*x), -e/(c^2*d)))*sqrt(-d))/(c*d^3*x^3)
```

3.146.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acsc}\,(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.146.8 Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{ex^2+dx^4}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.147 $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.147.1 Optimal result

Integrand size = 23, antiderivative size = 252

$$\begin{aligned}\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx &= \frac{bx\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}\\ &-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}\\ &+\frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}}-\frac{b(9c^2d-e)\,x\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{c^2x^2}}\end{aligned}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x))/e^3+8/3*b*c*d^(3/2)*x*arctan((e*x^2+
d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^3/(c^2*x^2)^(1/2)-1/6*b*(9*c^2*d-e)*
x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(c^2*x^
2)^(1/2)-d^2*(a+b*arccsc(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arccsc(c*x))*(
e*x^2+d)^(1/2)/e^3+1/6*b*x*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(c^2*x^
2)^(1/2)
```

3.147.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.53 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.04

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{16bd^2\sqrt{1+\frac{d}{ex^2}}(-1+c^2x^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+be(-9c^2d+e)\sqrt{}$$

input
```
Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(16*b*d^2*Sqrt[1 + d/(e*x^2)]*(-1 + c^2*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c
^2*x^2), -(d/(e*x^2))] + b*e*(-9*c^2*d + e)*Sqrt[1 - 1/(c^2*x^2)]*x^4*Sqrt
[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x
^2)/d)] + 2*x*(-1 + c^2*x^2)*(b*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2) - 2*
a*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*Ar
cCsc[c*x]))/(12*c*e^3*x*(-1 + c^2*x^2)*Sqrt[d + e*x^2])
```

3.147.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5762, 27, 7282, 2118, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 5762$$

$$\frac{bcx\int -\frac{-e^2x^4+4dex^2+8d^2}{3e^3x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\int \frac{-e^2x^4+4dex^2+8d^2}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{3e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{7282}$$

$$-\frac{bcx\int \frac{-e^2x^4+4dex^2+8d^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{2118}$$

$$-\frac{bcx\left(\frac{\int \frac{e\left(16c^2d^2+\left(9c^2d-e\right)ex^2\right)}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2e}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\left(\frac{\int \frac{16c^2d^2+\left(9c^2d-e\right)ex^2}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{175}$$

$$-\frac{bcx\left(\frac{16c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(9c^2d-e\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{66}$$

$$-\frac{bcx\left(\frac{16c^2d^2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(9c^2d-e\right)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2\left(a+b\csc^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{104}$$

3.147. $\int \frac{x^5\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

$$-\frac{bcx\left(\frac{32c^2d^2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e(9c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{bcx\left(\frac{2e(9c^2d-e)\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-32c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}$$

$$\Big\downarrow \text{221}$$

$$-\frac{d^2(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{(d+ex^2)^{3/2}(a+b\csc^{-1}(cx))}{3e^3}-\frac{bcx\left(\frac{\frac{2\sqrt{e}(9c^2d-e)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-32c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((d^2*(a + b*ArcCsc[c*x]))/(e^3*Sqrt[d + e*x^2])) - (2*d*Sqrt[d + e*x^2]*
(a + b*ArcCsc[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]))/(3*e^3)
 - (b*c*x*(-((e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/c^2) + (-32*c^2*d^(3/2
)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (2*(9*c^2*d - e)*
Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c)/(2*c
^2)))/(6*e^3*Sqrt[c^2*x^2])
```

3.147. $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.147. $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.147.4 Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 1480, normalized size of antiderivative = 5.87

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/24*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(8*c^
4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x
^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 16*(b
*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 -
8*(c^2*d^2 - d*e)*x^2 - 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e
*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 -
16*a*c^3*d^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*
x) + (b*c*e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*
x^2 + c^3*d*e^3), 1/24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*arctan(-1/2
*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*
e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - (9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e
- b*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e
 - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x
^2 + d)*sqrt(e) + e^2) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d
^2 + 2*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*arccsc(c*x) + (b*c*
e^2*x^2 + b*c*d*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*
d*e^3), 1/12*((9*b*c^2*d^2 - b*d*e + (9*b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*a
rctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt
(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 8*(b*c^3*d*e*x^2 + b
*c^3*d^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - ...
```

3.147.6 Sympy [F]

$$\int \frac{x^5(a + b\csc^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{x^5(a + b\operatorname{acsc}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**5*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.147.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.147.8 Giac [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^5\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.148 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

3.148.1 Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}-\frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{e^2\sqrt{c^2x^2}}+\frac{bx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{c^2x^2}}$$

output
```
b*x*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(c^2*x^2)
^(1/2)-2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e
^2/(c^2*x^2)^(1/2)+d*(a+b*arccsc(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arccsc(c*x
))*(e*x^2+d)^(1/2)/e^2
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.04

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\frac{-\frac{2bd\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx}-\frac{bce\sqrt{1-\frac{1}{c^2x^2}}x^3\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,c^2x^2,-\frac{ex^2}{d}\right.}{\sqrt{1-c^2x^2}}}{2e^2\sqrt{d+ex^2}}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
((-2*b*d*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*
x^2))])/(c*x) - (b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Sqrt[1 + (e*x^2)/d]*Appel
lF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x^2)/d)])/Sqrt[1 - c^2*x^2] + 2*(2*d + e
*x^2)*(a + b*ArcCsc[c*x]))/(2*e^2*Sqrt[d + e*x^2])
```

3.148.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5762, 27, 435, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+b \csc^{-1}(cx))}{(d+ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx \int \frac{ex^2+2d}{e^2x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}+\frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx \int \frac{ex^2+2d}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}+\frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{bcx \int \frac{ex^2+2d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}+\frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{175}$$

$$\frac{bcx\left(e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}+\frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{bcx\left(2d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^2}+\frac{d(a+b\csc^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{bcx\left(4d\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{bcx\left(2e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-4\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e^2\sqrt{c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{bcx\left(\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-4\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)\right)}{2e^2\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(d*(a + b*ArcCsc[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cCsc[c*x]))/e^2 + (b*c*x*(-4*Sqrt[d]*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[
-1 + c^2*x^2])] + (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[
d + e*x^2])])/c))/(2*e^2*Sqrt[c^2*x^2])
```

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.148. $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

3.148.4 Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 1072, normalized size of antiderivative = 6.87

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*
(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*
sqrt(e*x^2 + d)*sqrt(e) + e^2) + 2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*
d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*(
(c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(a*c*e*x
^2 + 2*a*c*d + (b*c*e*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*
x^2 + c*d*e^2), -1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x
^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^
2*d^2 - d*e)*x^2 - d^2)) - (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4
*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*
e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(a*c*e*x^2 + 2*a*c
*d + (b*c*e*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*
e^2), -1/2*((b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*
sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e
 - c*e^2)*x^2)) - (b*c*e*x^2 + b*c*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e +
 e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 -
 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d + (b
*c*e*x^2 + 2*b*c*d)*arccsc(c*x))*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), -
1/2*(2*(b*c*e*x^2 + b*c*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d -
 e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x...
```

3.148.6 Sympy [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^3(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.148.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.148.8 Giac [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.149 $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

3.149.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}$$

output
```
b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e/d^(1/2)/(c^2*x^2
)^(1/2)+(-a-b*arccsc(c*x))/e/(e*x^2+d)^(1/2)
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\frac{b\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)-2cx(a+b\csc^{-1}(cx))}{2cex\sqrt{d+ex^2}}$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(b*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))]
 - 2*c*x*(a + b*ArcCsc[c*x]))/(2*c*e*x*Sqrt[d + e*x^2])
```

3.149.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5760, 354, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5760}$$

$$-\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{354}$$

$$-\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{104}$$

$$-\frac{bcx\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}}{e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e\sqrt{d+ex^2}}$$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((a + b*ArcCsc[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*e*Sqrt[c^2*x^2])
```

3.149.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.149.4 Maple [F]

$$\int \frac{x(a+b\,\mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.58

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(bex^2+bd)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+e^2)x^4-8\,(c^2d^2-de)x^2+4\,\sqrt{c^2x^2-1}((c^2d-e)x^2-2\,d)\sqrt{ex^2+}}{x^4}\right.}{4\,(de^2x^2+d^2e)}\right.$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*e*x^2 + b*d)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(
c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^
2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*(b*d*arccsc(c*x) + a*d))
/(d*e^2*x^2 + d^2*e), 1/2*((b*e*x^2 + b*d)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^
2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2
*d^2 - d*e)*x^2 - d^2)) - 2*sqrt(e*x^2 + d)*(b*d*arccsc(c*x) + a*d))/(d*e^
2*x^2 + d^2*e)]
```

3.149.6 Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.149.7 Maxima [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-(sqrt(e*x^2 + d)*c^2*e*integrate(x*e^(-1/2*log(e*x^2 + d) + 1/2*log(c*x +
 1) + 1/2*log(c*x - 1))/(c^2*e*x^2 + (c^2*e*x^2 - e)*e^(log(c*x + 1) + log
(c*x - 1)) - e), x) + arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))*b/(sqrt(e*x
^2 + d)*e) - a/(sqrt(e*x^2 + d)*e)
```

3.149.8 Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.150 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

3.150.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.150.2 Mathematica [N/A]

Not integrable

Time = 8.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

3.150.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 5772$$

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.150.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.150.4 Maple [N/A] (verified)

Not integrable

Time = 2.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\operatorname{arccsc}\left(c x\right)}{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.150.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

3.150.6 Sympy [N/A]

Not integrable

Time = 75.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

3.150.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.150.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

3.150.9 Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.151 $\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.151.2 Mathematica [N/A]

Not integrable

Time = 11.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

3.151.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3 (d+ex^2)^{3/2}} \, dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{a+b\csc^{-1}(cx)}{x^3 (d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.151.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.151.4 Maple [N/A] (verified)

Not integrable

Time = 5.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsc}\left(c x\right)}{x^{3} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.151.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

3.151.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.151.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

3.151.9 Mupad [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.152 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

3.152.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Int}\left(\frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.152.2 Mathematica [N/A]

Not integrable

Time = 14.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

3.152.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5772}$$

$$\int \frac{x^4 (a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.152.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.152.4 Maple [N/A] (verified)

Not integrable

Time = 1.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4 (a+b \operatorname{arccsc}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.152.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

3.152.6 Sympy [N/A]

Not integrable

Time = 122.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^4(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.152.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.152.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

3.152.9 Mupad [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4\left(a+b\operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.153 $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.153.2 Mathematica [N/A]

Not integrable

Time = 5.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

3.153.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a + b \csc^{-1}(cx))}{(d + ex^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 5772$$

$$\int \frac{x^2 (a + b \csc^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.153.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.153.4 Maple [N/A] (verified)

Not integrable

Time = 1.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2 (a + b \operatorname{arccsc}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.153.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsc(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

3.153.6 Sympy [N/A]

Not integrable

Time = 32.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.153.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.153.8 Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex^2)^{3/2}}\, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

3.153.9 Mupad [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a + b\csc^{-1}(cx))}{(d + ex^2)^{3/2}}\, dx = \int \frac{x^2 \left(a + b \operatorname{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{3/2}}\, dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.154 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

3.154.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
x*(a+b*arccsc(c*x))/d/(e*x^2+d)^(1/2)+b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*
(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(
e*x^2+d)^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x(a+b\csc^{-1}(cx))}{d\sqrt{d+ex^2}}+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\left(-c+c^3x^2\right)\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x
*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))
])/(d*(-c + c^3*x^2)*Sqrt[d + e*x^2])
```

3.154.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5752, 27, 323, 323, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5752}$$

$$\frac{bcx\int \frac{1}{d\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d\sqrt{c^2x^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{bcx\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{d\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{bcx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{x\left(a+b\csc^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{bx\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d\sqrt{c^2x^2}\sqrt{c^2x^2-1}\sqrt{d+ex^2}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCsc[c*x]))/(d*Sqrt[d + e*x^2]) + (b*x*Sqrt[1 - c^2*x^2]*Sqrt[
1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(d*Sqrt[c^2*x^2]*Sqrt
[-1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 5752
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCsc[c*x])   u,
x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.154.4 Maple [F]

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{(e\,x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = -\frac{(bex^2+bd)\sqrt{-d}F\left(\arcsin(cx)\mid -\frac{e}{c^2d}\right)-(bcdx\,\text{arccsc}(cx)+acdx)\sqrt{ex^2+d}}{cd^2ex^2+cd^3}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
-((b*e*x^2 + b*d)*sqrt(-d)*elliptic_f(arcsin(c*x), -e/(c^2*d)) - (b*c*d*x*
arccsc(c*x) + a*c*d*x)*sqrt(e*x^2 + d))/(c*d^2*e*x^2 + c*d^3)
```

3.154.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\text{acsc}(cx)}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.154.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.154.8 Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.155 $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 275

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = -\frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{c^2x^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} + \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} - \frac{b(c^2d+2e)\,x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
(-a-b*arccsc(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arccsc(c*x))/d^2/(e*x^2+
d)^(1/2)-b*c*(c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*x^2)^(1/2)+b*c^2*x
*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(c
^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)-b*(c^2*d+2*e)*x*Elliptic
F(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d^2/(c^2*x^2)
^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.155.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.77

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{-bc\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\csc^{-1}(cx)}{d^2x\sqrt{d+ex^2}} + \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)\left|-\frac{e}{c^2d}\right.\right)-(c^2d+2e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right)}{\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
(-(b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) - a*(d + 2*e*x^2) - b*(d + 2*e
*x^2)*ArcCsc[c*x])/(d^2*x*Sqrt[d + e*x^2]) + (I*b*c*Sqrt[1 - 1/(c^2*x^2)]*
x*Sqrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d
))] - (c^2*d + 2*e)*EllipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sq
rt[-c^2]*d^2*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.155.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5762, 25, 27, 445, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5762}$$

$$\frac{bcx\int -\frac{2ex^2+d}{d^2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{bcx\int \frac{2ex^2+d}{d^2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{c^2x^2}} - \frac{2ex(a+b\csc^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bcx\int \frac{2ex^2+d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{445}$$

$$-\frac{bcx\left(\frac{\int \frac{de\left(2-c^2x^2\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\left(e\int \frac{2-c^2x^2}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{399}$$

$$-\frac{bcx\left(e\left(\frac{\left(c^2d+2e\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{323}$$

$$-\frac{bcx\left(e\left(\frac{\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{323}$$

$$-\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}\left(c^2d+2e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{321}$$

3.155. $\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

$$-\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{331}$$

$$-\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{330}$$

$$-\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2ex\left(a+b\csc^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\csc^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{bcx\left(e\left(\frac{\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}-\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}\right)+\frac{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcCsc[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcCsc[c*x]))
/(d^2*Sqrt[d + e*x^2]) - (b*c*x*((Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2])/x +
e*(-((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*
d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d])) + ((c^2*d + 2*e)*Sqrt[1
- c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*e*
Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))))/(d^2*Sqrt[c^2*x^2])
```

3.155. $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.155. $\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.155.4 Maple [F]

$$\int \frac{a + b\,\mathrm{arccsc}\,(cx)}{x^2\,(e\,x^2 + d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int \frac{a + b\csc^{-1}(cx)}{x^2\,(d + ex^2)^{3/2}}\,dx =$$

$$-\frac{(2\,acdex^2 + acd^2 + (2\,bcdex^2 + bcd^2)\,\mathrm{arccsc}\,(cx) + (bcdex^2 + bcd^2)\sqrt{c^2x^2 - 1}\big)\sqrt{ex^2 + d} + \big((bc^4dex^3 + bc}{cd^3ex^3 + cd^4}$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
-((2*a*c*d*e*x^2 + a*c*d^2 + (2*b*c*d*e*x^2 + b*c*d^2)*arccsc(c*x) + (b*c*
d*e*x^2 + b*c*d^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) + ((b*c^4*d*e*x^3 +
b*c^4*d^2*x)*elliptic_e(arcsin(c*x), -e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)*
x^3 + (b*c^4*d^2 + 2*b*d*e)*x)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-
d))/(c*d^3*e*x^3 + c*d^4*x)
```

3.155.6 Sympy [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\operatorname{acsc}(cx)}{x^2(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

3.155.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.155.8 Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.156 $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 243

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcdx\sqrt{-1+c^2x^2}}{3e^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3} - \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3e^3\sqrt{c^2x^2}} + \frac{bx\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{c^2x^2}}$$

output
```
-1/3*d^2*(a+b*arccsc(c*x))/e^3/(e*x^2+d)^(3/2)+b*x*arctanh(e^(1/2)*(c^2*x^
2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(c^2*x^2)^(1/2)-8/3*b*c*x*arctan((e*
x^2+d)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(c^2*x^2)^(1/2)+2*d*(a
+b*arccsc(c*x))/e^3/(e*x^2+d)^(1/2)+1/3*b*c*d*x*(c^2*x^2-1)^(1/2)/e^2/(c^2
*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arccsc(c*x))*(e*x^2+d)^(1/2)/e^
3
```

3.156.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.47 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{\frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d+e} + 2a(8d^2+12dex^2+3e^2x^4) + \frac{bc(d+ex^2)\left(-\frac{8d\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2}\right.}{c^2}\right.}{}}{6}$$

input
```
Integrate[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
((2*b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d + e) + 2*a*(8*d^2
+ 12*d*e*x^2 + 3*e^2*x^4) + (b*c*(d + e*x^2)*((-8*d*Sqrt[1 + d/(e*x^2)]*Ap
pellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -(d/(e*x^2))])/c^2 - (3*e*Sqrt[1 - 1/(
c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, c^2*x^2, -((e*x
^2)/d)])/Sqrt[1 - c^2*x^2]))/x + 2*b*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcC
sc[c*x])/(6*e^3*(d + e*x^2)^(3/2))
```

3.156.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5762, 27, 7282, 2117, 27, 175, 66, 104, 217, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{3e^3x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{c^2x^2}} - \frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}} + \frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow \text{7282}$$

$$\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{x^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx^2}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow \text{2117}$$

$$\frac{bcx\left(\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{2\int -\frac{d(dc^2+e)(3ex^2+8d)}{2x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d(c^2d+e)}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\int \frac{3ex^2+8d}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow \text{175}$$

$$\frac{bcx\left(3e\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+8d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow \text{66}$$

$$\frac{bcx\left(8d\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+6e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow \text{104}$$

3.156. $\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

$$\frac{bcx\left(6e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}+16d\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow 217$$

$$\frac{bcx\left(6e\int \frac{1}{c^2-ex^4}d\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}-16\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}$$

$$\Big\downarrow 221$$

$$-\frac{d^2(a+b\csc^{-1}(cx))}{3e^3(d+ex^2)^{3/2}}+\frac{2d(a+b\csc^{-1}(cx))}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\csc^{-1}(cx))}{e^3}+\frac{bcx\left(-16\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)+\frac{6\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}+\frac{2de\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(d^2*(a + b*ArcCsc[c*x]))/(e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*ArcCs
c[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]))/e^3
 + (b*c*x*((2*d*e*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) - 16*S
qrt[d]*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])] + (6*Sqrt[e]*A
rcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/c))/(6*e^3*Sqrt[
c^2*x^2])
```

3.156.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.156. $\int \frac{x^5 (a+b \csc^{-1}(cx))}{(d+ex^2)^{5/2}} \, dx$

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.156.4 Maple [F]

$$\int \frac{x^5(a+b\ \mathrm{arccsc}\left(cx\right))}{\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

input
```
int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(205) = 410$.

Time = 0.50 (sec) , antiderivative size = 2119, normalized size of antiderivative = 8.72

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
 + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*
d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(
e*x^2 + d)*sqrt(e) + e^2) + 8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*
e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2
*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)
*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(-d) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 + 8*a*
c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2
 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*
d^2*e + b*c*d*e^2)*x^2)*arccsc(c*x) + (b*c*d*e^2*x^2 + b*c*d^2*e)*sqrt(c^2
*x^2 - 1))*sqrt(e*x^2 + d))/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)
*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/12*(16*(b*c^3*d^3 + b*c*d^2*e +
(b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*arc
tan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)
/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 3*(b*c^2*d^3 + (b*c^2*d*e^2
+ b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*
e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 + 4*(2*c^3*e*x^2
 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) - 4*(8*a*
c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e +
a*c*d*e^2)*x^2 + (8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)...
```

3.156.6 Sympy [F]

$$\int \frac{x^5(a + b\csc^{-1}(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x^5(a + b\operatorname{acsc}(cx))}{(d + ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x**5*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

3.156.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.156.8 Giac [F]

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^5}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^5\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.157 $\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx\sqrt{-1+c^2x^2}}{3e(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{d(a+b\csc^{-1}(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}} + \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{c^2x^2}}$$

output
```
1/3*d*(a+b*arccsc(c*x))/e^2/(e*x^2+d)^(3/2)+2/3*b*c*x*arctan((e*x^2+d)^(1/
2)/d^(1/2)/(c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(c^2*x^2)^(1/2)+(-a-b*arccsc(c*x
))/e^2/(e*x^2+d)^(1/2)-1/3*b*c*x*(c^2*x^2-1)^(1/2)/e/(c^2*d+e)/(c^2*x^2)^(
1/2)/(e*x^2+d)^(1/2)
```

3.157.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{b(c^2d+e)\sqrt{1+\frac{d}{ex^2}}(d+ex^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right) - cx\left(bce\sqrt{1-\frac{}{c^2}}\right.}{3ce^2(c^2d+e)\,x\,(}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(b*(c^2*d + e)*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/
(c^2*x^2), -(d/(e*x^2))] - c*x*(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)
+ a*(c^2*d + e)*(2*d + 3*e*x^2) + b*(c^2*d + e)*(2*d + 3*e*x^2)*ArcCsc[c*x
]))/(3*c*e^2*(c^2*d + e)*x*(d + e*x^2)^(3/2))
```

3.157.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5762, 27, 435, 169, 25, 27, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b \csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx \int -\frac{3ex^2+2d}{3e^2x\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx \int \frac{3ex^2+2d}{x\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{3e^2\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$-\frac{bcx \int \frac{3ex^2+2d}{x^2\sqrt{c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx^2}{6e^2\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{169}$$

$$-\frac{bcx\left(\frac{2e\sqrt{c^2x^2-1}}{\left(c^2d+e\right)\sqrt{d+ex^2}}-\frac{2\int -\frac{d\left(dc^2+e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d\left(c^2d+e\right)}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{bcx\left(\frac{2\int \frac{d\left(dc^2+e\right)}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d\left(c^2d+e\right)}+\frac{2e\sqrt{c^2x^2-1}}{\left(c^2d+e\right)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\csc^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\left(2\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2+\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 104$$

$$-\frac{bcx\left(4\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}+\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 217$$

$$-\frac{a+b\csc^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\csc^{-1}(cx))}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{2e\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{4\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}}\right)}{6e^2\sqrt{c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(d*(a + b*ArcCsc[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsc[c*x])/(e
^2*Sqrt[d + e*x^2]) - (b*c*x*((2*e*Sqrt[-1 + c^2*x^2])/((c^2*d + e)*Sqrt[d
 + e*x^2]) - (4*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/Sqrt
[d]))/(6*e^2*Sqrt[c^2*x^2])
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.157.4 Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(137) = 274.

Time = 0.38 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.07

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de\ldots}{\ldots}\right.}{\ldots}\right.$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e +
 b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
- d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*s
qrt(-d) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*
e^2)*x^2 + (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arccs
c(c*x) + (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*
d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*
x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2
*e + b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2
- 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2))
- (2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 + (2*b*c^2*d^3
+ 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*arccsc(c*x) + (b*d*e^2*x^2 +
b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^4*e^2 + d^3*e^3 + (c^2
*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

3.157.6 Sympy [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

3.157.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.157.8 Giac [F]

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.158 $\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.158.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx\sqrt{-1+c^2x^2}}{3d\,(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\csc^{-1}(cx)}{3e\,(d+ex^2)^{3/2}} + \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2}}$$

output
```
1/3*(-a-b*arccsc(c*x))/e/(e*x^2+d)^(3/2)+1/3*b*c*x*arctan((e*x^2+d)^(1/2)/
d^(1/2)/(c^2*x^2-1)^(1/2))/d^(3/2)/e/(c^2*x^2)^(1/2)+1/3*b*c*x*(c^2*x^2-1)
^(1/2)/d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

3.158.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-\frac{2a}{e} + \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}\,x(d+ex^2)}{d(c^2d+e)} + \frac{b\sqrt{1+\frac{d}{ex^2}}\,(d+ex^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cdex} - \frac{2b\csc^{-1}(cx)}{e}}{6\,(d+ex^2)^{3/2}}$$

input
```
Integrate[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
((-2*a)/e + (2*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2))/(d*(c^2*d + e)) +
(b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, 1/(c^2*x^2), -
(d/(e*x^2))])/(c*d*e*x) - (2*b*ArcCsc[c*x])/e)/(6*(d + e*x^2)^(3/2))
```

3.158.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5760, 354, 107, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5760}$$

$$-\frac{bcx\int \frac{1}{x\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{bcx\int \frac{1}{x^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx^2}{6e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 107}$$

$$-\frac{bcx\left(\frac{\int \frac{1}{x^2\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d}-\frac{2e\sqrt{c^2x^2-1}}{d(c^2d+e)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 104}$$

$$-\frac{bcx\left(\frac{2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}}{d}-\frac{2e\sqrt{c^2x^2-1}}{d(c^2d+e)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}}-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{a+b\csc^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(-\frac{2\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{d^{3/2}}-\frac{2e\sqrt{c^2x^2-1}}{d(c^2d+e)\sqrt{d+ex^2}}\right)}{6e\sqrt{c^2x^2}}$$

input
```
Int[(x*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcCsc[c*x])/(e*(d + e*x^2)^(3/2)) - (b*c*x*((-2*e*Sqrt[-1 + c
^2*x^2])/(d*(c^2*d + e)*Sqrt[d + e*x^2]) - (2*ArcTan[Sqrt[d + e*x^2]/(Sqrt
[d]*Sqrt[-1 + c^2*x^2])])/d^(3/2)))/(6*e*Sqrt[c^2*x^2])
```

3.158.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5760
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsc[c*x])/(2*e*(p + 1))), x
] + Simp[b*c*(x/(2*e*(p + 1)*Sqrt[c^2*x^2]))   Int[(d + e*x^2)^(p + 1)/(x*S
qrt[c^2*x^2 - 1]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

3.158. $\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

3.158.4 Maple [F]

$$\int \frac{x(a+b\,\mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.158.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(114) = 228$.

Time = 0.37 (sec) , antiderivative size = 573, normalized size of antiderivative = 4.15

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(bc^2d^3+(bc^2de^2+be^3)x^4+bd^2e+2\,(bc^2d^2e+bde^2)x^2)\sqrt{-d}\log\left(\frac{(c^4d^2-6\,c^2de+}{}\right.}{}\right.$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e
+ b*d*e^2)*x^2)*sqrt(-d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2
 - d*e)*x^2 + 4*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x^2 - 2*d)*sqrt(e*x^2 + d)*
sqrt(-d) + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*ar
ccsc(c*x) - (b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c
^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^
3)*x^2), 1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*
d^2*e + b*d*e^2)*x^2)*sqrt(d)*arctan(-1/2*sqrt(c^2*x^2 - 1)*((c^2*d - e)*x
^2 - 2*d)*sqrt(e*x^2 + d)*sqrt(d)/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2
)) - 2*(a*c^2*d^3 + a*d^2*e + (b*c^2*d^3 + b*d^2*e)*arccsc(c*x) - (b*d*e^2
*x^2 + b*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 +
 (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

3.158.6 Sympy [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x(a+b\operatorname{acsc}(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*acsc(c*x))/(d + e*x**2)**(5/2), x)
```

3.158.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.158.8 Giac [F]

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.159 $\int \frac{a+b\csc^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\mathrm{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.159.2 Mathematica [N/A]

Not integrable

Time = 16.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

3.159.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{5772}$$

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.159.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.159.4 Maple [N/A] (verified)

Not integrable

Time = 3.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\operatorname{arccsc}\left(c x\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.159.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a + b\csc^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^
2*e*x^3 + d^3*x), x)
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.159.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.159.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

3.159.9 Mupad [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.160 $\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.160.2 Mathematica [N/A]

Not integrable

Time = 18.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

3.160.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 5772$$

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.160.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.160.4 Maple [N/A] (verified)

Not integrable

Time = 3.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsc}\left(c x\right)}{x^{3}\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.160.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a + b\csc^{-1}(cx)}{x^3 (d + ex^2)^{5/2}}\,dx = \int \frac{b\operatorname{arccsc}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\csc^{-1}(cx)}{x^3 (d + ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.160.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^{-1}(cx)}{x^3 (d + ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.160.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

3.160.9 Mupad [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\csc^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^3\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.161 $\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

3.161.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \operatorname{Int}\left(\frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.161.2 Mathematica [N/A]

Not integrable

Time = 16.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

3.161.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}} \, dx$$

$$\Big\downarrow \ 5772$$

$$\int \frac{x^6 (a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(x^6*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.161.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.161.4 Maple [N/A] (verified)

Not integrable

Time = 1.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6 (a+b \operatorname{arccsc}(cx))}{(e x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.161.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^6*arccsc(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.161.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.161.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^6}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

3.161.9 Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^6\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^6*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.162 $\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\text{Int}\left(\frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.162.2 Mathematica [N/A]

Not integrable

Time = 15.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2), x]
```

3.162.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 5772$$
$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}dx$$

input
```
Int[(x^4*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.162.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.162.4 Maple [N/A] (verified)

Not integrable

Time = 1.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\,\mathrm{arccsc}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.162.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)x^4}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arccsc(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.162.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\csc^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.162.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

3.162.9 Mupad [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.163 $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

3.163.1 Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx^2\sqrt{-1+c^2x^2}}{3d(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}} + \frac{x^3(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3de(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3de\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

output
```
1/3*x^3*(a+b*arccsc(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*c*x^2*(c^2*x^2-1)^(1/2)/
d/(c^2*d+e)/(c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)-1/3*b*c^2*x*EllipticE(c*x,(-e/
c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(e*x^2+d)^(1/2)/d/e/(c^2*d+e)/(c^2*x^2)^(
1/2)/(c^2*x^2-1)^(1/2)/(1+e*x^2/d)^(1/2)+1/3*b*x*EllipticF(c*x,(-e/c^2/d)^
(1/2))*(-c^2*x^2+1)^(1/2)*(1+e*x^2/d)^(1/2)/d/e/(c^2*x^2)^(1/2)/(c^2*x^2-1
)^(1/2)/(e*x^2+d)^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.67

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{x^2\left(a(c^2d+e)\,x+bc\sqrt{1-\frac{1}{c^2x^2}}(d+ex^2)+b(c^2d+e)\,x\csc^{-1}(cx)\right)}{3d\,(c^2d+e)\,(d+ex^2)^{3/2}} - \frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|-\frac{c^2d}{e}\right)}{3d\sqrt{-\frac{e}{d}}\,(c^2d+e)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^2*(a*(c^2*d + e)*x + b*c*Sqrt[1 - 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d +
 e)*x*ArcCsc[c*x]))/(3*d*(c^2*d + e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 - 1/
(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], -((c^2*
d)/e)])/(3*d*Sqrt[-(e/d)]*(c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.163.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {5762, 27, 373, 326, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int \frac{x^2}{3d\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{x^2}{\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3d\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{373}$$

$$\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{c^2x^2-1}}{\sqrt{ex^2+d}}dx}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 326$$

$$\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\left(c^2d+e\right)\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{e}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 323$$

$$\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 323$$

$$\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int\frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{e\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 321$$

$$\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx}{e}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 331$$

$$\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

3.163. $\int\frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}dx$

$$\Big\downarrow \ 330$$

$$\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}+\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \ 327$$

$$\frac{x^3\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bcx\left(\frac{x\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}-\frac{\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\middle|-\frac{e}{c^2d}\right)}{e\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}-\frac{\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\,\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{ce\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}\right)}{3d\sqrt{c^2x^2}}$$

input
```
Int[(x^2*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (b*c*x*((x*Sqrt[-1 + c
^2*x^2])/((c^2*d + e)*Sqrt[d + e*x^2]) - ((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*
x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(e*Sqrt[-1 + c^2*x^2]*Sqrt[1 +
(e*x^2)/d]) - ((c^2*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF
[ArcSin[c*x], -(e/(c^2*d))])/(c*e*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/(c^
2*d + e)))/(3*d*Sqrt[c^2*x^2])
```

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

3.163. $\int \frac{x^2\left(a+b\csc^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

3.163. $\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx$

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.163.4 Maple [F]

$$\int \frac{x^2(a + b \operatorname{arccsc}(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}dx$$

input
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.03

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{((bc^3d^2e + bcde^2)x^3 \operatorname{arccsc}(cx) + (ac^3d^2e + acde^2)x^3 + (bcde^2x^3 + bcd^2ex)\sqrt{c^2}}{}$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(((b*c^3*d^2*e + b*c*d*e^2)*x^3*arccsc(c*x) + (a*c^3*d^2*e + a*c*d*e^2
)*x^3 + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) +
 ((b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x)
, -e/(c^2*d)) - (b*c^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^
4*d^2*e + b*d*e^2)*x^2)*elliptic_f(arcsin(c*x), -e/(c^2*d)))*sqrt(-d))/(c^
3*d^5*e + c*d^4*e^2 + (c^3*d^3*e^3 + c*d^2*e^4)*x^4 + 2*(c^3*d^4*e^2 + c*d
^3*e^3)*x^2)
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.163.7 Maxima [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x
^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sq
rt(e*x^2 + d)), x)
```

3.163.8 Giac [F]

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arccsc}(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\csc^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,dx$$

input
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*asin(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.164 $\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

3.164.1 Optimal result

Integrand size = 20, antiderivative size = 296

$$\begin{aligned}\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx &= -\frac{bcex^2\sqrt{-1+c^2x^2}}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{d+ex^2}}+\frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}}\\ &+\frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{bc^2x\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{3d^2(c^2d+e)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}}\\ &+\frac{2bx\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}\end{aligned}$$

output
```
1/3*x*(a+b*arccsc(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccsc(c*x))/d^2/(e*x
^2+d)^(1/2)-1/3*b*c*e*x^2*(c^2*x^2-1)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/
(e*x^2+d)^(1/2)+1/3*b*c^2*x*EllipticE(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(
1/2)*(e*x^2+d)^(1/2)/d^2/(c^2*d+e)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(1+e*
x^2/d)^(1/2)+2/3*b*x*EllipticF(c*x,(-e/c^2/d)^(1/2))*(-c^2*x^2+1)^(1/2)*(1
+e*x^2/d)^(1/2)/d^2/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)/(e*x^2+d)^(1/2)
```

3.164.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.84

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x\left(-bce\sqrt{1-\frac{1}{c^2x^2}}x(d+ex^2)+a(c^2d+e)(3d+2ex^2)+b(c^2d+e)(3d+2ex^2)\csc^{-1}(cx)\right)}{3d^2(c^2d+e)(d+ex^2)^{3/2}} + \frac{ibc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right)|-\frac{e}{c^2d}\right)+2(c^2d+e)\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-c^2}x\right),-\frac{e}{c^2d}\right)\right)}{3\sqrt{-c^2}d^2(c^2d+e)\sqrt{1-c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(-(b*c*e*Sqrt[1 - 1/(c^2*x^2)]*x*(d + e*x^2)) + a*(c^2*d + e)*(3*d + 2*
e*x^2) + b*(c^2*d + e)*(3*d + 2*e*x^2)*ArcCsc[c*x]))/(3*d^2*(c^2*d + e)*(d
 + e*x^2)^(3/2)) + ((I/3)*b*c*Sqrt[1 - 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*
(c^2*d*EllipticE[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))] + 2*(c^2*d + e)*El
lipticF[I*ArcSinh[Sqrt[-c^2]*x], -(e/(c^2*d))]))/(Sqrt[-c^2]*d^2*(c^2*d +
e)*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])
```

3.164.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5752, 27, 402, 27, 399, 323, 323, 321, 331, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 5752$$

$$\frac{bcx\int \frac{2ex^2+3d}{3d^2\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{c^2x^2}}+\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{bcx\int \frac{2ex^2+3d}{\sqrt{c^2x^2-1}(ex^2+d)^{3/2}}dx}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{bcx\left(\frac{\int \frac{d\left(ex^2c^2+3dc^2+2e\right)}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{d(c^2d+e)}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{ex^2c^2+3dc^2+2e}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+2\left(c^2d+e\right)\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{323}$$

$$\frac{bcx\left(\frac{c^2\int \frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\sqrt{1-c^2x^2}\left(c^2d+e\right)\sqrt{\frac{ex^2}{d}+1}\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x\left(a+b\csc^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\csc^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{321}$$

3.164. $\int \frac{a+b\csc^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

$$\frac{bcx\left(\frac{c^2\int\frac{\sqrt{ex^2+d}}{\sqrt{c^2x^2-1}}dx+\frac{2\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{331}$$

$$\frac{bcx\left(\frac{\frac{c^2\sqrt{1-c^2x^2}\int\frac{\sqrt{ex^2+d}}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}}+\frac{2\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{330}$$

$$\frac{bcx\left(\frac{\frac{c^2\sqrt{1-c^2x^2}\sqrt{d+ex^2}\int\frac{\sqrt{\frac{ex^2}{d}+1}}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}+\frac{2\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}+\frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2x(a+b\csc^{-1}(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\csc^{-1}(cx))}{3d(d+ex^2)^{3/2}}+\frac{bcx\left(\frac{\frac{2\sqrt{1-c^2x^2}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\text{EllipticF}\left(\arcsin(cx),-\frac{e}{c^2d}\right)}{c\sqrt{c^2x^2-1}\sqrt{d+ex^2}}+\frac{c\sqrt{1-c^2x^2}\sqrt{d+ex^2}E\left(\arcsin(cx)\left|-\frac{e}{c^2d}\right.\right)}{\sqrt{c^2x^2-1}\sqrt{\frac{ex^2}{d}+1}}}{c^2d+e}-\frac{ex\sqrt{c^2x^2-1}}{(c^2d+e)\sqrt{d+ex^2}}\right)}{3d^2\sqrt{c^2x^2}}$$

input
```
Int[(a + b*ArcCsc[c*x])/(d + e*x^2)^(5/2),x]
```

3.164. $\int\frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}dx$

output
```
(x*(a + b*ArcCsc[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCsc[c*x])
)/(3*d^2*Sqrt[d + e*x^2]) + (b*c*x*(-((e*x*Sqrt[-1 + c^2*x^2])/((c^2*d + e
)*Sqrt[d + e*x^2])) + ((c*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2]*EllipticE[ArcS
in[c*x], -(e/(c^2*d))])/(Sqrt[-1 + c^2*x^2]*Sqrt[1 + (e*x^2)/d]) + (2*(c^2
*d + e)*Sqrt[1 - c^2*x^2]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(
c^2*d))])/(c*Sqrt[-1 + c^2*x^2]*Sqrt[d + e*x^2]))/(c^2*d + e)))/(3*d^2*Sqr
t[c^2*x^2])
```

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 331
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 5752
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCsc[c*x])   u,
x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 -
 1]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1
/2, 0])
```

3.164.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsc}\left(c x\right)}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}d x$$

input
```
int((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.18

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{(2\,(ac^3d^2e+acde^2)x^3+3\,(ac^3d^3+acd^2e)x+(2\,(bc^3d^2e+bcde^2)x^3+3\,(bc^3d^3+bcd}{}$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((2*(a*c^3*d^2*e + a*c*d*e^2)*x^3 + 3*(a*c^3*d^3 + a*c*d^2*e)*x + (2*(
b*c^3*d^2*e + b*c*d*e^2)*x^3 + 3*(b*c^3*d^3 + b*c*d^2*e)*x)*arccsc(c*x) -
(b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d) - ((b*c^4
*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*elliptic_e(arcsin(c*x), -e/(c^
2*d)) - (((b*c^4 - 3*b*c^2)*d*e^2 - 2*b*e^3)*x^4 + (b*c^4 - 3*b*c^2)*d^3 -
 2*b*d^2*e + 2*((b*c^4 - 3*b*c^2)*d^2*e - 2*b*d*e^2)*x^2)*elliptic_f(arcsi
n(c*x), -e/(c^2*d)))*sqrt(-d))/(c^3*d^6 + c*d^5*e + (c^3*d^4*e^2 + c*d^3*e
^3)*x^4 + 2*(c^3*d^5*e + c*d^4*e^2)*x^2)
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsc(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.164.7 Maxima [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(
e*x^2 + d)), x)
```

3.164.8 Giac [F]

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\csc^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*asin(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.165 $\int (fx)^m (d+ex^2)^3 (a+b\csc^{-1}(cx))\,dx$

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 585

$$\begin{aligned}
&\int (fx)^m (d+ex^2)^3 (a+b\csc^{-1}(cx))\,dx \\
&= \frac{be\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}} \\
&+ \frac{be^2(e(5+m)^2+3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1+c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{c^2x^2}} \\
&+ \frac{be^3x(fx)^{5+m}\sqrt{-1+c^2x^2}}{cf^5(6+m)(7+m)\sqrt{c^2x^2}} + \frac{d^3(fx)^{1+m}(a+b\csc^{-1}(cx))}{f(1+m)} \\
&+ \frac{3d^2e(fx)^{3+m}(a+b\csc^{-1}(cx))}{f^3(3+m)} \\
&+ \frac{3de^2(fx)^{5+m}(a+b\csc^{-1}(cx))}{f^5(5+m)} + \frac{e^3(fx)^{7+m}(a+b\csc^{-1}(cx))}{f^7(7+m)} \\
&+ \frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}+\frac{e(1+m)\left(e^2(15+8m+m^2)^2+3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{(3+m)(5+m)(7+m)}\right)x}{c^5f(1+m)(2+m)(4+m)(6+m)\sqrt{c^2x^2}\sqrt{-1+}}
\end{aligned}$$

output
```
d^3*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccsc(
c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccsc(c*x))/f^5/(5+m)+e^3*(f*x)^
(7+m)*(a+b*arccsc(c*x))/f^7/(7+m)+b*(c^6*d^3*(2+m)*(4+m)*(6+m)/(1+m)+e*(1+
m)*(e^2*(m^2+8*m+15)^2+3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m
^3+179*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*(f*x)^(1+m)*hypergeom([1/2
, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^5/f/(1+m)/(2+m)/(4+
m)/(6+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*(e^2*(m^2+8*m+15)^2+3*c^2*d
*e*(3+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)
^(1+m)*(c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(c^
2*x^2)^(1/2)+b*e^2*(e*(5+m)^2+3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(c^2*x^
2-1)^(1/2)/c^3/f^3/(4+m)/(5+m)/(6+m)/(7+m)/(c^2*x^2)^(1/2)+b*e^3*x*(f*x)^(
5+m)*(c^2*x^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(c^2*x^2)^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.69

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right) \, dx = x(fx)^m \left(\frac{ad^3}{1+m}+\frac{3ad^2ex^2}{3+m}+\frac{3ade^2x^4}{5+m}+\frac{ae^3x^6}{7+m}+\frac{bd^3\csc^{-1}(cx)}{1+m}+\frac{3bd^2ex^2\csc^{-1}(cx)}{3+m}+\frac{3bde^2x^4\csc^{-1}(cx)}{5+m}+\frac{be^3x^6\csc^{-1}(cx)}{7+m}-\frac{bcd^3\sqrt{1-\frac{1}{c^2x^2}}x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}}-\frac{3bcd^2e\sqrt{1-\frac{1}{c^2x^2}}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}}-\frac{3bcde^2\sqrt{1-\frac{1}{c^2x^2}}x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}}-\frac{bce^3\sqrt{1-\frac{1}{c^2x^2}}x^7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},c^2x^2\right)}{(7+m)^2\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5
+ m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcCsc[c*x])/(1 + m) + (3*b*d^2*e*x^2*
ArcCsc[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcCsc[c*x])/(5 + m) + (b*e^3*x^6*Ar
cCsc[c*x])/(7 + m) - (b*c*d^3*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/
2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) - (3*b*c*
d^2*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/
2, c^2*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) - (3*b*c*d*e^2*Sqrt[1 - 1/(c^2*
x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^
2*Sqrt[1 - c^2*x^2]) - (b*c*e^3*Sqrt[1 - 1/(c^2*x^2)]*x^7*Hypergeometric2F
1[1/2, (7 + m)/2, (9 + m)/2, c^2*x^2])/((7 + m)^2*Sqrt[1 - c^2*x^2]))
```

3.165.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 541, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5762, 2340, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (fx)^m \left(a+b\csc^{-1}(cx)\right) dx$$

$$\Big\downarrow \ 5762$$

$$\frac{bcx\int \frac{(fx)^m\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\csc^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \ 2340$$

$$\frac{bcx\left(\frac{\int \frac{(fx)^m\left(\frac{e^2\left(3d\left(m^2+13m+42\right)c^2+e(m+5)^2\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+6)x^2}{m+3}+\frac{c^2d^3(m+6)}{m+1}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+6)}+\frac{e^3\sqrt{c^2x^2-1}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}\right)}{\sqrt{c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\csc^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \ 1590$$

$$\frac{bcx\left(\frac{\frac{\int \frac{(fx)^m\left(\frac{d^3(m+4)(m+6)c^4}{m+1}+\frac{e\left(3d^2\left(m^4+22m^3+179m^2+638m+840\right)c^4+3de(m+3)^2\left(m^2+13m+42\right)c^2+e^2\left(m^2+8m+15\right)^2\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+4)}+\frac{e^2\sqrt{c^2x^2-1}(fx)\cdots}{c\cdots}}{c^2(m+6)}\cdots\right)}{\sqrt{c^2x^2}}+$$

$$\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\csc^{-1}(cx)\right)}{f^7(m+7)}$$

$\downarrow$ 363

$$\frac{bcx\left(\frac{\frac{\left(\frac{c^4d^3(m+4)(m+6)}{m+1}+\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)\int\frac{(fx)^m}{\sqrt{c^2x^2-1}}dx+\frac{e\sqrt{c^2x^2-\cdots}}{\cdots}}{c^2(m+4)}\cdots\right)}{\cdots}+$$

$$\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\csc^{-1}(cx)\right)}{f^7(m+7)}$$

$\downarrow$ 279

$$\frac{bcx\left(\frac{\frac{\sqrt{1-c^2x^2}\left(\frac{c^4d^3(m+4)(m+6)}{m+1}+\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)\int\frac{(fx)^m}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}}+\cdots}{c^2(m+4)}\cdots\right)}{\cdots}+$$

$$\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\csc^{-1}(cx)\right)}{f^7(m+7)}$$

$\downarrow$ 278

3.165. $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\, dx$

$$\frac{d^3(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\csc^{-1}(cx)\right)}{f^7(m+7)}+$$

$$bcx\left(\frac{e^3\sqrt{c^2x^2-1}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}+\frac{\frac{e^2\sqrt{c^2x^2-1}(fx)^{m+3}\left(3c^2d\left(m^2+13m+42\right)+e(m+5)^2\right)}{c^2f^3(m+4)(m+5)(m+7)}+\frac{e\sqrt{c^2x^2-1}(fx)^{m+1}\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)+3c^2 \ldots\right.}{c^2f(m+2)(m+3)(m+5)(\ldots}}{\ldots}\right.$$

input
```
Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsc[c*x]),x]
```

output
```
(d^3*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 +
m)*(a + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcC
sc[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsc[c*x]))/(f^7*(7
+ m)) + (b*c*x*((e^3*(f*x)^(5 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f^5*(6 + m)*(7
 + m)) + ((e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*(f*x)^(3 + m)*Sqr
t[-1 + c^2*x^2])/(c^2*f^3*(4 + m)*(5 + m)*(7 + m)) + ((e*(e^2*(15 + 8*m +
m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m +
179*m^2 + 22*m^3 + m^4))*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f*(2 + m)*
(3 + m)*(5 + m)*(7 + m)) + (((c^4*d^3*(4 + m)*(6 + m))/(1 + m) + (e*(1 + m
)*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*
d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/(c^2*(2 + m)*(3 + m)*(5 + m)*
(7 + m)))*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[-1 + c^2*x^2]))/(c^2*(4 + m)))/(c^2
*(6 + m))))/Sqrt[c^2*x^2]
```

3.165. $\int(fx)^m\left(d+ex^2\right)^3\left(a+b\csc^{-1}(cx)\right)\,dx$

3.165.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.165.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 \left(a+b \operatorname{arccsc}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x)
```

3.165.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^3 (b \operatorname{arccsc}\left(cx\right)+a)(fx)^m\, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccsc(c*x))*(f*x)^m, x)
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsc(c*x)),x)
```

output
```
Timed out
```

3.165.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^3 (b\,\mathrm{arccsc}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*
x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + (((b*e^3*f^m*m^3*arcta
n2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 9*b*e^3*f^m*m^2*arctan2(1, sqrt(c*x +
 1)*sqrt(c*x - 1)) + 23*b*e^3*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)
) + 15*b*e^3*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^7 + 3*(b*d*e^2
*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 11*b*d*e^2*f^m*m^2*arct
an2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 31*b*d*e^2*f^m*m*arctan2(1, sqrt(c*x
 + 1)*sqrt(c*x - 1)) + 21*b*d*e^2*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x -
1)))*x^5 + 3*(b*d^2*e*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 13
*b*d^2*e*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 47*b*d^2*e*f^m*
m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 35*b*d^2*e*f^m*arctan2(1, sqrt
(c*x + 1)*sqrt(c*x - 1)))*x^3 + (b*d^3*f^m*m^3*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1)) + 15*b*d^3*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) +
71*b*d^3*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 105*b*d^3*f^m*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x)*x^m + (m^4 + 16*m^3 + 86*m^2 + 17
6*m + 105)*integrate(-(b*d^3*f^m*m^3 + 15*b*d^3*f^m*m^2 + (b*e^3*f^m*m^3 +
 9*b*e^3*f^m*m^2 + 23*b*e^3*f^m*m + 15*b*e^3*f^m)*x^6 + 71*b*d^3*f^m*m + 1
05*b*d^3*f^m + 3*(b*d*e^2*f^m*m^3 + 11*b*d*e^2*f^m*m^2 + 31*b*d*e^2*f^m*m
+ 21*b*d*e^2*f^m)*x^4 + 3*(b*d^2*e*f^m*m^3 + 13*b*d^2*e*f^m*m^2 + 47*b*d^2
*e*f^m*m + 35*b*d^2*e*f^m)*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m/(m^4 + ...
```

3.165.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^3 (b\,\mathrm{arccsc}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\csc^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right)^3 \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((f*x)^m*(d + e*x^2)^3*(a + b*asin(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^3*(a + b*asin(1/(c*x))), x)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\, dx$

3.166.1 Optimal result

Integrand size = 23, antiderivative size = 371

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\, dx$$
$$= \frac{be(e(3+m)^2+2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1+c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}}$$
$$+ \frac{be^2x(fx)^{3+m}\sqrt{-1+c^2x^2}}{cf^3(4+m)(5+m)\sqrt{c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\csc^{-1}(cx)\right)}{f(1+m)}$$
$$+ \frac{2de(fx)^{3+m}\left(a+b\csc^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2(fx)^{5+m}\left(a+b\csc^{-1}(cx)\right)}{f^5(5+m)}$$
$$+ \frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2+2c^2d(20+9m+m^2)\right))\,x(fx)^{1+m}\sqrt{1-}}{c^3f(1+m)^2(2+m)(3+m)(4+m)(5+m)\sqrt{c^2x^2}\sqrt{-1+c}}$$

output
```
d^2*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccsc(c*
x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccsc(c*x))/f^5/(5+m)+b*(c^4*d^2*(2+m)
*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2+2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+
m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^3/
f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*(e
*(3+m)^2+2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c^3/f/(4+m)
/(5+m)/(m^2+5*m+6)/(c^2*x^2)^(1/2)+b*e^2*x*(f*x)^(3+m)*(c^2*x^2-1)^(1/2)/c
/f^3/(4+m)/(5+m)/(c^2*x^2)^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.79

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx$$
$$= x(fx)^m\left(\frac{ad^2}{1+m}+\frac{2adex^2}{3+m}+\frac{ae^2x^4}{5+m}+\frac{bd^2\csc^{-1}(cx)}{1+m}+\frac{2bdex^2\csc^{-1}(cx)}{3+m}+\frac{be^2x^4\csc^{-1}(cx)}{5+m}-\frac{bcd^2\sqrt{1-\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}}-\frac{2bcde\sqrt{1-\frac{1}{c^2x^2}}\,x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{(3+m)^2\sqrt{1-c^2x^2}}-\frac{bce^2\sqrt{1-\frac{1}{c^2x^2}}\,x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},c^2x^2\right)}{(5+m)^2\sqrt{1-c^2x^2}}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
 (b*d^2*ArcCsc[c*x])/(1 + m) + (2*b*d*e*x^2*ArcCsc[c*x])/(3 + m) + (b*e^2*
x^4*ArcCsc[c*x])/(5 + m) - (b*c*d^2*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric
2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2]) - (
2*b*c*d*e*Sqrt[1 - 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 +
 m)/2, c^2*x^2])/((3 + m)^2*Sqrt[1 - c^2*x^2]) - (b*c*e^2*Sqrt[1 - 1/(c^2*
x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, c^2*x^2])/((5 + m)^
2*Sqrt[1 - c^2*x^2]))
```

3.166.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5762, 27, 1590, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (fx)^m \left(a+b\csc^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int \frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{(m^3+9m^2+23m+15)\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{\sqrt{c^2x^2-1}}dx}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \text{1590}$$

$$\frac{bcx\left(\frac{\int \frac{(fx)^m\left(c^2(m+3)(m+4)(m+5)d^2+e(m+1)\left(2d\left(m^2+9m+20\right)c^2+e(m+3)^2\right)x^2\right)}{\sqrt{c^2x^2-1}}dx}{c^2(m+4)}+\frac{e^2(m+1)(m+3)\sqrt{c^2x^2-1}(fx)^{m+3}}{c^2f^3(m+4)}\right)}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \text{363}$$

$$\frac{bcx\left(\frac{\frac{\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{m+2}\right)\int \frac{(fx)^m}{\sqrt{c^2x^2-1}}dx}{c^2}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{c^2f(m+2)}}{c^2(m+4)}+\right.}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}$$

$$\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \text{279}$$

$$\frac{bcx\left(\frac{\frac{\sqrt{1-c^2x^2}\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{m+2}\right)\int \frac{(fx)^m}{\sqrt{1-c^2x^2}}dx}{c^2\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+e(m\right.}{c^2f(m+2)}}{c^2(m+4)}\right.}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}$$

$$\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \text{278}$$

3.166. $\int (fx)^m\left(d+ex^2\right)^2\left(a+b\csc^{-1}(cx)\right)\,dx$

$$\frac{d^2(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\csc^{-1}(cx)\right)}{f^5(m+5)}+$$
$$\frac{bcx\left(\frac{e^2(m+1)(m+3)\sqrt{c^2x^2-1}(fx)^{m+3}}{c^2f^3(m+4)}+\frac{\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}\left(2c^2d\left(m^2+9m+20\right)+e(m+3)^2\right)}{c^2f(m+2)}+\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\left(c^4d^2(m+3)(m+4)(m+5)+\right.}{}}{c^2(m+4)}\right.}{(m^3+9m^2+23m+15)\sqrt{c^2x^2}}$$

input
```
Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsc[c*x]),x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)
*(a + b*ArcCsc[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCsc[c*x
]))/(f^5*(5 + m)) + (b*c*x*((e^2*(1 + m)*(3 + m)*(f*x)^(3 + m)*Sqrt[-1 + c
^2*x^2])/(c^2*f^3*(4 + m)) + ((e*(1 + m)*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m
+ m^2))*(f*x)^(1 + m)*Sqrt[-1 + c^2*x^2])/(c^2*f*(2 + m)) + ((c^4*d^2*(3 +
 m)*(4 + m)*(5 + m) + (e*(1 + m)^2*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2)
))/(2 + m))*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, c^2*x^2])/(c^2*f*(1 + m)*Sqrt[-1 + c^2*x^2]))/(c^2*(4 + m))
))/((15 + 23*m + 9*m^2 + m^3)*Sqrt[c^2*x^2])
```

3.166.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.166.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 (a+b\,\mathrm{arccsc}\,(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x)
```

3.166.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2 (b\,\text{arccsc}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccsc(c*x))*(f*x)^m, x)
```

3.166.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \int (fx)^m \left(a+b\,\text{acsc}\,(cx)\right)\left(d+ex^2\right)^2\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**2*(a+b*acsc(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*acsc(c*x))*(d + e*x**2)**2, x)
```

3.166.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2 (b\,\text{arccsc}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*
d^2/(f*(m + 1)) + (((b*e^2*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))
 + 4*b*e^2*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*e^2*f^m*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^5 + 2*(b*d*e*f^m*m^2*arctan2(1, sq
rt(c*x + 1)*sqrt(c*x - 1)) + 6*b*d*e*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c
*x - 1)) + 5*b*d*e*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x^3 + (b*d
^2*f^m*m^2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*b*d^2*f^m*m*arctan2
(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 15*b*d^2*f^m*arctan2(1, sqrt(c*x + 1)*s
qrt(c*x - 1)))*x)*x^m + (m^3 + 9*m^2 + 23*m + 15)*integrate(-(b*d^2*f^m*m^
2 + 8*b*d^2*f^m*m + (b*e^2*f^m*m^2 + 4*b*e^2*f^m*m + 3*b*e^2*f^m)*x^4 + 15
*b*d^2*f^m + 2*(b*d*e*f^m*m^2 + 6*b*d*e*f^m*m + 5*b*d*e*f^m)*x^2)*sqrt(c*x
 + 1)*sqrt(c*x - 1)*x^m/(m^3 - (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c^2)*x
^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

3.166.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^2 (b\,\mathrm{arccsc}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\csc^{-1}(cx)\right)\,dx=\int (f\,x)^m\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^2*(a + b*asin(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^2*(a + b*asin(1/(c*x))), x)
```

3.167 $\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx$

3.167.1 Optimal result

Integrand size = 21, antiderivative size = 215

$$\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx = \frac{bex(fx)^{1+m}\sqrt{-1+c^2x^2}}{cf(6+5m+m^2)\sqrt{c^2x^2}} + \frac{d(fx)^{1+m}(a+b\csc^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m}(a+b\csc^{-1}(cx))}{f^3(3+m)} + \frac{b(e(1+m)^2+c^2d(2+m)(3+m))\,x(fx)^{1+m}\sqrt{1-c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{cf(1+m)^2(2+m)(3+m)\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}$$

output
```
d*(f*x)^(1+m)*(a+b*arccsc(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccsc(c*x))/f^
3/(3+m)+b*(e*(1+m)^2+c^2*d*(2+m)*(3+m))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c/f/(1+m)^2/(2+m)/(3+m)/(c^
2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)+b*e*x*(f*x)^(1+m)*(c^2*x^2-1)^(1/2)/c/f/(m^
2+5*m+6)/(c^2*x^2)^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) \, dx$$
$$= x(fx)^m \left(-\frac{bcd\sqrt{1-\frac{1}{c^2x^2}}x \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},c^2x^2\right)}{(1+m)^2\sqrt{1-c^2x^2}} + \frac{\frac{(3+m)\left(d(3+m)+e(1+m)x^2\right)\left(a+b\csc^{-1}(cx)\right)}{1+m} - \frac{bce\sqrt{1-\frac{1}{c^2x^2}}x^3 \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},c^2x^2\right)}{\sqrt{1-c^2x^2}}}{(3+m)^2}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
x*(f*x)^m*(-((b*c*d*Sqrt[1 - 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, c^2*x^2])/((1 + m)^2*Sqrt[1 - c^2*x^2])) + (((3 + m)*(d*(3
+ m) + e*(1 + m)*x^2)*(a + b*ArcCsc[c*x]))/(1 + m) - (b*c*e*Sqrt[1 - 1/(c^
2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, c^2*x^2])/Sqrt[1
- c^2*x^2])/(3 + m)^2)
```

3.167.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5762, 27, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right) (fx)^m \left(a+b\csc^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{5762}$$

$$\frac{bcx\int \frac{(fx)^m\left(e(m+1)x^2+d(m+3)\right)}{(m^2+4m+3)\sqrt{c^2x^2-1}}dx}{\sqrt{c^2x^2}} + \frac{d(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)} + \frac{e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{(fx)^m(e(m+1)x^2+d(m+3))}{\sqrt{c^2x^2-1}}dx}{(m^2+4m+3)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \text{363}$$

$$\frac{bcx\left(\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\int \frac{(fx)^m}{\sqrt{c^2x^2-1}}dx+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \text{279}$$

$$\frac{bcx\left(\frac{\sqrt{1-c^2x^2}\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\int \frac{(fx)^m}{\sqrt{1-c^2x^2}}dx}{\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \text{278}$$

$$\frac{d(fx)^{m+1}\left(a+b\csc^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\csc^{-1}(cx)\right)}{f^3(m+3)}+\frac{bcx\left(\frac{\sqrt{1-c^2x^2}(fx)^{m+1}\left(\frac{e(m+1)^2}{c^2(m+2)}+d(m+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},c^2x^2\right)}{f(m+1)\sqrt{c^2x^2-1}}+\frac{e(m+1)\sqrt{c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{c^2x^2}}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsc[c*x]),x]
```

output
```
(d*(f*x)^(1 + m)*(a + b*ArcCsc[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a +
b*ArcCsc[c*x]))/(f^3*(3 + m)) + (b*c*x*((e*(1 + m)*(f*x)^(1 + m)*Sqrt[-1 +
 c^2*x^2])/(c^2*f*(2 + m)) + (((e*(1 + m)^2)/(c^2*(2 + m)) + d*(3 + m))*(f
*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
 c^2*x^2])/(f*(1 + m)*Sqrt[-1 + c^2*x^2])))/((3 + 4*m + m^2)*Sqrt[c^2*x^2]
)
```

3.167. $\int (fx)^m \left(d+ex^2\right)\left(a+b\csc^{-1}(cx)\right)\,dx$

3.167.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5762
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcCsc[c*x])   u, x] + Simp[b*c*(x/Sqrt[c^2*x^2])   Int[SimplifyIn
tegrand[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m,
 p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) |
| (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m
 + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.167.4 Maple [F]

$$\int (fx)^m\left(e\,x^2+d\right)\left(a+b\ \mathrm{arccsc}\left(cx\right)\right)dx$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x)
```

3.167.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) \, dx = \int (ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*(f*x)^m, x)
```

3.167.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) \, dx = \int (fx)^m \left(a+b\,\mathrm{acsc}\,(cx)\right) \left(d+ex^2\right) \, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(a+b*acsc(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*acsc(c*x))*(d + e*x**2), x)
```

3.167.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\csc^{-1}(cx)\right) \, dx = \int (ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) + (((b*e*f^m*m*arc
tan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + b*e*f^m*arctan2(1, sqrt(c*x + 1)*sq
rt(c*x - 1)))*x^3 + (b*d*f^m*m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 3
*b*d*f^m*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*x)*x^m + (m^2 + 4*m + 3)
*integrate((b*d*f^m*m + 3*b*d*f^m + (b*e*f^m*m + b*e*f^m)*x^2)*sqrt(c*x +
1)*sqrt(c*x - 1)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 - m^2 - 4*m - 3), x)
)/(m^2 + 4*m + 3)
```

3.167.8 Giac [F]

$$\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx = \int (ex^2+d)(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m (d+ex^2) (a+b\csc^{-1}(cx)) \, dx = \int (f\,x)^m \left(e\,x^2+d\right) \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*asin(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a + b*asin(1/(c*x))), x)
```

3.168 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{d+ex^2}\,dx$

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx = \text{Int}\left(\frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{d+ex^2},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

3.168.2 Mathematica [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{d+ex^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2), x]
```

3.168.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{d+ex^2} dx$$

$$\Big\downarrow \text{ 5772}$$

$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{d+ex^2} dx$$

input
```
Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

3.168.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.168.4 Maple [N/A] (verified)

Not integrable

Time = 4.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b \,\mathrm{arccsc}\left(cx\right)\right)}{e\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x)
```

3.168.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.168.6 Sympy [N/A]

Not integrable

Time = 35.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(fx)^m (a+b\,\mathrm{acsc}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d),x)
```

output
```
Integral((f*x)**m*(a + b*acsc(c*x))/(d + e*x**2), x)
```

3.168.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{d+ex^2}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.168.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(b\operatorname{arccsc}(cx) + a)(fx)^m}{ex^2 + d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.168.9 Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{d + ex^2}\,dx = \int \frac{(f\,x)^m\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2 + d}\,dx$$

input
```
int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2),x)
```

output
```
int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2), x)
```

3.169 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{(d+ex^2)^2} dx$

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^2} dx = \text{Int}\left(\frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^2}, x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

3.169.2 Mathematica [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^2} dx = \int \frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^2} dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2, x]
```

3.169.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^2} \, dx$$

$$\Big\downarrow \; 5772$$

$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

3.169.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.169.4 Maple [N/A] (verified)

Not integrable

Time = 3.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\,\mathrm{arccsc}\left(cx\right)\right)}{\left(e\,x^2+d\right)^2}dx$$

input
```
int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x)
```

3.169.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2}\, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{(ex^2 + d)^2}\, dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2}\, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.169.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2}\, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)(fx)^m}{(ex^2 + d)^2}\, dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.169.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\operatorname{arccsc}(cx) + a)(fx)^m}{(ex^2 + d)^2}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.169.9 Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(f\,x)^m \left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^2}\,dx$$

input
```
int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^2, x)
```

3.170 $\int (fx)^m (d+ex^2)^{3/2} (a+b\csc^{-1}(cx)) \, dx$

3.170.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m (d+ex^2)^{3/2} (a+b\csc^{-1}(cx)) \, dx = \text{Int}\left((fx)^m (d+ex^2)^{3/2} (a+b\csc^{-1}(cx)), x\right)$$

output
```
Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

3.170.2 Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m (d+ex^2)^{3/2} (a+b\csc^{-1}(cx)) \, dx = \int (fx)^m (d+ex^2)^{3/2} (a+b\csc^{-1}(cx)) \, dx$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]), x]
```

3.170.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\csc^{-1}(cx)\right) \, dx$$
$$\Big\downarrow \text{5772}$$
$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\csc^{-1}(cx)\right) dx$$

input
```
Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsc[c*x]),x]
```

output
```
$Aborted
```

3.170.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.170.4 Maple [N/A] (verified)

Not integrable

Time = 2.73 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arccsc}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x)
```

3.170.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsc(c*x))*sqrt(e*x^2 + d)*(f*
x)^m, x)
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*acsc(c*x)),x)
```

output
```
Timed out
```

3.170.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b \operatorname{arccsc}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

3.170.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\,\mathrm{arccsc}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsc(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

3.170.9 Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\csc^{-1}(cx)\right) \, dx = \int (f\,x)^m \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asin(1/(c*x))), x)
```

3.171 $\int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$

3.171.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx = \mathrm{Int}\Big((fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx)),x\Big)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.171.2 Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx = \int (fx)^m \sqrt{d+ex^2}(a+b\csc^{-1}(cx))\,dx$$

input
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]), x]
```

3.171.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(fx)^m \left(a+b\csc^{-1}(cx)\right) dx$$

$$\Big\downarrow \ 5772$$

$$\int \sqrt{d+ex^2}(fx)^m \left(a+b\csc^{-1}(cx)\right) dx$$

input
```
Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsc[c*x]),x]
```

output
```
$Aborted
```

3.171.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.171.4 Maple [N/A] (verified)

Not integrable

Time = 1.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(a+b\ \mathrm{arccsc}\left(cx\right)\right) \sqrt{e\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x)
```

3.171.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

3.171.6 Sympy [N/A]

Not integrable

Time = 51.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int (fx)^m\,(a+b\,\mathrm{acsc}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(a+b*acsc(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acsc(c*x))*sqrt(d + e*x**2), x)
```

3.171.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

3.171.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m, x)
```

3.171.9 Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\csc^{-1}(cx)\right)\,dx=\int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asin(1/(c*x))), x)
```

3.172 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$

3.172.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.172.2 Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2], x]
```

3.172.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$
$$\Big\downarrow \ 5772$$
$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCsc[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.172.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.172.4 Maple [N/A] (verified)

Not integrable

Time = 1.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a+b\,\operatorname{arccsc}\left(cx\right)\right)}{\sqrt{e\,x^2+d}}dx$$

input
```
int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x)
```

3.172.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.172.6 Sympy [N/A]

Not integrable

Time = 21.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m (a+b\,\mathrm{acsc}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acsc(c*x))/sqrt(d + e*x**2), x)
```

3.172.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.172.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.172.9 Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m (a + b\csc^{-1}(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(f\,x)^m\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

input
```
int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.173 $\int \frac{(fx)^m(a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$

3.173.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.173.2 Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2), x]
```

3.173.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{5772}$$
$$\int \frac{(fx)^m \left(a+b\csc^{-1}(cx)\right)}{(d+ex^2)^{3/2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCsc[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.173.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.173.4 Maple [N/A] (verified)

Not integrable

Time = 2.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(fx)^m \left(a+b\,\mathrm{arccsc}\left(cx\right)\right)}{\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

input
```
int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x)
```

3.173.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsc(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

3.173.6 Sympy [N/A]

Not integrable

Time = 134.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(fx)^m (a+b\,\mathrm{acsc}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acsc(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((f*x)**m*(a + b*acsc(c*x))/(d + e*x**2)**(3/2), x)
```

3.173.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.173.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsc(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.173.9 Mupad [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m (a+b\csc^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int(((f*x)^m*(a + b*asin(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.174 $\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

3.174.1 Optimal result

Integrand size = 26, antiderivative size = 401

$$\begin{aligned}\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx &= -\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{7b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\\&-\frac{13b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{150c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{3b\sqrt{1-c^2x^2}(1+c^2x^2)^{7/2}}{70c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\\&-\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{9/2}}{90c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}}\\&+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{3c^{12}}\\&-\frac{(1-c^4x^4)^{5/2}(a+b\csc^{-1}(cx))}{10c^{12}}\\&+\frac{4b\sqrt{1-c^2x^2}\operatorname{arctanh}\left(\sqrt{1+c^2x^2}\right)}{15c^{13}\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

output
```
1/3*(-c^4*x^4+1)^(3/2)*(a+b*arccsc(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b
*arccsc(c*x))/c^12+7/90*b*(c^2*x^2+1)^(3/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1
/c^2/x^2)^(1/2)-13/150*b*(c^2*x^2+1)^(5/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/
c^2/x^2)^(1/2)+3/70*b*(c^2*x^2+1)^(7/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2
/x^2)^(1/2)-1/90*b*(c^2*x^2+1)^(9/2)*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2/x^
2)^(1/2)+4/15*b*arctanh((c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/c^13/x/(1-1/
c^2/x^2)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1-1/c^2
/x^2)^(1/2)-1/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

3.174.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.48

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx =$$

$$-\frac{105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}(768+36c^2x^2+78c^4x^4+5c^6x^6+35c^8x^8)}{-1+c^2x^2}+105b\sqrt{1-c^4x^4}(}{3150c^{12}}$$

input
```
Integrate[(x^11*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/3150*(105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1
 - 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6
*x^6 + 35*c^8*x^8))/(-1 + c^2*x^2) + 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^
4 + 3*c^8*x^8)*ArcCsc[c*x] + 840*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^2)]*x)/Sqrt
[1 - c^4*x^4]])/c^12
```

3.174.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.57, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {5770, 27, 7272, 1388, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$$

$$\Big\downarrow \text{5770}$$

$$\frac{b\int -\frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{30c^{12}\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{30c^{13}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{7272}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{x\sqrt{1-c^2x^2}}dx}{30c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}\left(3c^8x^8+4c^4x^4+8\right)}{x}dx}{30c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2331}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}\left(3c^8x^8+4c^4x^4+8\right)}{x^2}dx^2}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2123}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \left(3c^2\left(c^2x^2+1\right)^{7/2}-9c^2\left(c^2x^2+1\right)^{5/2}+13c^2\left(c^2x^2+1\right)^{3/2}-7c^2\sqrt{c^2x^2+1}+\frac{8\sqrt{c^2x^2+1}}{x^2}\right)dx^2}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\csc^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2009}$$

3.174. $\int \frac{x^{11}\left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

$$-\frac{(1-c^4x^4)^{5/2}\,(a+b\csc^{-1}(cx))}{10c^{12}}+\frac{(1-c^4x^4)^{3/2}\,(a+b\csc^{-1}(cx))}{3c^{12}}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^{12}}-\frac{b\sqrt{1-c^2x^2}\left(-16\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{2}{3}(c^2x^2+1)^{9/2}-\frac{18}{7}(c^2x^2+1)^{7/2}+\frac{26}{5}(c^2x^2+1)^{5/2}-\frac{14}{3}(c^2x^2+1)^{3/2}+}{60c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^11*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/c^12 + ((1 - c^4*x^4)^(3/2)*(
a + b*ArcCsc[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(a + b*ArcCsc[c*x]))/(
10*c^12) - (b*Sqrt[1 - c^2*x^2]*(16*Sqrt[1 + c^2*x^2] - (14*(1 + c^2*x^2)^
(3/2))/3 + (26*(1 + c^2*x^2)^(5/2))/5 - (18*(1 + c^2*x^2)^(7/2))/7 + (2*(1
 + c^2*x^2)^(9/2))/3 - 16*ArcTanh[Sqrt[1 + c^2*x^2]]))/(60*c^13*Sqrt[1 - 1
/(c^2*x^2)]*x)
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.174.4 Maple [F]

$$\int \frac{x^{11}(a + b \operatorname{arccsc}(cx))}{\sqrt{-c^4x^4+1}}dx$$

input
```
int(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx =$$
$$-\frac{(35\,bc^8x^8+5\,bc^6x^6+78\,bc^4x^4+36\,bc^2x^2+768\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-840\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c}}{\sqrt{c^2}}\right.}{}$$

input
```
integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3150*((35*b*c^8*x^8 + 5*b*c^6*x^6 + 78*b*c^4*x^4 + 36*b*c^2*x^2 + 768*b
)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1) - 840*(b*c^2*x^2 - b)*arctan(sqrt(-
c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + 105*(3*a*c^10*x^10 - 3*a*c^8*x^8 + 4*a*c
^6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 + (3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^
6*x^6 - 4*b*c^4*x^4 + 8*b*c^2*x^2 - 8*b)*arccsc(c*x) - 8*a)*sqrt(-c^4*x^4
+ 1))/(c^14*x^2 - c^12)
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(a + b \csc^{-1}(cx))}{\sqrt{1 - c^4x^4}} \, dx = \text{Timed out}$$

input
```
integrate(x**11*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

3.174.7 Maxima [F]

$$\int \frac{x^{11}(a + b \csc^{-1}(cx))}{\sqrt{1 - c^4x^4}} \, dx = \int \frac{(b \operatorname{arccsc}(cx) + a)x^{11}}{\sqrt{-c^4x^4 + 1}} \, dx$$

input
```
integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*s
qrt(-c^4*x^4 + 1)/c^12) + 1/30*(30*c^12*integrate(1/30*(3*c^10*x^11 + 3*c^
8*x^9 + 4*c^6*x^7 + 4*c^4*x^5 + 8*c^2*x^3 + 8*x)*e^(-1/2*log(c^2*x^2 + 1)
+ 1/2*log(c*x - 1))/(c^10*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x +
1)) + sqrt(-c*x + 1)*c^10), x) - (3*c^8*x^8*arctan2(1, sqrt(c*x + 1)*sqrt(
c*x - 1)) + 4*c^4*x^4*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 8*arctan2(
1, sqrt(c*x + 1)*sqrt(c*x - 1)))*sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x
 + 1))*b/c^12
```

3.174.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a + b \csc^{-1}(cx))}{\sqrt{1 - c^4x^4}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^11*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^{11}\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^11*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^11*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.175 $\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

3.175.1 Optimal result

Integrand size = 26, antiderivative size = 268

$$\begin{aligned}\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = &-\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}+\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{3/2}}{18c^9\sqrt{1-\frac{1}{c^2x^2}}x}\\&-\frac{b\sqrt{1-c^2x^2}(1+c^2x^2)^{5/2}}{30c^9\sqrt{1-\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}\\&+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}\\&+\frac{b\sqrt{1-c^2x^2}\,\text{arctanh}\left(\sqrt{1+c^2x^2}\right)}{3c^9\sqrt{1-\frac{1}{c^2x^2}}x}\end{aligned}$$

output
```
1/6*(-c^4*x^4+1)^(3/2)*(a+b*arccsc(c*x))/c^8+1/18*b*(c^2*x^2+1)^(3/2)*(-c^
2*x^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/30*b*(c^2*x^2+1)^(5/2)*(-c^2*x^
2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)+1/3*b*arctanh((c^2*x^2+1)^(1/2))*(-c^
2*x^2+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2
+1)^(1/2)/c^9/x/(1-1/c^2/x^2)^(1/2)-1/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/
2)/c^8
```

3.175.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.59

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx =$$

$$-\frac{15a\sqrt{1-c^4x^4}(2+c^4x^4)+\frac{bc\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(28+c^2x^2+3c^4x^4)}{-1+c^2x^2}+15b\sqrt{1-c^4x^4}(2+c^4x^4)\csc^{-1}(cx)+30b\,\mathrm{a}}{90c^8}$$

input
```
Integrate[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/90*(15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 - 1/(c^2*x^2)]*x
*Sqrt[1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c^2*x^2) + 15*b*Sqrt[
1 - c^4*x^4]*(2 + c^4*x^4)*ArcCsc[c*x] + 30*b*ArcTan[(c*Sqrt[1 - 1/(c^2*x^
2)]*x)/Sqrt[1 - c^4*x^4]])/c^8
```

3.175.3 Rubi [A] (warning: unable to verify)

Time = 1.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5770, 27, 7272, 1388, 1579, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}dx$$

$$\Big\downarrow \ 5770$$

$$\frac{b\int -\frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{6c^8\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{6c^9}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\csc^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^8}$$

$$\Big\downarrow \ 7272$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{x\sqrt{1-c^2x^2}}dx}{6c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}(c^4x^4+2)}{x}dx}{6c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{1579}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}(c^4x^4+2)}{x^2}dx^2}{12c^9x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{517}$$

$$-\frac{b\sqrt{1-c^2x^2}\int -\frac{x^4(c^4x^8-2c^4x^4+3c^4)}{1-x^4}d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\sqrt{1-c^2x^2}\int \frac{x^4(c^4x^8-2c^4x^4+3c^4)}{1-x^4}d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{1584}$$

$$\frac{b\sqrt{1-c^2x^2}\int \left(-c^4x^8+c^4x^4-2c^4+\frac{2c^4}{1-x^4}\right)d\sqrt{c^2x^2+1}}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}+\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-c^4x^4)^{3/2}(a+b\csc^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^8}-\frac{b\sqrt{1-c^2x^2}\left(-2c^4\text{arctanh}\left(\sqrt{c^2x^2+1}\right)+\frac{c^4x^{10}}{5}-\frac{c^4x^6}{3}+2c^4\sqrt{c^2x^2+1}\right)}{6c^{13}x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^7*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

3.175. $\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/c^8 + ((1 - c^4*x^4)^(3/2)*(a
 + b*ArcCsc[c*x]))/(6*c^8) - (b*Sqrt[1 - c^2*x^2]*(-1/3*(c^4*x^6) + (c^4*x
^10)/5 + 2*c^4*Sqrt[1 + c^2*x^2] - 2*c^4*ArcTanh[Sqrt[1 + c^2*x^2]]))/(6*c
^13*Sqrt[1 - 1/(c^2*x^2)]*x)
```

3.175.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.175. $\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.175.4 Maple [F]

$$\int \frac{x^7(a + b\ \mathrm{arccsc}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

input
```
int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = -\frac{(3\,bc^4x^4+bc^2x^2+28\,b)\sqrt{-c^4x^4+1}\sqrt{c^2x^2-1}-30\,(bc^2x^2-b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right)+15\,(ac^6x^6-ac^4x^4}{90\,(c^{10}x^2-c^8)}$$

input
```
integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/90*((3*b*c^4*x^4 + b*c^2*x^2 + 28*b)*sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 -
1) - 30*(b*c^2*x^2 - b)*arctan(sqrt(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + 15*
(a*c^6*x^6 - a*c^4*x^4 + 2*a*c^2*x^2 + (b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^
2 - 2*b)*arccsc(c*x) - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^8)
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

3.175.7 Maxima [F]

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*(c^8*x^8
*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)) + 6*sqrt(c^2*x^2 + 1)*sqrt(c*x +
1)*sqrt(-c*x + 1)*c^8*integrate(1/6*(c^6*x^7 + c^4*x^5 + 2*c^2*x^3 + 2*x)*
e^(-1/2*log(c^2*x^2 + 1) + 1/2*log(c*x - 1))/(c^6*e^(log(c*x + 1) + log(c*
x - 1) + 1/2*log(-c*x + 1)) + sqrt(-c*x + 1)*c^6), x) + c^4*x^4*arctan2(1,
 sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))
*b/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8)
```

3.175.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^7\,\left(a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^7*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^7*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.176 $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

3.176.1 Optimal result

Integrand size = 26, antiderivative size = 126

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=-\frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}+\frac{bx\arctan\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1+c^2x^2}}\right)}{2c^3\sqrt{c^2x^2}}$$

output
```
1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(c^2*x^2-1)^(1/2))/c^3/(c^2*x^2)^(1/2)-1
/2*(a+b*arccsc(c*x))*(-c^4*x^4+1)^(1/2)/c^4-1/2*b*x*(-c^4*x^4+1)^(1/2)/c^3
/(c^2*x^2)^(1/2)/(c^2*x^2-1)^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\frac{\left(a-bc\sqrt{1-\frac{1}{c^2x^2}}x-ac^2x^2\right)\sqrt{1-c^4x^4}-b(-1+c^2x^2)\sqrt{1-c^4x^4}\csc^{-1}(cx)+(b-bc^2x^2)\arctan\left(\frac{c\sqrt{1-}}{\sqrt{1-}}\right.}{2c^4\left(-1+c^2x^2\right)}$$

input
```
Integrate[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
((a - b*c*Sqrt[1 - 1/(c^2*x^2)]*x - a*c^2*x^2)*Sqrt[1 - c^4*x^4] - b*(-1 +
 c^2*x^2)*Sqrt[1 - c^4*x^4]*ArcCsc[c*x] + (b - b*c^2*x^2)*ArcTan[(c*Sqrt[1
 - 1/(c^2*x^2)]*x)/Sqrt[1 - c^4*x^4]])/(2*c^4*(-1 + c^2*x^2))
```

3.176.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5770, 27, 1896, 1388, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\csc^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}dx$$

$$\Big\downarrow \text{5770}$$

$$\frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1-\frac{1}{c^2x^2}}x^2}dx}{2c^5}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{1896}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1-c^2x^2}}dx}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}}{x}dx}{2c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{b\sqrt{1-c^2x^2}\int \frac{\sqrt{c^2x^2+1}}{x^2}dx^2}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}\left(a+b\csc^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{60}$$

$$-\frac{b\sqrt{1-c^2x^2}\left(\int \frac{1}{x^2\sqrt{c^2x^2+1}}dx^2+2\sqrt{c^2x^2+1}\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{b\sqrt{1-c^2x^2}\left(\frac{2\int \frac{1}{\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{c^2x^2+1}}{c^2}+2\sqrt{c^2x^2+1}\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{1-c^4x^4}(a+b\csc^{-1}(cx))}{2c^4}-\frac{b\sqrt{1-c^2x^2}\left(2\sqrt{c^2x^2+1}-2\text{arctanh}\left(\sqrt{c^2x^2+1}\right)\right)}{4c^5x\sqrt{1-\frac{1}{c^2x^2}}}$$

input
```
Int[(x^3*(a + b*ArcCsc[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcCsc[c*x]))/c^4 - (b*Sqrt[1 - c^2*x^2]*(2
*Sqrt[1 + c^2*x^2] - 2*ArcTanh[Sqrt[1 + c^2*x^2]]))/(4*c^5*Sqrt[1 - 1/(c^2
*x^2)]*x)
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1896
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Simp[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q])   Int[x^(m + mn*q)*(1 + d*(1/(x^
mn*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] &&
EqQ[n2, -2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

rule 5770
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHide
[u, x]}, Simp[(a + b*ArcCsc[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegran
d[v/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]]
/; FreeQ[{a, b, c}, x]
```

3.176. $\int \frac{x^3(a+b\csc^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx$

3.176.4 Maple [F]

$$\int \frac{x^3(a + b \operatorname{arccsc}(cx))}{\sqrt{-c^4x^4 + 1}}dx$$

input
```
int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = -\frac{\sqrt{-c^4x^4 + 1}\sqrt{c^2x^2 - 1}b - (bc^2x^2 - b)\arctan\left(\frac{\sqrt{-c^4x^4+1}}{\sqrt{c^2x^2-1}}\right) + \sqrt{-c^4x^4 + 1}(ac^2x^2 + (bc^2x^2 - b)\operatorname{arccsc}(cx}{2\,(c^6x^2 - c^4)}$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-c^4*x^4 + 1)*sqrt(c^2*x^2 - 1)*b - (b*c^2*x^2 - b)*arctan(sqrt
(-c^4*x^4 + 1)/sqrt(c^2*x^2 - 1)) + sqrt(-c^4*x^4 + 1)*(a*c^2*x^2 + (b*c^2
*x^2 - b)*arccsc(c*x) - a))/(c^6*x^2 - c^4)
```

3.176.6 Sympy [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{x^3(a + b\operatorname{acsc}(cx))}{\sqrt{-(cx - 1)(cx + 1)(c^2x^2 + 1)}}\,dx$$

input
```
integrate(x**3*(a+b*acsc(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acsc(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

3.176.7 Maxima [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*c^4*integrate(1/2*(c^2*x^3 + x)*e^(-1/2*log(c^2*x^2 + 1) + 1/2*log(
c*x - 1))/(c^2*e^(log(c*x + 1) + log(c*x - 1) + 1/2*log(-c*x + 1)) + sqrt(
-c*x + 1)*c^2), x) - sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan
2(1, sqrt(c*x + 1)*sqrt(c*x - 1)))*b/c^4 - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

3.176.8 Giac [F]

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{(b\,\mathrm{arccsc}\,(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}}\,dx$$

input
```
integrate(x^3*(a+b*arccsc(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\csc^{-1}(cx))}{\sqrt{1 - c^4x^4}}\,dx = \int \frac{x^3\,\left(a + b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1 - c^4\,x^4}}\,dx$$

input
```
int((x^3*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^3*(a + b*asin(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.177 $\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

3.177.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

3.177.2 Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

3.177.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

$$\Big\downarrow \text{5772}$$

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

3.177.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.177.4 Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\mathrm{arccsc}\,(cx)}{x\sqrt{-c^4x^4+1}}dx$$

input
```
int((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

3.177.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arccsc(c*x) + a)/(c^4*x^5 - x), x)
```

3.177.6 Sympy [N/A]

Not integrable

Time = 11.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{acsc}\,(cx)}{x\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
 x)
```

3.177.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*int
egrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x
 + 1)*sqrt(-c*x + 1)*x), x)
```

3.177.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

3.177.9 Mupad [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\csc^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.178 $\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

3.178.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\text{Int}\left(\frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

output
```
Unintegrable((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

3.178.2 Mathematica [N/A]

Not integrable

Time = 8.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

3.178.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$
$$\Big\downarrow \text{ 5772}$$
$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}dx$$

input
```
Int[(a + b*ArcCsc[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

3.178.3.1 Defintions of rubi rules used

rule 5772
```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCsc[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.178.4 Maple [N/A] (verified)

Not integrable

Time = 3.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\operatorname{arccsc}\left(cx\right)}{x^5\sqrt{-c^4x^4+1}}dx$$

input
```
int((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

3.178.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a + b\csc^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x^5}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arccsc(c*x) + a)/(c^4*x^9 - x^5), x)
```

3.178.6 Sympy [N/A]

Not integrable

Time = 83.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a + b\csc^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\mathrm{acsc}\,(cx)}{x^5\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*acsc(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*acsc(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
)), x)
```

3.178.7 Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.31

$$\int \frac{a + b\csc^{-1}(cx)}{x^5\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\mathrm{arccsc}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x^5}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) +
2*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(arctan2(1, sqrt(c*x + 1)*sqrt(c*
x - 1))/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^5), x)
```

3.178.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\operatorname{arccsc}(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arccsc(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsc(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

3.178.9 Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\csc^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\mathrm{asin}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*asin(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*asin(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```